

Malati

Mathematics learning and teaching initiative

ALGEBRA

MODULE 6

Linear equations and inequalities

Grades 8 and 9

Teacher document

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Introduction and overview of Module 6

Our starting point in developing the formal notion of an equation is based on:

1. The knowledge that pupils have of equations through our intervention in dealing with the structure of numerical and algebraic sentences and in finding functional rules¹ in our algebra Modules 1 to 5.
2. The intuitive procedures that pupils use to solve equations ([see this research paper](#)).

In dealing with the role of the letter in algebraic sentences, pupils intuitively discussed the truth set at a *semantic level*. From a conceptual perspective the notion of a solution is those numbers that will make the algebraic sentence a true numerical sentence.

In their experiences with functional tables pupils dealt with two kinds of situations (see the [algebra rationale](#) document and learner activities in [Module 3](#)):

- In the first kind of situation the input number was given and the output number had to be determined.
- In the second first kind of situation the output number was given and the input number had to be determined

The latter situation is in fact the first step towards the notion of an equation and the notion of the solution of an equation.

When required to find the input number given the output number, the pupils use informal ways, mainly guess-and-check (*trial-and-improvement*) and inverse operations. For example, when asked to find the input number of a functional rule,

$t = 2x + 5$, $t = 75$ pupils will usually go through the following steps:

$$\begin{aligned}75 &= 2x + 5 \\ \Rightarrow 2x &= 75 - 5 \\ \Rightarrow 2x &= 70 \\ \Rightarrow x &= 70 \div 2 \\ \Rightarrow x &= 35\end{aligned}$$

Note that in the above example, conceptually, pupils **do not operate on both sides** of the equal sign.

¹ We use the terms algebraic rule / functional rule / a formula, interchangeably since they have the same meaning.

Our aim is to transfer this intuitive strategy to the *formal strategy* that is traditionally taught (*the “balancing” method*) where one operates on both sides of the equal sign.

The approach we take is to make the pupils realise that in their intuitive strategy they were in fact operating on both sides of the equation.

In the first stage we will deal only with equations where the unknown appears on one side of the equal sign. In the second stage we will deal with equations where the unknown appears on both sides of the equal sign. These two separate stages are important since the pupils do not intuitively use inverse operations on algebraic terms. The subtraction of an algebraic term which is, for example, on the right-hand side of an equation, from an algebraic term on the left-hand side of an equation (which is spontaneously done on numerical terms and not on algebraic terms) cannot be motivated by inverse operations. Thus we distinguish between two kinds of linear equations:

1. $ax + b = c$ (sometimes referred to as **arithmetical equations**)
2. $ax + b = cx + d$ (sometimes referred to as **algebraic equations**)

Another dimension to our teaching of equations will be an emphasis on the importance of the validity of the solution set of the equation. The establishment of validation procedures will therefore be dealt with to strengthen the notion of a solution.

Note: The approach we present will only make sense to those pupils who have had experiences with functional tables. This approach as mentioned earlier is based on the following two assumptions:

1. That the pupils can make sense out of the functional rule.
2. That the pupils can produce a functional rule from a table of values or from a situation in words.

It is therefore necessary before starting this module, to prepare a short diagnostic assessment to identify those pupils who have not mastered the above and to revisit these ideas with them.

ACTIVITY 1

Mr Bean buys a plant at the nursery when it is 12 cm high.

He measures the height of the plant regularly and records it in the following table:

Week	0	1	2	3	4	5	6	7							n
Height (cm)	12	15	18	21	24										

1. What will the height of the plant be after n weeks? Write your rule in the algebraic language.
2. What will the height of the plant be after 15 weeks?
3. After how many weeks will the height of the plant be 90 cm?
4. After how many weeks will the height of the plant be 114 cm?



Teacher Notes: Activity 1

This activity revisits the functional tables in which the output and input values are determined. In question 3 and question 4, the concept of an equation is intuitively being dealt with.

Do not teach the pupils how to solve the equation through a formal approach. Allow and encourage the pupils to use their intuitive strategies at this point. The adding and subtracting of numerical terms from both sides of an equation or dividing both sides of an equation by a numerical term is motivated by reflection on the inverse operations and not given directly as a formal method. This will be dealt with later on.

Note: In question 2 we deal with the finding of output values.

In questions 3 and 4 we deal with the finding of input values.

Discuss the differences between these.

Although we do not use formal terminology like equation and solution at this point, we encourage the use of algebraic notation that represents the solution *process*, for example,

$$\begin{aligned}h &= 12 + 3x, & h &= 90 \\90 &= 12 + 3x \\90 - 12 &= 3x \\78 &= 3x \\26 &= x\end{aligned}$$

Class Discussion

Let pupils *reflect* on and *compare* the strategies that they used in finding the solution.

Do a few examples with the pupils in which the number appears on both sides of the equation, for example $12 + 3x = 90$ and $90 = 12 + 3x$.

ACTIVITY 2

The supervisor of a community garden project organises volunteers to help dig out weeds. The supervisor has found that the more people they have, the more weeds get pulled. That's not surprising, but the results get even better than one might think as shown in the following table:

Number of people	1	2	3	4	5	6	7					n
Number of bags of weed pulled in a day	5	9	13	17								

1. How many bags of weed will be pulled by n people in a day?
Write your rule in the algebraic language.
2. How many bags of weed will be pulled by 53 people in a day?
3. The supervisor estimates that there are 77 bags of weed to be pulled. How many volunteers would the supervisor need to get the job done in a day?
4. How many volunteers will the supervisor need to get the job done in a day if there are 153 bags of weed to be pulled?

Teacher Notes: Activity 2

Refer to the [notes in Activity 1](#).

Identify those pupils who use the guess-and-improve strategy and who are not able to make sense of the inverse operation strategy. Before moving to Activity 3 we would like all the pupils to be on the level where they use the *inverse operation strategy* and write the solution process in the algebraic notation as described in the teacher [notes in Activity 1](#).

ACTIVITY 3

What is the *input* value for the given *output* value for each of the following algebraic rules?

		<u>Output value</u>
1.	$5t + 3 = n$	$n = 118$
2.	$a = 4b + 4$	$a = -100$
3.	$5d - 3 = c$	$c = 77$
4.	$s = -2 + 7t$	$s = -79$
5.	$y = -3x - 8$	$y = 34$
6.	$-5 - 2c = p$	$p = -37$
7.	$m = 2n - 5$	$m = -23$
8.	$-7t + 12 = k$	$k = 47$

Teacher Notes: Activity 3

Class Discussion and reflection:

- Ensure that all the pupils are using inverse operations. Do not introduce the formal “balancing” method at this point. The pupils need however to be able to write the solution process in algebraic notation, for example:

$$y = -3x - 8 \qquad y = 34$$

$$34 = -3x - 8$$

$$34 + 8 = -3x$$

$$42 = -3x$$

$$-14 = x$$

- Ensure that the pupils are *making sense* of what the input and output is.

After Activity 3, the terminology *equation* and *solution* can be introduced. The formal structural definition of an equation is postponed until algebraic equations are introduced. The teacher may say, for example, $2x + 5 = 17$ and $39 = 2x + 5$ are called *equations* and the input value (x) that makes both sides of an equation numerically equivalent is called the *solution of the equation*.

A short assessment can be given now for the purpose of seeing how pupils use the inverse operation strategy and their ability to use the algebraic notation.

ACTIVITY 4

Note: This activity is planned for a whole-class setting. This activity is not designed as a pupil hand-out but as guide for the teacher in leading the discussion.

Thando is given R5 for his birthday. Thando decides to save this R5. Thereafter he also decides to save R3 every week from the money that he earns from delivering newspapers in his neighbourhood.

