"SPATIAL SENSE": TRANSLATING CURRICULUM INNOVATION INTO CLASSROOM PRACTICE

Kate Bennie – Mathematics Learning and Teaching Initiative (MALATI)

Sarie Smit – Centre for Education Development, University of Stellenbosch (CENEDUS)

Introduction:

The question often asked is, "What is geometry?" Geometry as traditionally taught in South African schools paints a particular picture of geometry, namely, the classification of different plane figures (for example, triangles and parallelograms), the study and comparison of the properties of these figures, and the use of deduction to obtain some properties from others. This is done within the context of a specific axiomatic deductive organisation of the concepts. The geometry studied at the primary school is intended to prepare learners for the formal geometry of the secondary school.

While this approach to geometry has merit, there is much more to geometry than this. Curriculum 2005 certainly paints a broader picture of geometry. Specific Outcome 7 for Mathematics, Mathematical Literacy and Mathematical Sciences claims that:

> *Mathematics enhances and helps to formalise the ability to be able to grasp, visualise and represent the space in which we live. In the real world, space and shape do not exist in isolation from motion and time. Learners should be able to display an understanding of spatial sense and motion in time.*

This outcome requires that learners:

> *…describe and represent experiences with shape, space, time and motion, using all available senses.*

> (Department of Education, 1997)

This recognition that geometry is important as a body of knowledge that supports interaction in space is also reflected in curriculum reform elsewhere in the world. Standard 3 of the NCTM Draft "Standards 2000" Document proposes that mathematics instruction programmes should pay attention to geometry and spatial sense so that all students, among other things, "use visualisation and spatial reasoning to solve problems both within and outside of mathematics".

The rationale for including this aspect of geometry in the school curriculum is that it is regarded as important in supporting everyday interaction in space. Recent research also suggests links between spatial sense and performance in geometry and mathematics in general. Clements and Battista (1992) provide a summary of the work done in this area, but warn that results do vary and that the relationship between spatial skills and the learning of non-geometric concepts is not straightforward.

Mathematics educators in South Africa are presently attempting to translate this aspect of Curriculum 2005 into practice. Observations of such attempts by the presenters suggest that teachers have difficulty precisely because the new curriculum paints such a different picture of geometry to that suggested by the traditional curriculum. Teachers thus appear to have difficulty reconciling materials developed for Curriculum 2005 with their existing understanding of geometry. The aim of this workshop is to provide teachers with an opportunity to explore these materials, identifying the geometry inherent in them as well as the links to Curriculum 2005.

What is Spatial Sense?

Review of the literature in this field indicates that a variety of terminology can be used, for example, spatial ability, spatial orientation, spatial reasoning and spatial insight, and that there is little consensus on the definition of the concept. However, by considering the kind of interaction required in order to interact in space, one can gain a picture of the kind of skills necessary for this interaction. Smit (1998) suggests that without spatial skills it would be difficult to exist in the world as one would not be able to communicate about position and relationships between objects, give and receive directions and imagine changes taking place in the position or size of shapes. Human (1999) has identified the following forms of interaction:

- Observing spatial objects (two- and three-dimensional) and their properties in a discriminating way, for example, changes in position, size, shape, the relationships between the objects
- Generating information that cannot be directly observed, for example, determining distances, elevations, areas and volumes
- Representing spatial objects in two dimensions
- Interpreting two-dimensional representations of spatial objects.

The Development of Spatial Sense:

Spatial sense cannot be taught, but must be developed over a period of time. A number of researchers refer to the importance of learners engaging with concrete spatial activities before being able to form and to manipulate visual images. Van Niekerk (1995) identifies different levels of interaction in space: She distinguishes between "spatial orientation" ("basically a three-dimensional experience") and "spatial insight" ("a two-dimensional experience"). She suggests that an individual must have experiences with a three-dimensional cube before being able to describe it verbally, make a mental image of the cube, or make a two-dimensional drawing of the cube. Owens and Gould (1998) have developed a framework of strategies used by students in solving spatial problems and list these in the order in which they are likely to emerge.[1] These range from manipulation in the physical world and perception, to the use of strategies using visual imagery only, to the use of this imagery in problem solving.

