- **physics of the very small:**

High energy physics & String theory

- **physics of the very large:**

Astrophysics & Cosmology

- **physics of the very complex:**

Condensed matter physics

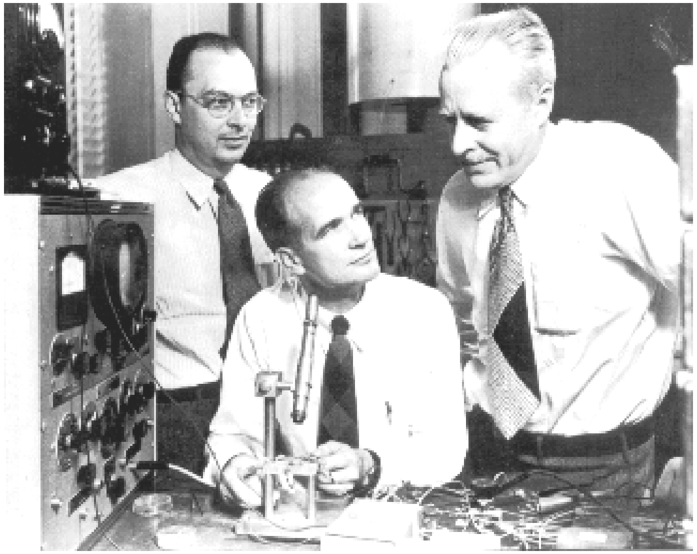
Condensed Matter Physics

Complex behaviour of systems of many interacting particles

"Emergent properties"

The collective behaviour of a system is qualitatively different from that of its constituents

Complexity arises from simple local interactions

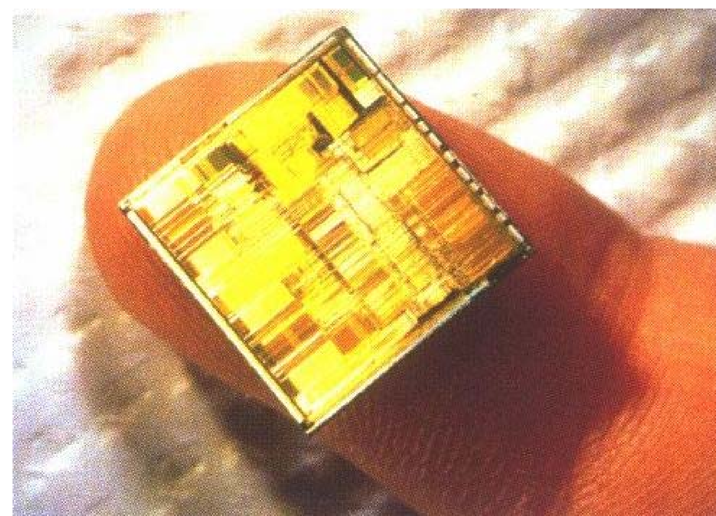


J. Bardeen, W. Shockley
& W. Brattain

The first transistor (1947)