The functional rule for the amount of money that Thando saves after x weeks is $y = 3x + 5$.

After how many weeks will Thando have saved R41?

The equation is $3x + 5 = 41$.

The pupils will solve the equation in the familiar way:

$$3x + 5 = 41$$

$$3x = 41 - 5$$

$$3x = 36$$

$$x = 36 \div 3$$

$$x = 12.$$

The solution process will be written on the board.

In the following steps the same solution process will be written in a table while the terminology left-hand side (LHS) and right-hand side (RHS) is introduced in order to show the pupils that they were operating on both sides of the equation.

1. Set up a table:

LHS	RHS
$3x + 5$	41

2. What was the first step that you did?

LHS	RHS		
$3x + 5$	41		
$3x$	$41 - 5$	36	5 was subtracted from 41

3. The result was

LHS	RHS
$3x + 5$	41
$3x$	$41 - 5$
$3x$	36

4. 36 was then divided by 3

LHS	RHS		
$3x + 5$	41		
$3x$	$41 - 5$	36	
$3x$	36		
x	$36 \div 3$	12	we found the solution is 12

Teacher Notes: Activity 4

Module 5 is not a prerequisite for this module. If this is the case the pupils will not be familiar with the algebraic notation $y = 3x + 5$ without a table, to which they can usually connect the meaning of y which in this case represents the amount of money saved.

These pupils need to be given the algebraic rule $y = 3x + 5$ as well as the table:

Weeks	1	2	3	4	5	6				x
Money saved	8	11	14	17						

The pupils need to be able to interpret the solution semantically as well, which means that after 12 weeks Thando would have saved R41.

In this activity we are not abandoning the intuitive inverse operation approach. We build on the intuitive approach, towards the formal approach of operating on both sides of the equation.

The focus of the discussion after the solution process was written in the table must now be that 5 was not only subtracted from the RHS of the equation, it was actually also subtracted from the LHS of the equation. Refer to the table in step 2 above.

NOTE: 5 is also subtracted from $3x + 5$

LHS	RHS	
$3x + 5$	41	-5
-5	-5	
$3x$	$41 - 5$	

Emphasise that 5 was subtracted from the RHS (41) and from the LHS ($3x + 5$)
When using inverse operations the pupils focus only on subtracting from the 41. In fact the 5 was subtracted from both sides. It is this that has to be made clear here.

Continue to emphasise this in all the other steps:

NOTE:

What was divided by 3?

36 was divided by 3, but

also $3x$ was divided by 3

LHS	RHS	
$3x$	36	$\div 3$
x	12	

Now we will write all of this in the algebraic notation:

$$3x + 5 = 41 \quad /-5$$

$$3x = 36 \quad /\div 3$$

$$x = 12$$

Now do another example, building the tables and discussing and reflecting on all the steps.

ACTIVITY 5 (Whole-class setting)

Let's consider the equation $\frac{x}{2} - 8 = 52$

The pupils solve the equation in the familiar way:

$$\frac{x}{2} - 8 = 52$$

$$\frac{x}{2} = 52 + 8$$

$$\frac{x}{2} = 60$$

$$x = 60 \times 2$$

$$x = 120$$

Set up the tables as in Activity 4:

1.

LHS	RHS
$\frac{x}{2} - 8$	52

2. What was the first step that you did?

LHS	RHS	
$\frac{x}{2} - 8$	52	
+8	+8	
$\frac{x}{2}$	52 + 8	60 8 was added to 52

Point out that **8 was also added to** $\frac{x}{2} - 8$.

3. The result was:

LHS	RHS
$\frac{x}{2} - 8$	52
$\frac{x}{2}$	$52 + 8$
$\frac{x}{2}$	60

4. 60 was then multiplied by 2:

LHS	RHS	
$\frac{x}{2} - 8$	52	
$\frac{x}{2}$	$52 + 8$	60
$\frac{x}{2}$	60	
$\times 2$	$\times 2$	
x	60×2	

we found the solution is 120

Point out that $\frac{x}{2}$ **was also multiplied by 2.**

Now write the solution in algebraic notation:

$$\begin{aligned} \frac{x}{2} - 8 &= 52 & / +8 \\ \frac{x}{2} &= 60 & / \times 2 \\ x &= 120 \end{aligned}$$

ACTIVITY 6

Solve the following equations:

1. $5x - 2 = 13$

2. $2x + 7 = 17$

3. $-3x - 8 = 20 + 14$

4. $47 = -10a + 3a + 12$

5. $\frac{x}{6} - 5 = 12$

6. $-4 - \frac{a}{3} = -115$

7. $3(x - 2) = 42$

8. $\frac{3b+5}{-2} = -13$

9. $-2x - 5 - (x + 4) = -9$

10. $22 = 3(2x + 1) - (x - 6)$

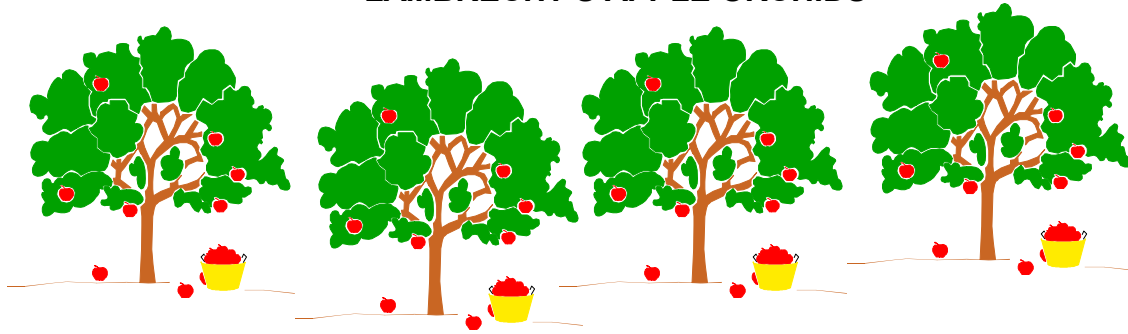
Teacher Notes: Activity 6

The aim of this activity is to practise solving these equations using the balancing method. Note that questions 3 and 4 help the pupils realise that it makes sense to group the like terms on the same side of the equation first. It might be a good idea to do an example like question 7, 8 or 10 that involves simplification of the algebraic expression first.

A short assessment on the solving of arithmetical equations can be given at this point. The purpose of this assessment is to identify those pupils who still have problems with the balancing method in the context of arithmetical equations. Consolidation work needs to be prepared for these pupils.

ACTIVITY 7

LAMBRECHT'S APPLE ORCHIDS



Farmer Lambrecht initially planted 120 Golden Delicious apple trees in one of his orchids and 460 Granny Smith apple trees in another orchid.

He then decided to plant 15 Golden Delicious (G-D) apple trees every day thereafter as shown in the table below:

Number of days	0	1	2	3	4	5	6				x
Number of G-D trees	120	135	150	165							

He also decided to plant 5 Granny Smith (G-S) apple trees every day thereafter as shown in the table below:

Number of days	0	1	2	3	4	5	6				x
Number of G-S trees	460	465	470	475							

1. Write down an algebraic rule for finding the number of G-D apple trees after x days.
2. Write down an algebraic rule for finding the number of G-S apple trees after x days.
3. On what day will the number of apple trees in the two orchids be the same?

ACTIVITY 8

Let's consider some other situations for planting apple trees in farmer Lambrecht's orchids so that the number of apple trees in the two orchids will always be the same on the 34th day as it was in the first situation (Activity 7).