It is thus the role of foundation and intermediate phase teachers to provide learners with rich concrete experiences and to encourage reflection on these experiences to ensure progression to a more abstract level.

Geometric Thought:

The importance of geometry as the study of space has been discussed. We can also identify geometry as an important domain of purely **mathematical activity**. This view is reflected in Specific Outcome 10 for Mathematical Literacy, Mathematics and Mathematical Sciences in Curriculum 2005:

> *Reasoning is fundamental to mathematical activity. Active learners question, examine, conjecture and experiment. Mathematics programmes should provide opportunities for learners to develop and employ their reasoning skills. Learners need varied experiences to construct arguments in problem settings and to evaluate the arguments of others.*

(Department of Education, 1997)

This outcome requires that learners are required to "use various logical processes to formulate, test and justify conjectures" (ibid.).

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[1] The strategies are classified (in the order in which they are said to emerge) as "emergent strategies", "perceptual strategies", "pictorial imagery strategies", "pattern and dynamic imagery strategies" and "efficient strategies".

Curriculum 2005 also emphasises the importance of communication of mathematical ideas, concepts and thought processes.

Geometry provides a rich context for learners to experience mathematical activity and the communication of this activity. This can be achieved in both the primary and secondary school. Hoffer (1981) identifies five geometric skills: visual, verbal, drawing, logical and application. De Villiers (1997) refers to the following processes in the study of geometry: axiomatising, proving, defining, experimenting, refuting, pattern finding, generalising, specialising, classifying and theorem finding.

The Development of Geometric Thought:

Teachers of geometry will be familiar with the following scenarios: A learner is able to recognise a square, but is not able to define it; a learner does not recognise a square as a special type of rectangle; a learner does not see the need to prove something that is regarded as obvious (Smit, 1998).

The van Hiele's developed a theory to explain responses of this nature. This theory states that geometric thought progresses through hierarchical levels: A learner must have achieved thinking on one level in order to be able to perform with understanding on the next. The van Hiele's identifed five levels of thought (later revised to three), three of which are appropriate in this context:

- The Visual Level: A learner operates on a figure according to its appearance as a whole. Figures are grouped together because they "look alike" and a figure is identified as a rectangle because it has a "long shape".

- The Analysis Level: A learner analyses a figure according to its properties and the relationship between the properties of a figure. These properties are determined empirically, for example, by measuring or cutting and folding. A learner on this level would identify a figure as a rectangle because it has "opposite sides equal, opposite sides parallel and all angles equal to 90°. At this level the relationship between **figures** is not perceived.

- The Ordering / Informal Deduction Level: A learner can formulate and use precise definitions, can give informal arguments and can compare properties of figures and classes of figures. For example, a learner on this level can state that "a rectangle is a parallelogram with one angle equal to 90°". This can be used to conclude that the diagonals of a rectangle are equal.

The traditional school geometry curriculum requires that learners in grade 10 are on the van Hiele ordering level. Research conducted in South Africa (de Villiers & Njisane, 1987; Smith, 1987), however, indicates that many secondary learners are on the visual or analysis level. The school geometry thus has little meaning for these learners and they resort to memorisation.

Most foundation and intermediate phase learners are on the visual level and teachers in these phases have an important role to play in providing learners with appropriate activities to encourage movement to the analysis level. Teachers moreover need to realise that development through these levels, although hierarchical, is not age-related.

Curriculum 2005 and Geometry:

The diagram on the following page provides one way of conceptualising the geometry interspersed amongst a number of specific outcomes in Mathematics, Mathematical Literacy and Mathematical Sciences. Dotted lines have been used to indicate that these outcomes should not be seen in isolation and can be linked in the same activity. For example, the study of measurement required in Specific Outcome 5 could include a study of the historical development of the topic (Specific Outcome 3). Specific Outcomes 9 and 10 have been drawn "across" the other outcomes as these include the geometric and communication skills that will be required in all geometric activity.