ideas

technology

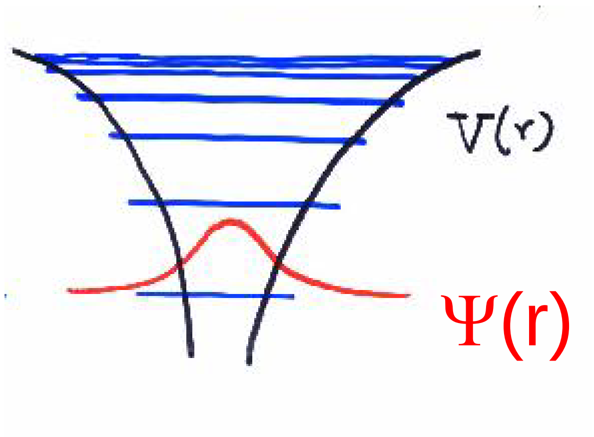


5 million transistors in a Pentium chip

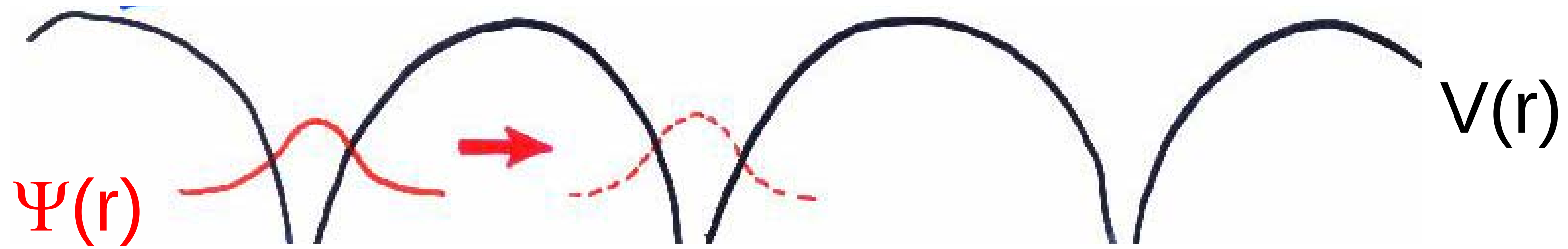
WHAT ARE SOME OF THE BIG QUESTIONS CONDENSED MATTER PHYSICISTS ARE TRYING TO ANSWER???

HOW DO MANY INTERACTING ELECTRONS ORGANISE THEMSELVES AT LOW TEMPERATURE (T=0)??

Isolated Atom:
Discrete energy levels
of bound states



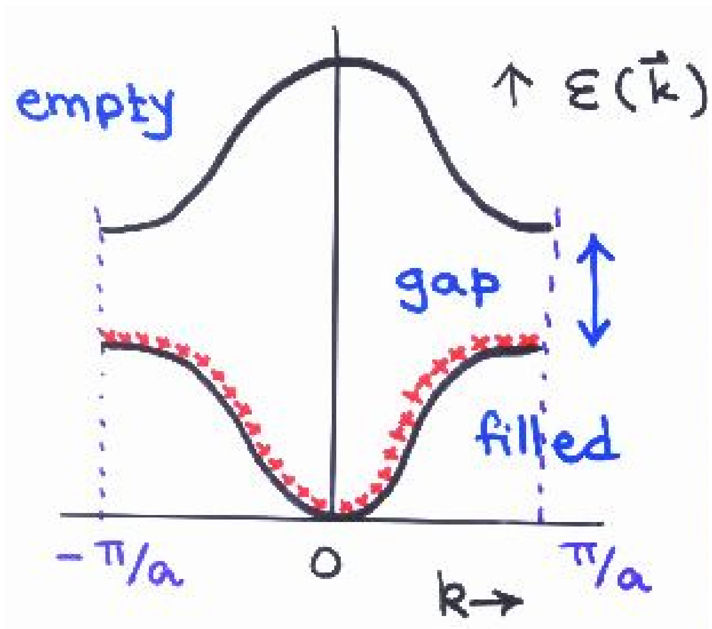
Crystal: periodic array of atoms



- **quantum tunneling**
- **propagating waves**

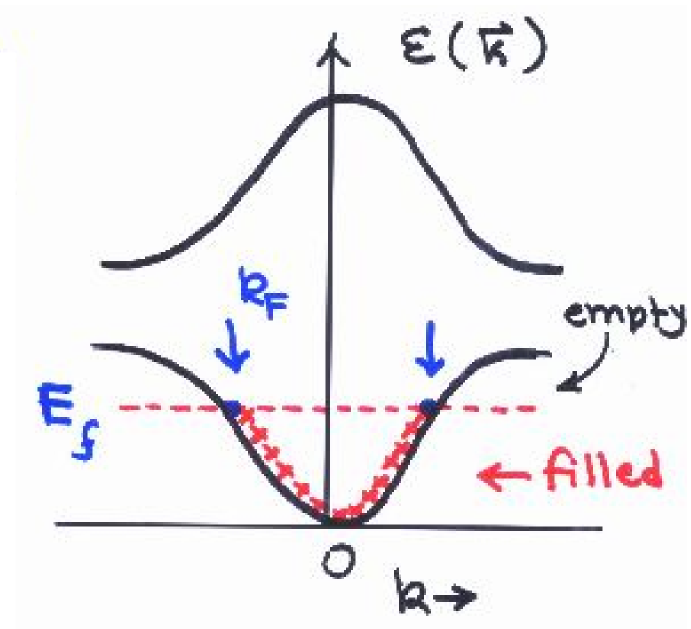
Band Theory: Insulators v/s. Metals

- solve the quantum mechanics of a single electron in a periodic potential
- obtain “bands” and “gaps” of energy levels
- fill up states consistent with Pauli exclusion