In each of the situations described below decide what farmer Lambrecht must do so that the number of apple trees in the two orchids will be the same on the 34th day.

Situation 2

Farmer Lambrecht decides to plant 7 more new G-D apple trees every day than in the first situation in which he planted only 15 new trees every day. How many new G-S apple trees should he plant everyday so that there will be the same number of apple trees in the two orchids on the 34th day?

Situation 3

Farmer Lambrecht decides to plant 10 more G-D apple trees everyday than in the first situation in which he planted only 15 new trees every day. How many new G-S apple trees should he plant everyday so that there will be the same number of apple trees in the two orchids on the 34th day?

Situation 4

Farmer Lambrecht decides to plant 460 G-S apple trees initially but then does not plant any new trees to the G-S apple orchid. How many new G-D apple trees should he plant everyday so that there will be the same number of apple trees in the two orchids on the 34 day?

Class Discussion: Activity 7 and 8

Discuss the informal methods that the pupils use.

Make the idea of “balancing” explicit in the context used. In other words emphasise that if one decides to plant 7 more new apple trees to the G-D orchid, one has to plant the same number of new apple trees to the G-S orchid to keep the number of trees in the two orchids the same, on the same day as previously.

Refer to the teacher notes on Activities 7 and 8.

Teacher Notes for Activity 7 and 8

The aim of Activities 7 and 8 is to construct the notion of an algebraic equation (an equation with the unknown on both sides of the equality symbol) and to develop an understanding of equivalent equations.

In Activity 7 the pupils construct the equation, $120 + 15x = 460 + 5x$, the solution of which is 34.

In Activity 8, we provide the pupils with experiences in which they are intuitively dealing with the idea of “balancing”. The pupils need to realise that in order to keep the solution of the new equation the same as $120 + 15x = 460 + 5x$, a change in the one apple orchid implies the **same change** in the other apple orchid. This is usually referred to as, a change on the left-hand side of the equation must imply the same change to the right-hand side of the equation, if the solution is to remain the same. Reflection on the establishment of the new equations of the four situations in Activities 7 and 8:

In the class discussion following Activity 7 and 8 the pupils need to reflect on what the four situations (for planting the apple trees) have in common. The pupils need to describe the “balancing” of the situations verbally. For example, the pupils need to say that if there are 5 more trees planted in the G-S orchid every day then 5 more trees need to be planted in the G-D orchid every day for the two orchids to have the same number of trees on the 34th day.

After this process, the pupils need to reflect on how we can express these changes (“balancing”) in the algebraic language.

	G-D APPLE ORCHID	G-S APPLE ORCHID	The solution
Situation 1	$120 + 15x$	$460 + 5x$	$x = 34$
Situation 2	$120 + 15x$ +7x $120 + 15x + 7x$	$460 + 5x$ +7x $460 + 5x + 7x$	$x = 34$
Situation 3	$120 + 15x$ +10x $120 + 15x + 10x$	$460 + 5x$ +10x $460 + 5x + 10x$	$x = 34$
Situation 4	$120 + 15x$ -5x $120 + 15x - 5x$	$460 + 5x$ -5x $460 + 5x - 5x$	$x = 34$

Note: Do not push for the implementation of the idea of “balancing” to the formal “balancing” method at this point. Now we only want the pupils to reflect on the “balancing” of the equation and **to check that the solution is in fact 34, through substitution**. There is no need to focus on the algebraic solution process of the different situations yet.

The terminology of equivalent equations can be dropped by the end of this discussion by pointing out that the following equations

$$\begin{aligned} 120 + 15x &= 460 + 5x \\ 120 + 15x + 7x &= 460 + 5x + 7x \\ 120 + 15x + 10x &= 460 + 5x + 10x \\ 120 + 15x - 5x &= 460 + 5x - 5x \end{aligned}$$

are called **equivalent equations** because they have the **same solution**.

Another situation which can be looked at is when the farmer reduces the number of G-S apples trees initially to 450 while still planting the same number of new trees to the G-S apple orchid everyday, namely

$$\begin{aligned} 120 + 15x &= 460 + 5x \\ &\quad -10 \\ -10 & \\ 120 - 10 + 15x &= 450 + 5x \qquad x = 34 \end{aligned}$$

ACTIVITY 9

Godfrey must solve each of the following equations:

1. $12x - 7 = -31$

2. $56 = -6x + 8$

3. $350 - 15x = 200$

What must Godfrey do to either side of the equation in order to keep the solution the same if the equations above are changed as shown below:

1. $7x - 7 = -31$

2. $56 = -6x + 24$

3. $350 - 8x = 200$

Check your solution.

Teacher Notes: Activity 9

The pupils need to be able to solve arithmetical equations using the balancing method.

The pupils must use algebraic notation to show the changes, for example:

$$1. \quad 12x - 7 = -31 \quad /-5x$$

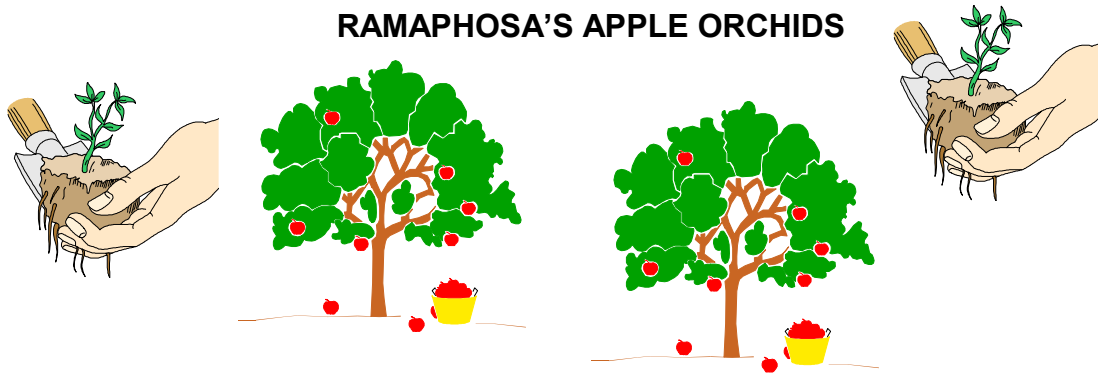
$$12x - 5x - 7 = -31 - 5x$$

$$7x - 7 = -31 - 5x$$

The pupils can substitute the solution $x = 2$ into the equivalent $7x - 7 = -31 - 5x$ to check.

ACTIVITY 10

RAMAPHOSA'S APPLE ORCHIDS



Farmer Ramaphosa decides to plant Starking apple trees in one of his orchids. He decides to plant 465 Starking apple trees initially and 5 more new trees every day thereafter as shown in the table below:

Number of days	0	1	2	3	4	5	6			x
Number of Starking trees	465	470	475	480						

Farmer Ramaphosa decides to plant Golden Delicious (G-D) apple trees in his other orchid. He initially plants 225 G-D apple trees and 15 more new trees every day thereafter as shown in the table below:

Number of days	0	1	2	3	4	5	6			x
Number of G-D trees	225	240	255	270						

1. What is the functional rule for finding the number of Starking apples planted in x days?
2. What is the functional rule for finding the number of G-D apples planted in x days?
3. On what day will there be the same number of Starking apples and G-D apples in the two orchids?

Teacher Notes: Activity 10

It is not spontaneous for the pupils to transfer the idea of “balancing” to the solving of the equation $225 + 15x = 465 + 5x$. The pupils will probably work through numerical methods using tables or substitution.