SO2	SO3	SO4	SO5	SO7	SO8
Manipulate number and number patterns in different ways	Demonstrate an understanding of the historical development of mathematics in various social and cultural contexts	Critically analyse how numerical relationships are used in social political and economic relations	Measure with competence and confidence in a variety of contexts	Describe and represent experiences with shape, space, time and motion, using all available senses	Analyse natural forms, cultural products and processes as representations of shape, space and time
SO9: Use mathematical language to communicate mathematical ideas, concepts generalisations, and thought processes					
SO10: Use various logical processes to formulate, test and justify conjectures					

In an attempt to co-ordinate the aspects of geometry in the different specific outcomes, mathematics educators (Smit, 1998; van Niekerk, 1999; GICD) have found it useful to identify three "strands" that run across the study of geometry, namely, **shape**, **vision** and **position** (location).

Workshop Outline:

Workshop participants will be given the opportunity to engage with MALATI and CENEDUS activities which have been designed specifically for Curriculum 2005. They will be encouraged to

- identify the strands shape, vision and position in the activities
- relate the activities to the Curriculum 2005 document
- relate performance on these activities to the van Hiele levels
- decide on which geometric skills are developed, for example, visual, verbal, drawing, logical and/or application
- reflect on the relationship between these activities and what has traditionally been termed "geometry" for these grades
- consider ways in which this aspect of the curriculum can be assessed.

Consider, for example, the following activity:

This is a photo of two boxes:
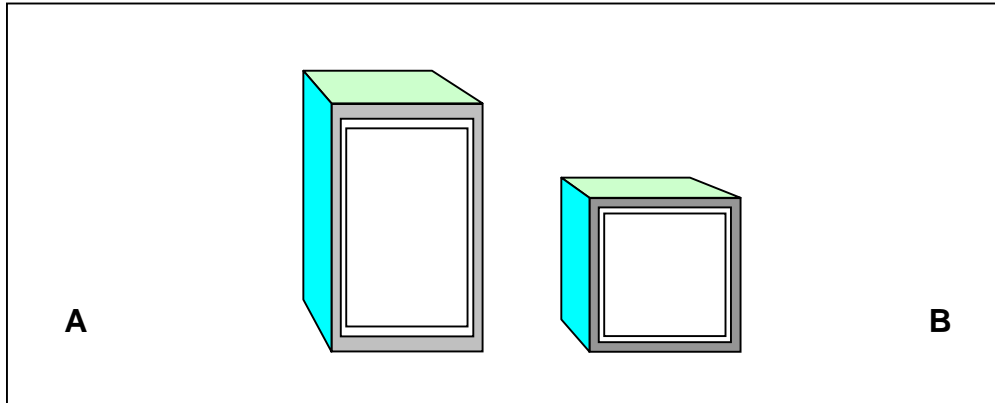
(a) Draw the aerial view of the two boxes.

(b) Draw the side view of the two boxes (from point A on the left).

(c) Draw the side view of the two boxes (from point B on the right).

(d) What would you see if you were looking at the two boxes from directly behind.

This activity can be characterised as follows:

CURRICULUM 2005	Specific Outcome 7	Describe and represent experiences with shape, space, time and motion, using all available senses	
	Assessment Criteria	Descriptions of orientation of an object in space	Demonstrate an understanding of the connectedness between shape, space and time
	Range Statements	Show understanding of the concept of point of reference in 2D and 3D Show understanding of perceptions by an observer from different reference points	Visualise and represent objects from various spatial orientations
	Performance Indicators	Explain one's position as origin Explain changes of appearance of objects when reference point (origin) changes Draw a diagram of the observed appearance of the shape	Observe and realise that objects viewed from different perspectives look different with respect to size, shape and their orientation in space
STRAND	Vision, Position		
GEOMETRIC THINKING	Visual, Drawing, Logical		
VAN HIELE LEVEL	Visual Level		

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