Excitation gap

- Insulator



Gapless excitations

- Metal

HIGH TEMPERATURE SUPERCONDUCTIVITY :

INSIGHTS INTO EXPERIMENTS FROM PROJECTED WAVE FUNCTIONS

High Tc Superconductors:

- **Highest transition temperatures** **(still low)**
- **Potential for applications**

Challenge the **basic paradigms** of
20th century solid state physics

- **band theory of metals & insulators**
- **Landau's theory of Fermi-liquids**
- **BCS theory of superconductivity**

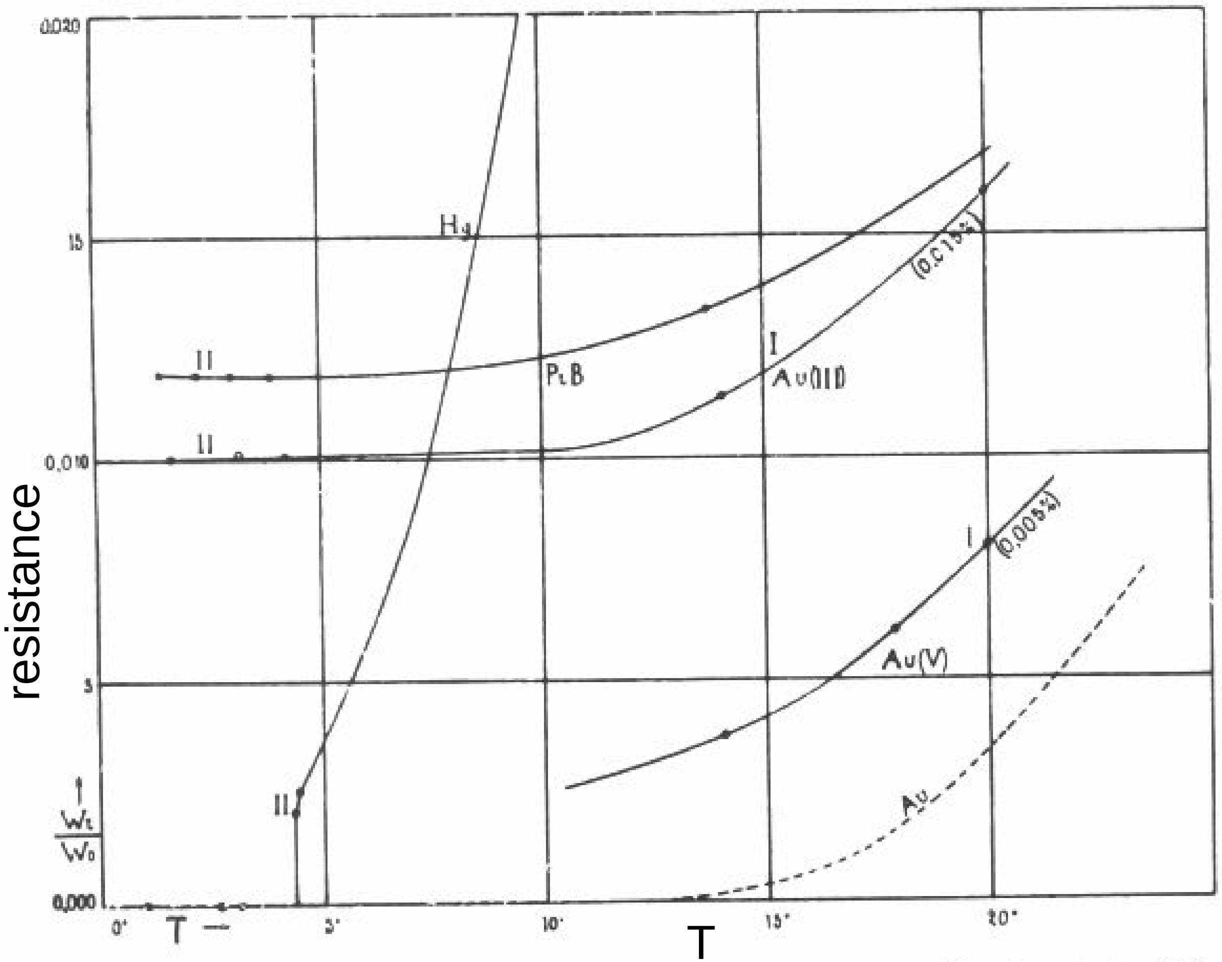


Figure 3.1 Resistance (vertical axis) versus temperature (horizontal axis) for different alloys. The dashed line is Onnes' extrapolation for a pure gold wire. Au stands for gold, Pt for platinum. Also plotted is the result for mercury (Hg) wires. (Courtesy of the Kamerlingh Onnes Laboratorium, Leiden. Copyright permission obtained from Elsevier Press.)