For example:

	$225 + 15x$	$465 + 5x$
$x =$		
$x =$		
.		
.		
.		
$x =$		

The role of the teacher is to ensure that the mathematical notation $225 + 15x = 465 + 5x$ makes sense to the pupils, namely that the equation represents the situation when the number of apple trees in the two orchids are the same. The solution tells you on what day the number of apple trees in the two orchids is the same.

Allow the pupils to present their solutions. If none of the pupils used the “balancing” method the teacher needs to write down the algebraic equation $225 + 15x = 465 + 5x$ and to transfer the idea of “balancing” to the formal “balancing” method of solving equations.

The conclusion of this activity is that whatever changes are made to the left-hand side of the equation must also be made to the right-hand side of the equation if we want to keep the solution the same and we can use this in order to develop a solution process.

In the class discussion the teacher needs to use the equation $25 + 15x = 465 + 5x$ to lead the pupils through the solution process:

$225 + 15x = 465 + 5x$	$/-5x$	(1)
$225 + 10x = 465$	$/-225$	(2)
$10x = 240$	$/+10$	(3)
$x = 24$		(4)

The teacher needs to point out that there are four equivalent equations in the solution process. The teacher can also show by substituting the solution into any of the four equivalent equations that *the solution is true for any of the four equations*. Hence the equations are equivalent.

ACTIVITY 11

Let's go back to Farmer Ramaphosa who planted Starking and G-D apple trees.

Let's consider some other situations for planting apple trees in farmer Ramaphosa's orchids so that the number of apple trees in the two orchids will always be the same on the 24th day as it was in the first situation (Activity10).

Situation 2

He decides to plant 20 more new trees in the G-D orchid instead of 15.

How many more trees must be planted in the Starking apple orchid so that the number of trees in the two orchids will be the same on the 24th day?

Situation 3

He decides to start off by planting 250 G-D apple trees instead of 225.

What must the number of Starking apple trees that he planted initially be, for the number of apples trees in the two orchids to be the same on the 24th day?

Teacher Notes: Activity 11

After the report back the pupils must be pushed to write the solution in the algebraic notation, for example

$$225 + 25 + 15x = 465 + 25 + 5x$$

Here once again it needs to be pointed out that if we want to keep the solution the same, whatever we do to one side of the equation we must also do to the other side of the equation. In doing so we create new equations (set of equivalent equations) while keeping the solution the same.

ACTIVITY 12

In 1970 the municipality opened the crèche, “Happy Days”, in Fairview. The crèche started with 35 children. From 1990 the number of children in the “Happy Days” crèche dropped by 3 children every year. In 1991 the municipality opened the crèche, “Ragamuffins”, in the newly built suburb, Greenfields. The “Ragamuffins” crèche started with 15 children. The number of children in the “Ragamuffin” crèche increased by 2 children every year.

1. The municipality decided that since the number of children attending the “Happy Days” crèche was dropping, it would close the crèche if it has fewer than 12 children. In what year was the “Happy Days” crèche closed by the municipality?
2. The municipality also decided to build another crèche in Greenfields once the number of children attending the “Ragamuffin” crèche was greater than 44. In what year was the second crèche built in Greenfields?
3. In what year did the “Happy Days” crèche and the “Ragamuffins” crèche have the same number of children?

Teacher Notes: Activity 12

The aim of this activity is for the pupils to build an algebraic equation that involves a subtraction of the variable, $35 - 3x = 15 + 2x$.

In the class discussion that follows, the “balancing” method can be referred to. The “balancing” method can be generalised at this point, for example,

$$\begin{array}{ll} 35 - 3x = 15 + 2x & \text{We may add } 3x \text{ to both sides of the equation to get} \\ & \text{an equivalent equation.} \\ 35 - 3x = 15 + 2x & / +3x \\ 35 = 15 + 5x & \end{array}$$

Since $35 - 3x = 15 + 2x$ and $35 = 15 + 5x$ are equivalent equations, they have the same solution.

We may now subtract 15 from both sides of $35 = 15 + 5x$

$$\begin{array}{ll} 35 = 15 + 5x & / - 15 \\ 20 = 5x & \end{array}$$

$20 = 5x$ is equivalent to $35 - 3x = 15 + 2x$ and $35 = 15 + 5x$ and therefore have the same solution.

Both sides of the equation $20 = 5x$ is divided by 5:

$$\begin{array}{ll} 20 = 5x & / \div 5 \\ 4 = x & \end{array}$$

$x = 4$ is equivalent to the equations $35 - 3x = 15 + 2x$, $35 = 15 + 5x$ and $20 = 5x$ which have the same solution. ($x = 4$)

The blackboard will look like this:

$35 - 3x = 15 + 2x$	$/ +3x$
$35 = 15 + 5x$	$/ - 15$
$20 = 5x$	$/ \div 5$
$4 = x$	

We suggest that $x = 4$ be substituted into the three equations

$$35 - 3x = 15 + 2x$$

$$35 = 15 + 5x$$

$$20 = 5x$$

to check their equivalence.

ACTIVITY 13

Solve each of the following equations by creating simpler equivalent equations. Check your solution by substituting into **all** the equivalent equations created.

- | | |
|----|------------------------|
| 1. | $3x - 2 = 5x + 4$ |
| 2. | $16x - 15 = 5x - 4$ |
| 3. | $12 - 4x = 3x - 9$ |
| 4. | $5(1 - x) = 4(x - 10)$ |
| 5. | $4(1 - 2x) = 12 - 12x$ |

ACTIVITY 14

Solve each of the following equations by creating simpler equivalent equations. Check your solution by substituting into **only two** of the equivalent equations created.

- | | |
|----|-------------------------|
| 1. | $5x - 9 = 2x + 6$ |
| 2. | $8(1 - 3x) = 5(4x + 6)$ |
| 3. | $3(x - 2) = 4(x + 1)$ |
| 4. | $4x + 9 = 6 - 3x$ |
| 5. | $3 - x = x - 3$ |

ACTIVITY 15

Solve each of the following equations by creating simpler equivalent equations. Check your solution by substituting into **the original** equation.

- | | |
|----|---------------------------------------|
| 1. | $3(x + 2) - 15 = 2(x - 1)$ |
| 2. | $2x - 3(3 + x) = 5x + 9$ |
| 3. | $\frac{2}{3}x + 5 = \frac{1}{2}x + 9$ |
| 4. | $\frac{1}{2}(2x - 3) = 5$ |
| 5. | $2,5x = 0,5(x + 10)$ |

Teacher Notes : Activities 13,14 &15

Let the pupils show *their method* for solving the equations. If the pupils come up with different solution processes for solving the equation $4(1 - 2x) = 12 - 12x$:

SOLUTION 1	SOLUTION 2
$4(1 - 2x) = 12 - 12x$	$4(1 - 2x) = 12 - 12x$ $/\div 4$
$4 - 8x = 12 - 12x$ $/+12x$	$1 - 2x = 3 - 3x$ $/ -3$
$4 - 8x + 12x = 12$ $/-4$	$-2 - 2x = -3x$ $/+2x$
$4x = 8$ $/\div 4$	$-2 = -x$ $/\div -2$
$x = 2$	$2 = x$

Point out to the pupils that there are many “balancing” routes that they could have followed to arrive at the correct solution.

Note: If the pupils do not come up with different methods it is necessary to show this to them.

REVIEW ON:

SOLVING AN EQUATION SOLUTION OF AN EQUATION VERIFYING THE SOLUTION EQUATION

Class Discussion

Open the discussion by asking pupils:

What does it mean to solve an equation?

Answers that pupils may typically give are:

- to simplify and simplify until we get the solution
- to add or subtract to both sides of the equation until we get the solution

The pupils often therefore refer to the *procedure* of solving an equation and not to the *concept* of what it means to solve an equation. The discussion should lead to the notion that to solve an equation means to find the value for the unknown that makes the equation true (or that produces equivalent numerical expressions on either side of the equality symbol).

