

# LECTURE SEVEN: The Optical Indicatrix 1

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- Optical Indicatrix
- Isotropic Indicatrix
- Uniaxial Indicatrix
- Ordinary and Extraordinary Rays
- Optic sign
- Use of the Indicatrix
- Biaxial Indicatrix
- Optic Sign
- Crystallographic Orientation of Indicatrix Axes
- Use of the Indicatrix

## THE OPTICAL INDICATRIX

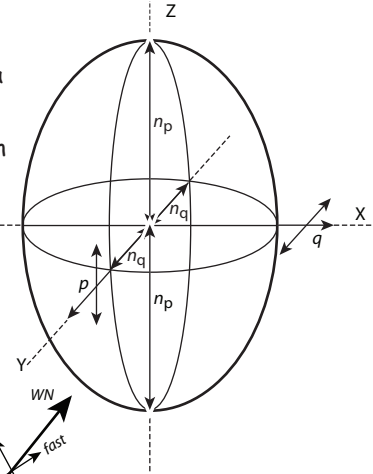
A geometric figure that shows the index of refraction and vibration direction for light passing in any direction through a material is called an optical indicatrix.

The indicatrix is constructed by plotting indices of refraction as radii parallel to the vibration direction of the light.

Ray p, propagating along Y, vibrates parallel to the Z-axis so its index of refraction ( $n_p$ ) is plotted as radii along Z.

Ray q, propagating along X, vibrates parallel to Y so its index of refraction ( $n_q$ ) is plotted as radii along Y.

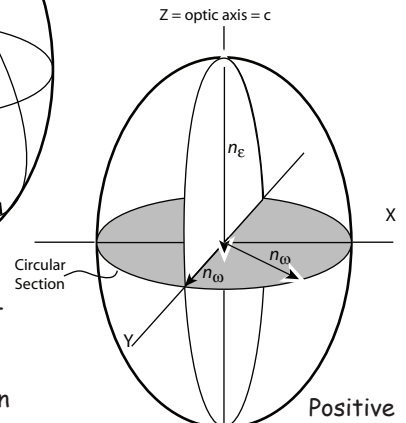
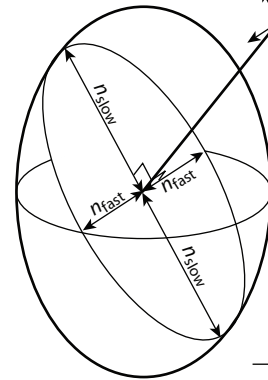
If the indices of refraction for all possible light rays are plotted in a similar way, the surface of the indicatrix is defined. The shape of the indicatrix depends on mineral symmetry.



## CONSTRUCTION of an OPTICAL INDICATRIX

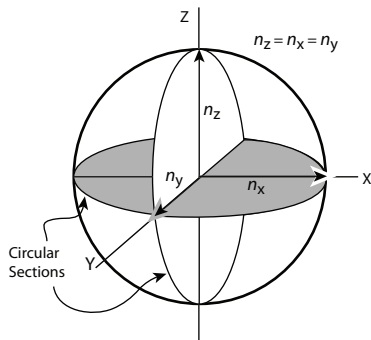
The primary use of the indicatrix is to determine the indices of refraction and vibration directions of the slow and fast rays given the wave normal direction followed by the light through the mineral.

The axes of the elliptical section are parallel to the vibration directions of the slow and fast rays and the lengths of radii parallel to those axes are equal to the indices of refraction.



## ISOTROPIC INDICATRIX

One unit cell dimension ( $a$ ) is required to describe the unit cell and one index of refraction ( $n$ ) is required to describe the optical properties because light velocity is uniform in all directions for a particular wavelength of light and therefore birefringence is zero.