Kammerlingh Onnes
Leiden 1911

Phase transition at Tc:
new state of matter for T<Tc
with zero resistance

SUPERCONDUCTOR

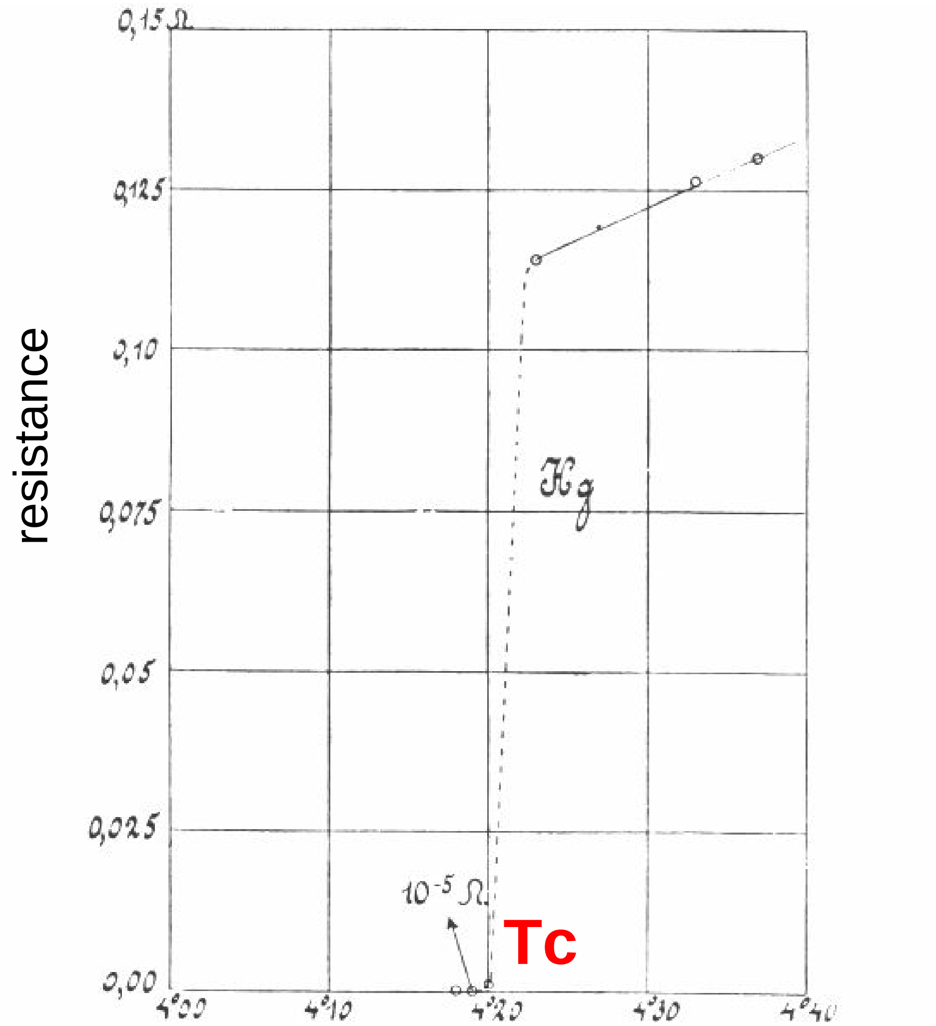


Figure 3.2 Resistance (vertical axis, in units of ohm) versus temperature (horizontal axis, in degrees Kelvin) for mercury (Hg) wires. (Courtesy of the Kamerlingh Onnes Laboratorium, Leiden. Copyright permission obtained from Elsevier Science Publishers.)