Now get the pupils to reflect on all the different methods (inverse operations, guess and check, “balancing” method) that were used to solve the equation. Point out that all the methods are different but they are all methods for solving equations.

What is a solution of an equation?

A typical response might be

- numbers that we find at the end

The discussion here should lead to the notion of solution as the number that makes the equation true.

What does it mean to check the solution?

A typical response might be

- to check if it is correct or gives the right answer

Here it must be pointed out that the most common and reliable way of checking the solution of an equation is by substituting into the original equation and checking to see if it makes the equation true.

Teacher Notes: On Assessment

A short assessment can be given on the solving of both arithmetical and algebraic equations as well their understanding of equivalent equations. The purpose of this assessment is to identify those pupils who have problems with the balancing method in the context of both arithmetical and algebraic equations.

Consolidation work needs to be prepared for these pupils.

ACTIVITY 16

The Grade 9D class is given the following equation to solve:

$$15 - 5(3 - 2x) = 8x + 3(x - 5)$$

Renuka's solution was 15, Bingo's solution was -15 and Karin's solution was 10.

Who is correct?

ACTIVITY 17

A. The Grade 9E class was given the following equation to solve:

$$2x - (3x + 5) = 5x - 2 - 3(2x + 1)$$

Alex's solution was -10 , Dumi's solution was 12 and Kim's solution was -3 .

Who is correct?

B. Can you find a value for x that does *not* make the equation

$$2x - (3x + 5) = 5x - 2 - 3(2x + 1) \text{ true?}$$

How will you convince someone of your answer?

ACTIVITY 18

Judy was given R20 for her 14th birthday.
Jamie was given R30 for his 14th birthday.

Judy decided to open a savings account at the **Bestgro Bank** that offered the following savings plan:

BESTGRO BANK

The local branch gave R3 for every month that money was banked with a bonus of 2 months. In addition to this the central office gave R2 for every month that the money was banked plus a fixed amount of R4.

The formula for determining the amount of money that Judy saved at the Bestgro bank therefore is

$$20 + 3(x + 2) + 2x + 4$$

where x is the number of months the money is banked.

Jamie decided to open a savings account at the **Fastgro Bank** that offered the following savings plan:

FASTGRO BANK

R5 for every month that the money is banked.

The formula for determining the amount of money that Jamie saved at the Fastgro Bank therefore is

$$30 + 5x$$

where x is the number of months the money is banked.

After how many months will Judy and Jamie have the same amount of money in their savings account?

Teacher Notes: Activities 16,17 & 18

The main aim of Activities 16, 17 and 18 is to confront the pupils with the notion of an equation and an algebraic identity.

In Module 4 the pupils were introduced to the notion of an algebraic identity as a mathematical way to express that two algebraic expressions are equivalent:

We can say two algebraic expressions $(13x + 17) - (6x + 14)$ and $7x + 3$ are equivalent because they have the same values for any value of the input variable x . An algebraic sentence like $(13x + 17) - (6x + 14) = 7x + 3$ which is true for all values of the input variables, is called an ***algebraic identity***.

We revisit this notion from a different perspective in which the identity is viewed as a special kind of equation that is true for any number to distinguish it from equations that are only true for some numbers.

ACTIVITY 19

A. Which of the following equations are identities?

- | | |
|----|---|
| 1. | $(3x + 2) + (7x - 5) = (7x - 5) + (3x + 2)$ |
| 2. | $(7x + 3) + (2x + 7) = (5x + 2) + (4x + 8)$ |
| 3. | $(13x - 8) + (15 - 7x) = 2(x + 1)$ |
| 4. | $3x - 7 - (13 + 5x) = 3x - 13 - (5x + 7)$ |
| 5. | $3(x - 4) - 2x = -12 + x$ |
| 6. | $10x - (5x - 3) = (3 - 5x) + 10x$ |
| 7. | $(7x - 8) - (7x - 11) = 3$ |
| 8. | $(4x + 12) - (4x + 15) = (15 + 8x) - (12 + 8x)$ |

B. For each of the expressions on the left-hand side, choose an expression

on the right-hand side that will make

1. an identity
2. an equation that is not an identity

$(3x + 5) + (2x + 7)$	$(4x + 8) + (6x - 6)$
$12x + (7 - 3x)$	$4x + 5$
$(8x - 3) + (2x + 5)$	$(2x + 8) + (3x + 4)$
$7x - (3x + 5)$	$7 + 9x$

Teacher Notes: Activity 19

For expression $7x - (3x + 5)$ we have deliberately chosen not to give an equivalent expression that will make an identity.

In the class discussion get the pupils to create an expression so that an identity is formed.

Review on equivalent equations:

Whole - Class Discussion

Start the discussion by asking the pupils the following question:

Can you give me an example of equivalent equations?

Write down the equations given by the pupils on the board.

Now ask the pupils why these equations are equivalent.

Some typical responses could be

- we can transform one equation into another by manipulation.

The discussion need to be directed to seeing equivalent equations as equations that have the same solution set.

Through the discussion it needs to be pointed out that **all the identities are equivalent equations.**

Teacher Notes: Activities 20, 21 & 22

Get the pupils to discuss in their groups why they say the equations that they created are equivalent to each other.

ACTIVITY 20

Build three equivalent equations for each of the following equations:

- | | |
|----|-------------------------|
| 1. | $x = 5$ |
| 2. | $x = -7$ |
| 3. | $2x = -25$ |
| 4. | $2x - 4 = 5x + 3$ |
| 5. | $3x = 2x$ |
| 6. | $4x - 5 = 4(x - 1) - 1$ |

ACTIVITY 21

Decide whether the following groups of equations are equivalent. Explain.

A.

$$\begin{aligned}x &= 3 \\ 3(x - 2) &= 2x - 3 \\ 2x + 1 &= 6\end{aligned}$$

B.

$$\begin{aligned}x &= -4 - 3x \\ 8x &= -8 \\ 2x &= -5 - 2x\end{aligned}$$

C.

$$\begin{aligned}3(x - 1) &= -2(x + 2) \\ x &= 5 \\ 3(x - 2) &= -21\end{aligned}$$

ACTIVITY 22

Write down 3 different equations that have the same solution as given below:

- | | |
|----|-------------------------------|
| 1. | The solution is -4 |
| 2. | The solution is all numbers |
| 3. | The solution is 0 |
| 4. | The solution is 14 |
| 5. | The solution is $\frac{2}{3}$ |

ACTIVITY 23A

Paula and Donald have to determine whether the following equations are equivalent:

$$-10(y + 1) + 5(y - 10) = 10 \quad \text{and} \quad 8(y + 4) - 6(y - 2) = 16$$

Paula first solves $-10(y + 1) + 5(y - 10) = 10$

$$\begin{aligned} -10y - 10 + 5y - 50 &= 10 \\ -5y - 60 &= 10 && /+60 \\ -5y &= 70 && /:-5 \\ y &= -14 \end{aligned}$$

Then Paula substitutes $y = -14$ into $8(y + 4) - 6(y - 2) = 16$

$$\begin{aligned} 8(-14 + 4) - 6(-14 - 2) &= 16 \\ 8(-10) - 6(-16) &= 16 \\ -80 + 96 &= 16 \\ 16 &= 16 \end{aligned}$$

Donald first solves $-10(y + 1) + 5(y - 10) = 10$

$$\begin{aligned} -10y - 10 + 5y - 50 &= 10 \\ -5y - 60 &= 10 && /+60 \\ -5y &= 70 && /:-5 \\ y &= -14 \end{aligned}$$

And then Donald solves $8(y + 4) - 6(y - 2) = 16$

$$\begin{aligned} 8y + 32 - 6y + 12 &= 16 \\ 2y + 44 &= 16 && /-44 \\ 2y &= -28 && /:2 \\ y &= -14 \end{aligned}$$

Discuss Paula and Donald's methods.