## UNIAXIAL INDICATRIX

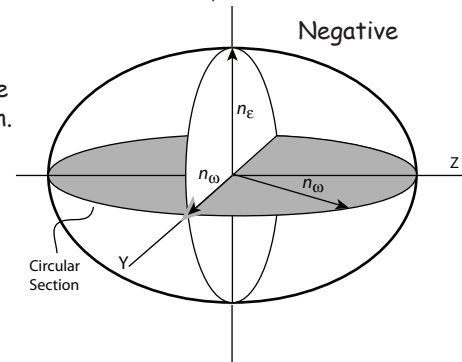
Minerals that crystallize in the tetragonal and hexagonal crystal systems have two different unit cell dimensions ( $a$  and  $c$ ) and two indices of refraction are required to define the dimensions of the indicatrix, which is an ellipsoid of revolution whose axis is the  $c$  crystal axis. The axis of the indicatrix measured parallel to the  $c$  axis is called  $n_\epsilon$ , and the radius at right angles is called  $n_\omega$ . The maximum birefringence of uniaxial minerals is always  $[n_\epsilon - n_\omega]$ . All vertical sections through the indicatrix that include the  $c$  axis are called principal sections. Random sections are ellipses whose dimensions are  $n_\omega$  and  $n_\epsilon'$  where  $n_\epsilon'$  is between  $n_\omega$  and  $n_\epsilon$ . The section at right angles to the  $c$  axis is a circular section (optic axis section) whose radius is  $n_\omega$ . Because hexagonal and tetragonal minerals have a single optic axis, they are called optically uniaxial.

## OPTIC SIGN

In optically positive minerals,  $n_\epsilon$  is greater than  $n_\omega$  and thus the extraordinary rays are slow rays.

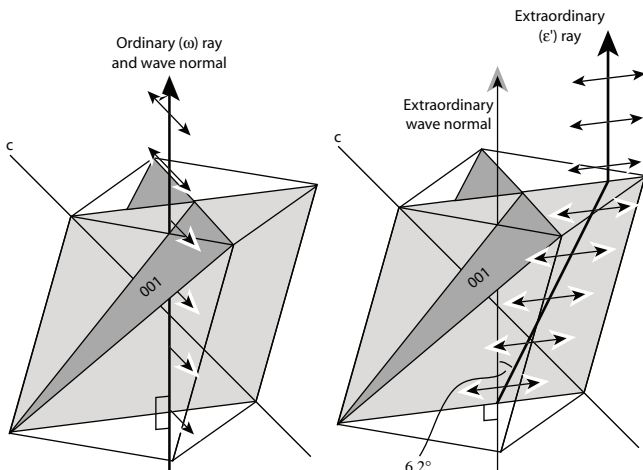
In optically negative minerals,  $n_\epsilon$  is less than  $n_\omega$  and thus extraordinary rays are fast rays.

X = optic axis = c



## ORDINARY RAYS

Ordinary rays or  $\omega$  rays have waves vibrating in the basal plane, which is represented by a sphere and thus all the waves travel the same distance in the same time. They thus behave in an ordinary manner and all have the same velocity and thus the same index of refraction.



## EXTRAORDINARY RAYS

Extraordinary rays or  $\epsilon$  rays have waves vibrating in a principal section, which is represented by an ellipsoid and thus the waves travel different distances in the same time depending on the orientation of the incident beam. For any propagation direction except perpendicular to the  $c$  axis, the index of refraction of the extraordinary ray is designated  $n_\epsilon'$  and is between  $n_\omega$  and  $n_\epsilon$ .

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## BIAXIAL INDICATRIX

Minerals that crystallise in the orthorhombic, monoclinic and triclinic crystal systems require three dimensions (a, b and c) to describe their unit cells and three indices of refraction to define the shape of their indicatrix.

The three principal indices of refraction are  $n_{\alpha}$ ,  $n_{\beta}$  and  $n_{\gamma}$  where  $n_{\alpha} < n_{\beta} < n_{\gamma}$ .

The maximum birefringence of a biaxial mineral is always  $n_{\gamma} - n_{\alpha}$ .