Many metals when cooled become Superconducting!

e.g. Al, Pb, Hg, …

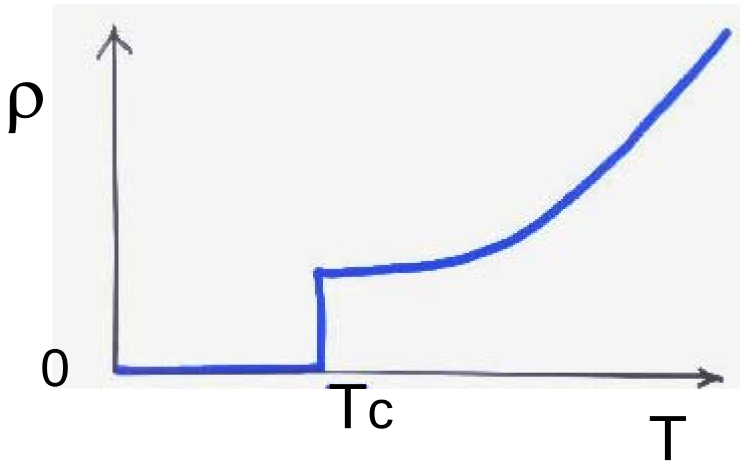
Phase transition at temperature **T_c**

$T > T_c$: normal metal / Landau Fermi liquid

$T < T_c$: Superconductor

typical T_c ~ 1 – 10 K

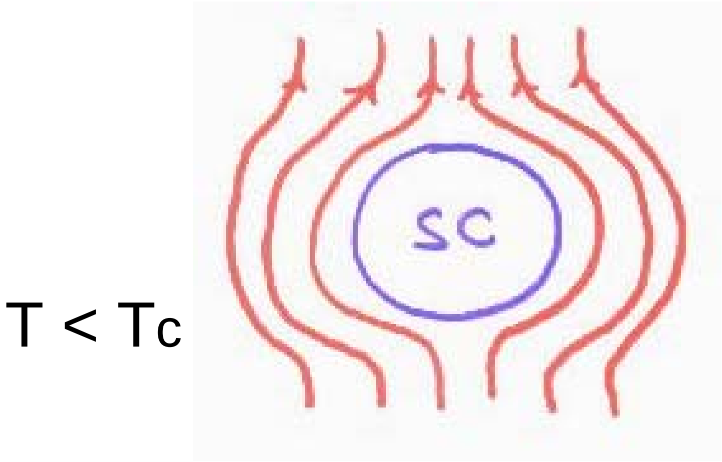
Kammerlingh Onnes (1911)



- **Perfect Conductor**

  $\rho = 0$

Meissner & Oschenfeld (1933)



- **Perfect Diamagnet**

  $B = 0$

BCS Theory of Superconductivity (1957)

J. Bardeen, L. N. Cooper & J. R. Schrieffer

PHYSICAL REVIEW VOLUME 108, NUMBER 5 DECEMBER 1, 1957

Theory of Superconductivity*

J. BARDEEN, L. N. COOPER,† AND J. R. SCHRIEFFER‡
Department of Physics, University of Illinois, Urbana, Illinois

(Received July 8, 1957)

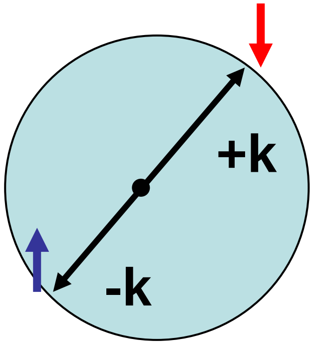
A theory of superconductivity is presented, based on the fact that the interaction between electrons resulting from virtual exchange of phonons is attractive when the energy difference between the electrons states involved is less than the phonon energy, $\hbar\omega$. It is favorable to form a superconducting phase when this attractive interaction dominates the repulsive screened Coulomb interaction. The normal phase is described by the Bloch individual-particle model. The ground state of a superconductor, formed from a linear combination of normal state configurations in which electrons are virtually excited in pairs of opposite spin and momentum, is lower in energy than the normal state by amount proportional to an average $(\hbar\omega)^2$, consistent with the isotope effect. A mutually orthogonal set of excited states in one-to-one correspondence with those of the normal phase is obtained by specifying occupation of certain Bloch states and by using the rest to form a linear combination of virtual pair configurations. The theory yields a second-order phase transition and a Meissner effect in the form suggested by Pippard. Calculated values of specific heats and penetration depths and their temperature variation are in good agreement with experiment. There is an energy gap for individual-particle excitations which decreases from about $3.5kT_c$ at $T=0°$K to zero at T_c. Tables of matrix elements of single-particle operators between the excited-state superconducting wave functions, useful for perturbation expansions and calculations of transition probabilities, are given.