ACTIVITY 23B

Paula has to determine whether the following equations are equivalent:

$$-10(y + 1) + 5(y - 10) = 10 \quad \text{and} \quad 3(y + 2) - (y - 4) - 2y - 7 = 3$$

Paula still uses her method:

$$\begin{aligned} \text{She solves } -10(y + 1) + 5(y - 10) &= 10 \\ -10y - 10 + 5y - 50 &= 10 \\ -5y - 60 &= 10 && /+60 \\ -5y &= 70 && /:-5 \\ \dots\dots y &= -14 \end{aligned}$$

She then substitutes $y = -14$ into $3(y + 2) - (y - 4) - 2y - 7 = 3$

$$\begin{aligned} 3(-14 + 2) - (-14 - 4) - 2 \cdot -14 - 7 &= 3 \\ 3(-12) - (-18) - (-28) - 7 &= 3 \\ -36 + 18 + 28 - 7 &= 3 \\ -43 + 46 &= 3 \\ 3 &= 3 \end{aligned}$$

Paula now concludes that the two equations are equivalent.

Do you agree with Paula?

Teacher Notes: Activities 23A and 23 B

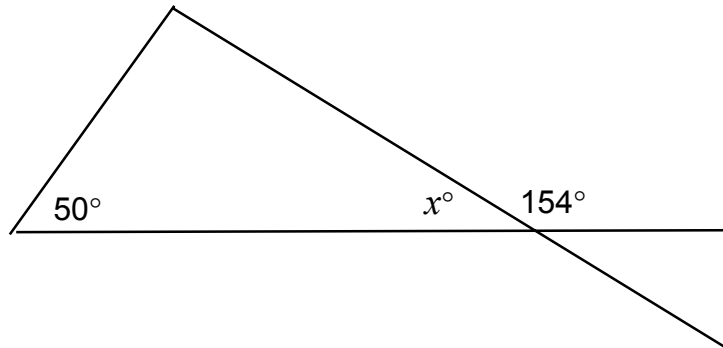
Discuss the different approaches.

Pupils need to realise that Paula's method has a limitation in that we need to know how many solutions each of the equations have. The pupils are confronted with this limitation in Activity 23B. If the pupils all agree with Paula it might be necessary to hint that they also try Donald's method. The pupils will now be confronted with different solution sets for the two equations, hence the two equations are not equivalent.

ACTIVITY 24

The Grade 8E class is given the following problem:

Write an equation to find the size of the angle x in the figure below.



Here are two of the equations that some of the pupils wrote:

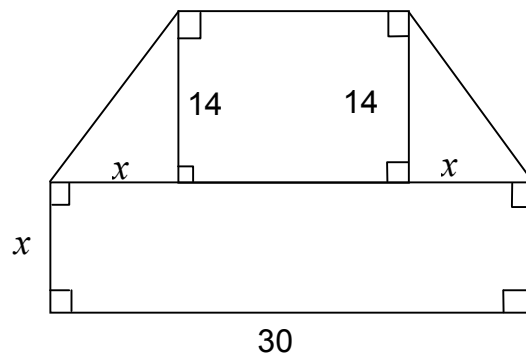
$$x + 50 + (154 - 50) = 180$$

$$360 - 2x = 2 \cdot 154$$

Discuss whether these equations are correct or not and whether they are equivalent.

ACTIVITY 25

The diagram below shows the plan of one of the playing grounds at a school.



If the area of the playing ground is 720 square units, what is the value of x ?

Here are some of the equations that Grade 8 pupils wrote to solve the problem:

$$30(x + 14) - 2(7x) = 720$$

$$14(30 - 2x) + 30x + 2(7x) = 720$$

$$(x + 14)(30 - 2x) + 2(2x) + 2(x \cdot x) = 720$$

Discuss whether these equations are correct or not and whether they are equivalent.

Teacher Notes : Review on Equivalent Expressions

Whole-Class Discussion

Open the discussion with a reflection on equivalent equations.

Remind the pupils that we also dealt with equivalent expressions. Refer to the definition of equivalent expressions as given in Module 4:

We say algebraic expressions are equivalent if they produce the same output numbers for the same input numbers

It needs to be made explicit here that we can check the equivalence of algebraic expressions either through simplifying or by substituting numbers into the expressions.

ACTIVITY 26

Are the following pairs of expressions equivalent?

Check by simplification and by substitution.

1. $(12x - 5) - (7x - 8)$ and $5x - 13$
2. $(13x - 7x) - (12x + 8x)$ and $1 - 15x$
3. $(6x^2 - 3x + 2) + (3x^2 + 8x - 5)$ and $9x^2 + 5x - 3$

ACTIVITY 27

Which of the following pairs of expressions are equivalent?

1. $-6(2x + 3) - 5(3 - 2x)$ and $-2x - 33$
2. $13x + 2 - (7x - 1)$ and $6x + 1$
3. $(20 - 3x^2 - 5x) - (4x^2 - 6x + 20)$ and $-7x^2 + x$
4. $\frac{x^2 + 3x - 2}{x + 2}$ and $x + 1$

ACTIVITY 28

Write down **two** equivalent expressions for each of the following expressions:

1. $4(x - 3) - 2(x + 3)$
2. $6x + 20$
3. $x^2 - x(3 - x)$

ACTIVITY 29

A. Godfrey wanted to simplify the expression $(x+1)(x+2) - x^2$.

He wrote down $3x + 2$.

Is Godfrey correct? Explain.

B. Godfrey solved the equation $3x + 2 = 35$.

He found the solution to be 11.

Vusi must solve the equation $(x+1)(x+2) - x^2 = 35$.

Godfrey suggests to him to simply solve the equation $3x + 2 = 35$.

Do you agree with Godfrey? Explain!

Teacher Notes: Activity 29

Whole-Class Discussion

When we equate two algebraic expressions that are equivalent we get an equation that is an identity. When we equate two algebraic expressions that are not equivalent we get an equation that is not an identity.

When we are given an equation, how can we judge whether it is an identity or not an identity?

Write down an equation on the board, for example:

$$(15 + x) - (10 - 3x) = (1 - x) + (4 + 5x)$$

One way to judge if the equation is an identity or not is to check whether the two expressions on either side of the equal sign are equivalent or not. This can be done by either substituting or simplifying.

Are there any other ways of judging?

Make up a few examples in which the equations are not identities.

Teacher Notes: Activities 26 , 27, 28 & 29

In this discussion we are revisiting the notion of equivalent algebraic expressions through equations. If the two algebraic expressions on the two sides of the equality sign are equivalent, it is an algebraic identity and the solution set should include all the numbers. So by solving the equation we can also decide whether the two algebraic expressions on both sides were equivalent or not. The discussion should lead to the latter notion. The pupils should also make sense that when they reach the stage in the solution process where the two algebraic expressions are obviously equivalent, for example, $3x + 2 = 3x + 2$, that the original equation was an identity.

The solution process might be viewed as a simplification process of both expressions until they can be recognised as being equivalent.

ACTIVITY 30



The supervisor of the Cape Town Garden and Claremont Garden organised volunteers to help dig out weeds.