Construction of a biaxial indicatrix requires that three indices of refraction are plotted. However, while three indices of refraction are required to describe biaxial optics, light that enters biaxial minerals is still split into two rays.

As we shall see, both of these rays behave as extraordinary rays for most propagation paths through the mineral.

The wave normal and ray diverge like the extraordinary ray in uniaxial minerals and their indices of refraction vary with direction, the same of the fast and slow rays that we are familiar with.

The index of refraction of the fast ray is identified as  $n_{\alpha}'$  where  $n_{\alpha} < n_{\alpha}' < n_{\beta}$  and the index of refraction of the slow ray is  $n_{\gamma}'$  where  $n_{\beta} < n_{\gamma}' < n_{\gamma}$ .

## BIAXIAL INDICATRIX

The biaxial indicatrix contains three principal sections, the YZ, XY and XZ planes.

The XY section is an ellipse with axes  $n_{\alpha}$  and  $n_{\beta}$ , the XZ section is an ellipse with axes  $n_{\alpha}$  and  $n_{\gamma}$  and the YZ section is an ellipse with axes  $n_{\beta}$  and  $n_{\gamma}$ .

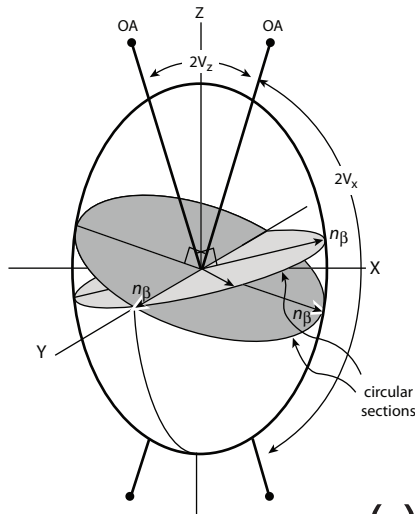
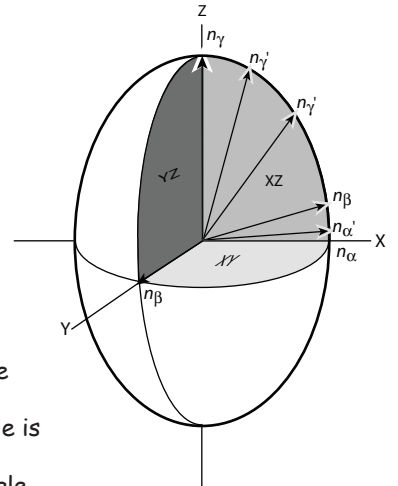
Random sections through the indicatrix are ellipses whose axes are  $n_{\alpha}'$  and  $n_{\gamma}'$ .

The indicatrix has two circular sections with radius  $n_{\beta}$  that intersect the Y axis.

The XZ plane is an ellipse whose radii vary between  $n_{\alpha}$  and  $n_{\gamma}$ . Therefore radii of  $n_{\beta}$  must be present.

Radii shorter than  $n_{\beta}$  are  $n_{\alpha}'$  and those that are longer are  $n_{\gamma}'$ .

The radius of the indicatrix along the Y axis is also  $n_{\beta}$ .



## OPTIC SIGN - BIAXIAL INDICATRIX

The acute angle between the optic axes is called the optic angle or  $2V$  angle.

The axis (either X and Z) that bisects the acute angle is the acute bisectrix or Bxa.

The axis (either Z or X) that bisects the obtuse angle between the optic axes is the obtuse bisectrix or Bxo.

The optic sign of biaxial minerals depends on whether the Z or X indicatrix axis bisects the acute angle between the optic axes.

● If the acute bisectrix is the X axis, the mineral is optically negative and  $2V_x$  is less than  $90^\circ$

● If the acute bisectrix is the Z axis, the mineral is optically positive and  $2V_z$  is less than  $90^\circ$

● If  $2V$  is exactly  $90^\circ$  so neither X nor Z is the acute bisectrix, the mineral is optically neutral.