I. INTRODUCTION

THE main facts which a theory of superconductivity must explain are (1) a second-order phase transition at the critical temperature, T_c, (2) an electronic specific heat varying as $\exp(-T_0/T)$ near $T=0°$K and other evidence for an energy gap for individual particle-like excitations, (3) the Meissner-Ochsenfeld effect ($\mathbf{B}=0$), (4) effects associated with infinite conductivity ($\mathbf{E}=0$), and (5) the dependence of T_c on isotopic mass, $T_c\sqrt{M}=\text{const}$. We present here a theory which accounts for all of these, and in addition gives good quantitative agreement for specific heats and penetration depths and their variation with temperature when evaluated from experimentally determined parameters of the theory.

basic. F. London[4] suggested a quantum-theoretic approach to a theory in which it was assumed that there is somehow a coherence or rigidity in the superconducting state such that the wave functions are not modified very much when a magnetic field is applied. The concept of coherence has been emphasized by Pippard,[5] who, on the basis of experiments on penetration phenomena, proposed a nonlocal modification of the London equations in which a coherence distance, ξ_0, is introduced. One of the authors[6,7] pointed out that an energy-gap model would most likely lead to the Pippard version, and we have found this to be true of the present theory. Our theory of the diamagnetic aspects thus follows along the general lines suggested by London and by Pippard.[7]

BCS Theory of Superconductivity



Pairing and Condensation

Weak attraction 'g' between electrons
(electron-phonon interaction in conventional SC's)

instability of the Fermi liquid:

S = **0, L** = **0 Pairs**

Binding energy of pairs **= Energy gap**

$\Delta_0 = \omega_0 \exp(-1/g)$

2nd order phase transition at Tc ~ Δ_0

Experimental observation of gap Δ**:**

Thermodynamics**: C(T);** $\chi(T)$

Spectroscopy**: tunneling, NMR, optics**

Order parameter:

$\Delta = |\Delta_0| \exp(i\,\theta)$

gap

Pairing and Condensation

Macroscopic number ($\sim 10^{20}$) of pairs condense into a single quantum state

(cf. Bose-Einstein condensation)

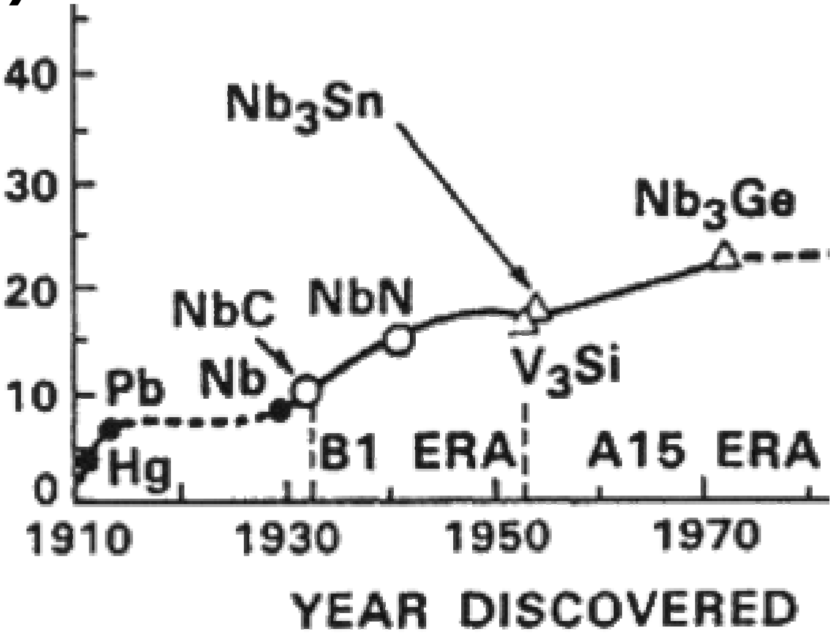
Superfluid stiffness

Experimental manifestation of θ:

- **superconductivity: $\rho = 0$**
- **Flux expulsion: $B = 0$**
- **Josephson Effects**

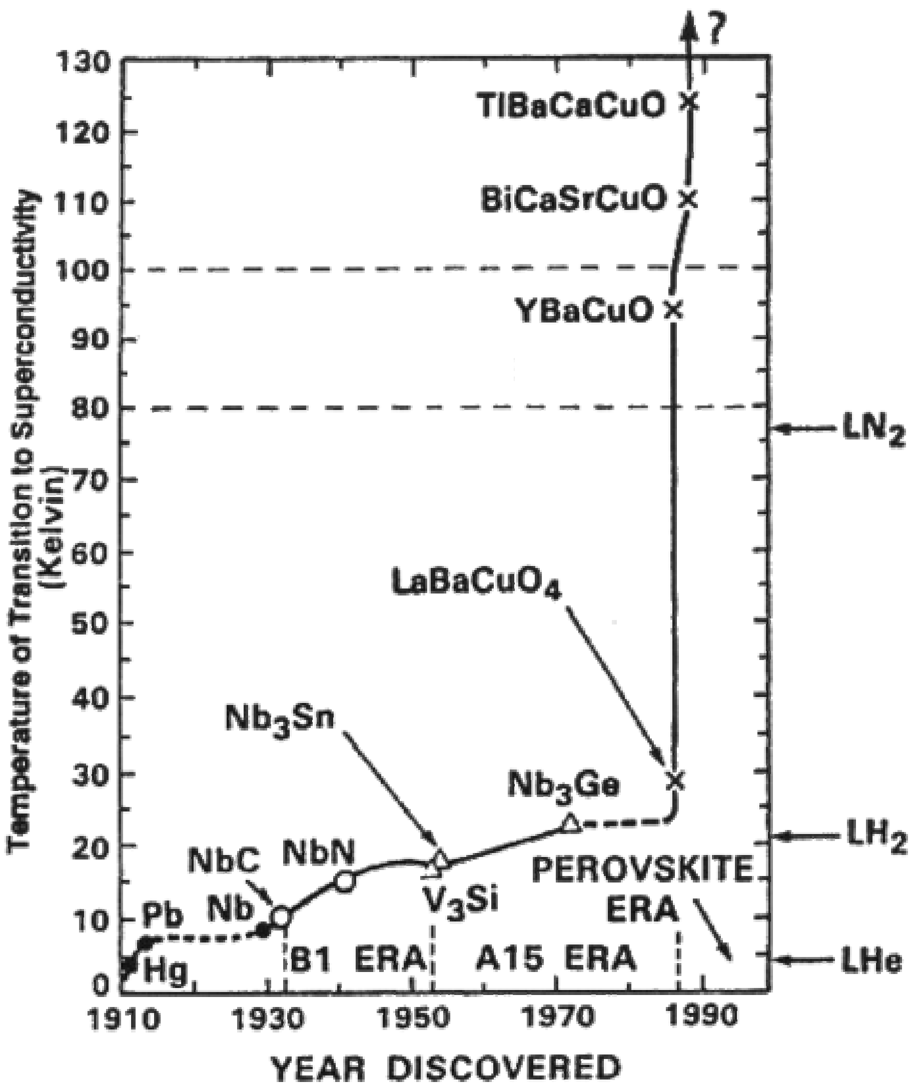
By the late 1970's
BCS and Ginzburg-Landau Theories solved Superconductivity

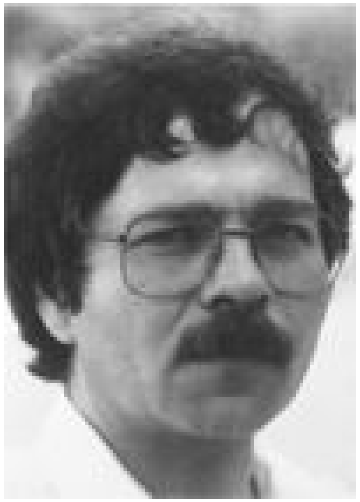
Tc(K)