The tables below show the number of people who volunteered to pull out weeds and the number of bags of weed pulled in a day in the two gardens:

Claremont Garden

Number of people	1	2	3	4	5	6					x
Number of bags of weed pulled in a day	5	9	13	17							

Cape Town Garden

Number of people	1	2	3	4	5	6					x
Number of bags of weed pulled in a day	7	11	15	19							

1. Write down the algebraic rules for the number of bags of weed pulled by x people in the Claremont Garden and in the Cape Town Garden.
2. How many people need to pull weeds, for the number of bags pulled in a day to be the same in both gardens?

Teacher Notes: Activity 30

The aim of this activity is to provide the pupils with a context to make sense of the algebraic model $1 + 4x = 3 + 4x$.

The pupils need to realise that the two expressions cannot be simplified any further, so the equation is not an identity. Furthermore if the solution process is continued through the “balancing” method, a contradiction will be reached, for example

$$1 + 4x = 3 + 4x \quad / - 4x$$

$$\Rightarrow 1 = 3$$

Hence there is no solution.

It is now also necessary to reflect on the role of the coefficient of x and the constant in the two algebraic expressions, in the context of the activity. For example, the number of people that pulled out weeds every day grows by 4 in both gardens (rate of change). The starting point, which is the number of bags pulled by one person, (the constant) is different.

ACTIVITY 31**CLASSIFYING EQUATIONS**

Equation	Solution	Identity (true for any x)	Contradiction (no solution)	(1 solution)
$2 - 5x = 3(1 - x) + 5$				
$5(x - 1) + 2x = 7(x + 5)$				
$2(x + 1) + 5x = 7(x + 2) - 12$				
$6(x - 2) - 5 = 3(x - 2) + 7$				
$3(2x + 1) - 5 = 6(x - 1) + 4$				
$2x + 5 - x = 3x + 2x + 5$				
$5(2 - x) = 6(1 - x) + 4 + x$				
$5(x - 1) = 5x + 3$				

ACTIVITY 32

HOW MANY SOLUTIONS?

Simplify the following equations, and say HOW MANY solutions each equation has.

1. $5x - 15 = 7x + 2$

2. $6x + 5 = 3(2x + 4) - 7$

3. $3x + 5x + 11 = 8x + 3$

4. $10 + 7(1 - 2x) = 5x + 2$

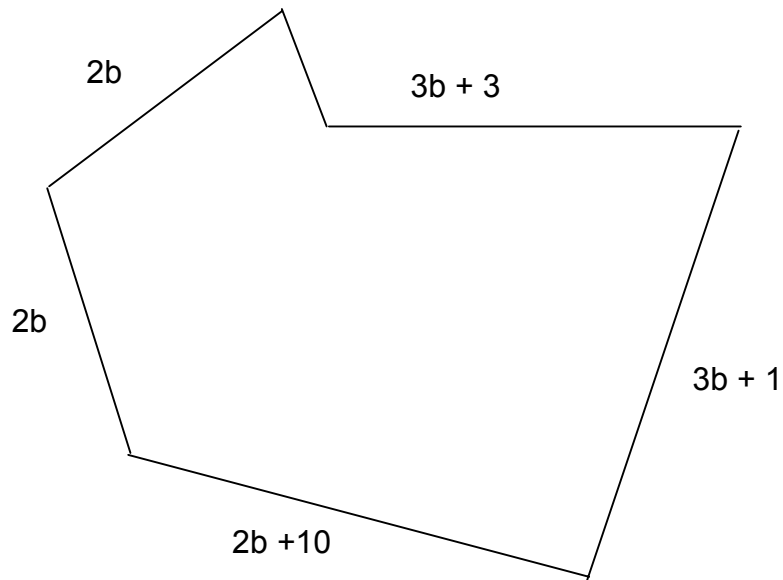
5. $100 - 15x + 2(x - 10) = 60 - 13x + 20$

6. $65 - 20x + 10 = 5(2 - x)$

7. $4(x + 1) - 2x = 5x + 11 - 3x$

8. $7x + 9(x - 3) = 8(x - 2) - 11$

ACTIVITY 33



1. Write down an expression for the perimeter of the figure. Write the expression in its simplest form.
2. If the perimeter of the figure is equal to 40, write down an equation that shows this.
3. For what value of b is the perimeter of the figure 40?
4. Verify that your solution is correct!

ACTIVITY 34

Larry takes a number and adds 1, then multiplies the result by 3, then adds 2 to the result.

1. Write an expression for Larry's procedure (use x to represent the number):

2. Rafiek performs a different operation on a number. He subtracts the number from 10, then multiplies the result by 2.

Write an expression for Rafiek's procedure (use x to represent the number):

3. Find a number for which Larry and Rafiek will get the *same* result.

4. Find a number for which Larry and Rafiek will get the same result by writing an equation.

5. Solve the equation.

INEQUALITIES

ACTIVITY 36

The tool rental company, TOOL-4-U charges a basic fee of R230 as well as R10 per day to rent a motorised water pump. Their competitors, CHEAP TOOLS, only charge R50 for the basic fee, but have a daily rate of R30 per day.

1. Write down an algebraic expression for the cost of renting from the TOOL-4-U company. Use the letter x to represent the number of days.
2. Write down an algebraic expression for the cost of renting from the CHEAP TOOLS company. Use the letter x to represent the number of days.
3. How does the cost at TOOLS-4-U compare with the cost at CHEAP TOOLS if we want to rent the water pump for 15 days?
4. How does the cost at TOOLS-4-U compare with the cost at CHEAP TOOLS if we want to rent the water pump for 2 days?
5. For what number of days would renting the pump from CHEAP TOOLS be a better deal than renting from TOOLS-4-U?
6. For what number of days would it make no difference in cost for renting the water pump from either CHEAP TOOLS or TOOL-4-U?

Teacher Notes: Activities 36, 37 & 38

The aim of Activities 36, 37 and 38 are to develop the notion of an inequality based on the pupils' knowledge of equations.

The pupils must be given the opportunity to compare the two algebraic expressions, using their own strategies. Many pupils will probably revert to numerical methods such as substitution. Eventually however the discussion should lead to the introduction of the symbols $>$ and $<$.

In Activity 36, for example, once the pupils have written down the algebraic expressions for the cost [$230 + 10x$ and $50 + 30x$] they can be shown how to express the comparison in the algebraic notation: $230 + 10x > 50 + 3x$.

ACTIVITY 37

The city of Durban has two fishponds in the same park. The one at the north end of the park has 20 cm of water in it, and the one at the south end of the park has 500 cm of water in it. At the beginning of the working day, the north pond begins to be filled with water at a rate of 5 cm a minute, while at the same time the pond at the south end begins to be drained at a rate of 3 cm a minute.

1. Write down an algebraic expression for the height of the water level in the north pond. Use the letter x to represent the number of minutes.
2. Write down an algebraic expression for the height of the water level in the south pond. Use the letter x to represent the number of minutes.
3. Read this question, but do not answer it:

After how many minutes is the water level in both ponds the same?

Write an **equation or inequality** that will enable us to solve the question.

4. Read, but do not answer the following question:

For what length of time will the water level in the south pond remain higher than the water level in the north pond?

Write an **equation or inequality** that will enable us to solve the question.

ACTIVITY 38

Mr James class and Mr Campbell's class are both saving money to go on a class trip. Mr James class has R40 left over from another trip, and Mr Campbell's class has R90 from the sales of chocolate bars. Each week Mr James class brings in R15, and Mr Campbell's class brings in R10. How long will it take until Mr James class has more money collected than Mr Campbell's class?

1. Write an expression to show the savings at any time for Mr James class:

2. Write an expression to show the savings at any time for Mr Campbell's class:

3. Write an inequality using these expressions to represent the question in the story:

4. What is the answer to the question in the story?

5. Answer the question in words:

ACTIVITY 39

This lesson is a whole-class discussion.

1.

$$2x + 4 > 158$$

What does it mean to *solve* this?

What does the solution mean?

2.

$$6(x - 2) - 5 < 155$$

What does it mean to *solve* this?

What does the solution mean?

3. What does it mean to *solve* the inequalities below?

What does the solution mean?

$$8 + x > 5x - 4$$

$$5(x - 1) + 2x < 3(x + 5)$$

$$2x + 3(x + 1) > 2(x + 2) - 7$$

Teacher Notes: Activity 39

We expect the discussion for question 1 to lead to the idea that we are looking for numbers (x) so that the left-hand side is greater than 158.

Let's see if we can apply the "balancing" method that we used for solving equations. If we now subtract or add a number to both sides of the inequality the relationship ($=$, $>$ or $<$) between the numbers on either side of the equality is unchanged, for example:

$$2x + 4 > 158 \quad /-4$$

$$2x > 154$$

At this point the pupils may continue with the "balancing" method:

$$2x > 154 \quad /\div 2$$

$$x > 77$$

What does the solution mean?

Get the pupils to verbalise that it means for all numbers (x) greater than 77, the left-hand side is greater than 158.

Now take some numerical examples to check:

$$x = 78 \quad 2(78) + 4 = 160 \quad 160 > 158$$

$$x = 110 \quad 2(110) + 4 = 224 \quad 224 > 158$$

$$x = 27 \quad 2(27) + 4 = 58 \quad 58 < 158$$

$$x = 77 \quad 2(77) + 4 = 158 \quad 158 = 158$$

Note: We are deliberately delaying the negative coefficient at this point.

Do another example, $6(x - 2) - 5 < 155$ and repeat the discussion as outlined including the numerical verification.

Now move to algebraic inequalities:

$$8 + x > 5x - 4$$

$$5(x - 1) + 2x < 3(x + 5)$$

$$2x + 3(x + 1) > 2(x + 2) - 7$$

Activity 40 is a fortification activity which can be given for homework.

ACTIVITY 40

Solve the following equations and inequalities. Then verify your solution.

1. $2x + 5 - x < 3x + 2x + 5$

2. $7(x - 1) = 5x + 3$

3. $2x + 5x + 1 = 7(x - 1)$

4. $6x + 2(1 - x) < 8 - 2x$

5. $5(2 - x) = 6(1 - x) + 4 + x$

6. $5(x - 2) > x + 2$

7. $7(x - 1) > 5x + 3$

ACTIVITY 41

Complete the following tables.

Write down your observations.

TABLE A

x	-3	-2	-1	0	1	2	3	4	5	6	7	8
$x + 12$												
$2x + 7$												

TABLE B

x	-3	-2	-1	0	1	2	3	4	5	6	7	8
$3(x + 12)$												
$3(2x + 7)$												

TABLE C

x	-3	-2	-1	0	1	2	3	4	5	6	7	8
$-3(x + 12)$												
$-3(2x + 7)$												

Teacher Notes: Activity 41

In this activity we want to highlight the effect of a negative coefficient on the “balancing” method in the solution process of an inequality. In other words, the effect of dividing or multiplying by a negative number when finding the solution of an inequality.

Once the pupils have completed TABLE A they need to realise that the expression $2x + 7$ has a smaller value than the expression $x + 12$ for values of x up until 5. For values of x greater than 5, $2x + 7$ has a bigger value than $x + 12$.

From TABLE B the pupils need to realise that when multiplying the expression by 3, the expression $3(2x + 7)$ is still smaller than $3(x + 12)$ for values of x up until 5. For values of x greater than 5, $3(2x + 7)$ has a bigger value than $3(x + 12)$.

From TABLE C the pupils will realise that the situation is now reversed since the value of the expression $-3(2x + 7)$ is bigger than $-3(x + 12)$ for values of x up to 5. For values of x greater than 5, $-3(2x + 7)$ is smaller than $-3(x + 12)$.

Class Discussion

Reflect on the observations in the tables. It is not necessary to link these observations to the solution procedure immediately. The discussion can revolve around numerical inequalities, for example,

$$3 > -4$$

$$\times 2 \text{ (if we multiply both sides by 2)} \quad 6 > -8 \text{ (LHS of the inequality is **still greater** than the RHS).}$$

$$\times -2 \text{ (if we multiply both sides by -2)} \quad -6 < 8 \text{ (LHS of the inequality is **now smaller** than the RHS).}$$

The discussion here needs to lead to the generalisation that this also applies to division. Do a few numerical examples.

Now we can connect this to the solution procedure of inequalities:

For example:

Let's solve $-4(x + 3) > 140$

Write down $-4x - 12 > 140$ $/ +12$

$-4x > 152$ $/\div -4$

At this point reflect on the previous activity.

Allow the pupils to verbalise the solution procedure. For example, the pupils need to say that the result of the number $-4x$ when divided by -4 will be smaller than the result of 152 when divided by -4 .

The final step in the solution procedure can be written after this, namely,

$$x < -38$$

The pupils can now verify the solution in the second last step:

$$-4x > 152$$

$$-4(-39) > 152$$

$$156 > 152$$

Depending on the level of the class, this inequality can be solved again, this time starting the "balancing" method by dividing by 4:

$$-4(x + 3) > 140 \quad /\div -4$$

$$(x + 3) < -35 \quad / - 3$$

$$x < -38$$

Note:

Activities 42, 43 and 44 are Fortification Activities that can be done in group settings or given as homework.

ACTIVITY 42

Solve the following inequalities. Check your solution.

1. $3(x + 5) > 24$

2. $(4x - 2) - (6x + 4) < 12$

3. $-2(x + 1) - 3x < 15$

4. $\frac{x}{5} - 25 > 68$

5. $\frac{3x}{4} + 15 > -15$

6. $-14 < -14(-y)$

7. $12 - \frac{2x}{5} < 0$

8. $-45 + \frac{1}{4}(x - 5) > 46\frac{1}{4}$

9. $-2(x - 14) < 74$

10. $\frac{5(x + 3)}{2} - 7\frac{1}{2}x < -3\frac{1}{2}$

ACTIVITY 43

Solve the following inequalities. Check your solution.

1. $4x > 21 + x$

2. $9x - 2 < 5x + 23$

3. $-5x - 28 < 2x + 70$

4. $27x - (17x - 8) < 10x - 8$

5. $2n - (5n + 64) > 44 - 5(6 - 2n)$

6. $\frac{y}{3} - 2 > \frac{y}{9}$

7. $\frac{x}{6} < \frac{x}{5}$

8. $\frac{1}{3}x - \frac{5}{3} < -\frac{2}{3}x + \frac{1}{3}$

9. $\frac{3y}{4} - y > \frac{2}{3} + \frac{y}{6}$

10. $2m < \frac{7m+5}{3} + \frac{1}{6}$

ACTIVITY 44

Solve the following equations and inequalities. Check your solution.

1. $122 - 11x = 11x - 120$

2. $4x - 3 + 2x = 2(3x - 1)$

3. $3x > 9x - (2x - 8)$

4. $3(2x - 1) = 4x - 3 + 2x$

5. $-4(2x - 5) < -4x + 10$

6. $5x + 8 > 8x - 18$

7. $\frac{2}{5}x - 2 = \frac{1}{5}x + 4$

8. $5 - \frac{x}{7} < \frac{2x}{7} - 4$

9. $\frac{3x + 40}{12} = \frac{2x - 15}{6}$

10. $\frac{4 - x}{-4} = \frac{5 - 2x}{9}$