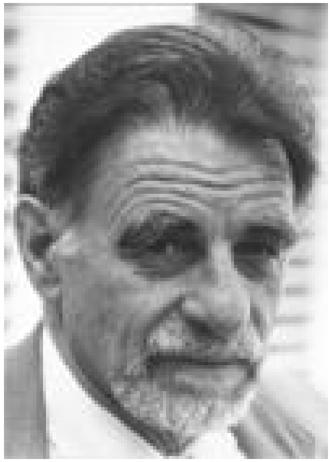
Plateau in
the search for
higher Tc materials

Superconductivity in cuprates (1986)





J. Bednorz



K. A. Muller

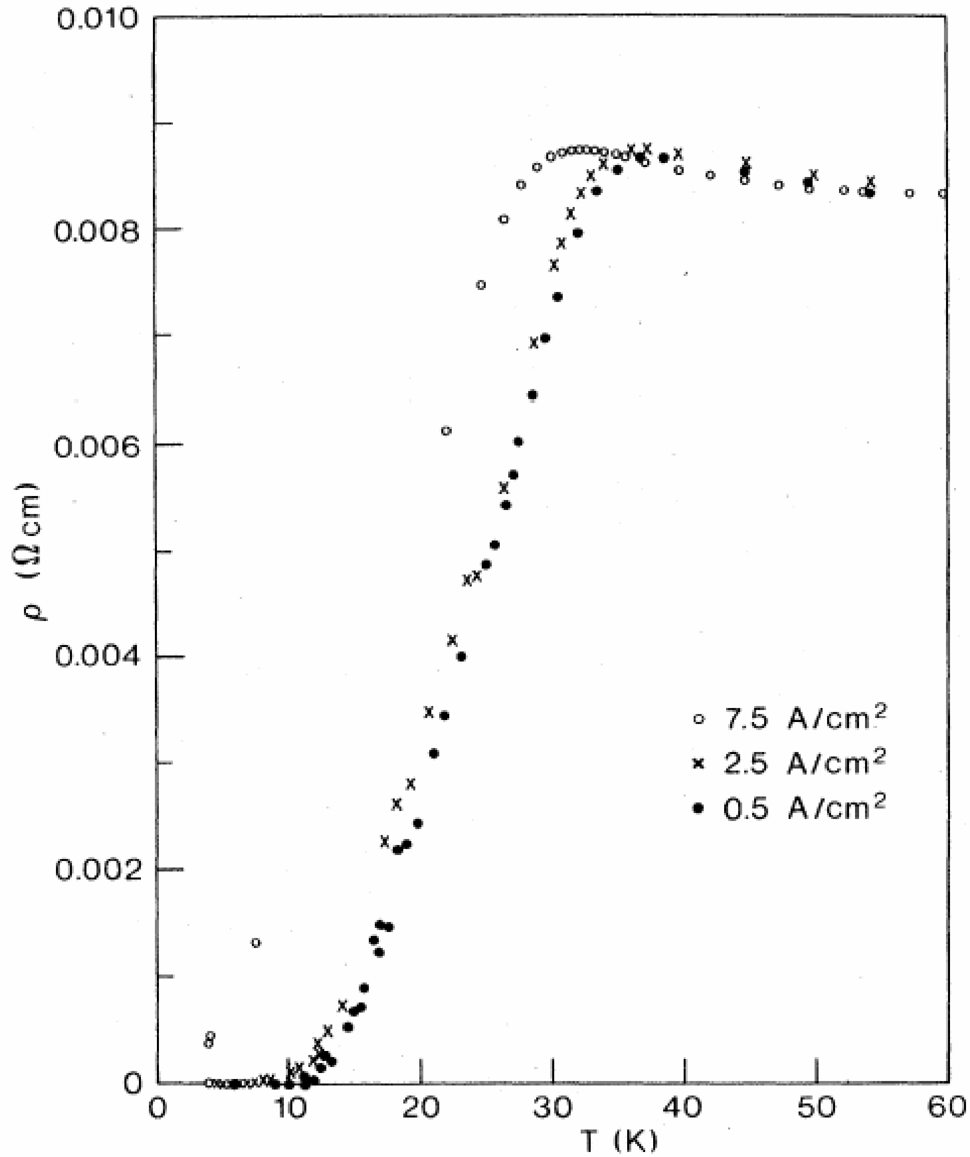


FIG. 5. Low-temperature resistivity of a sample with x(Ba)=0.75, recorded for different current densities. From Bednorz and Müller (1986), © Springer-Verlag 1986.

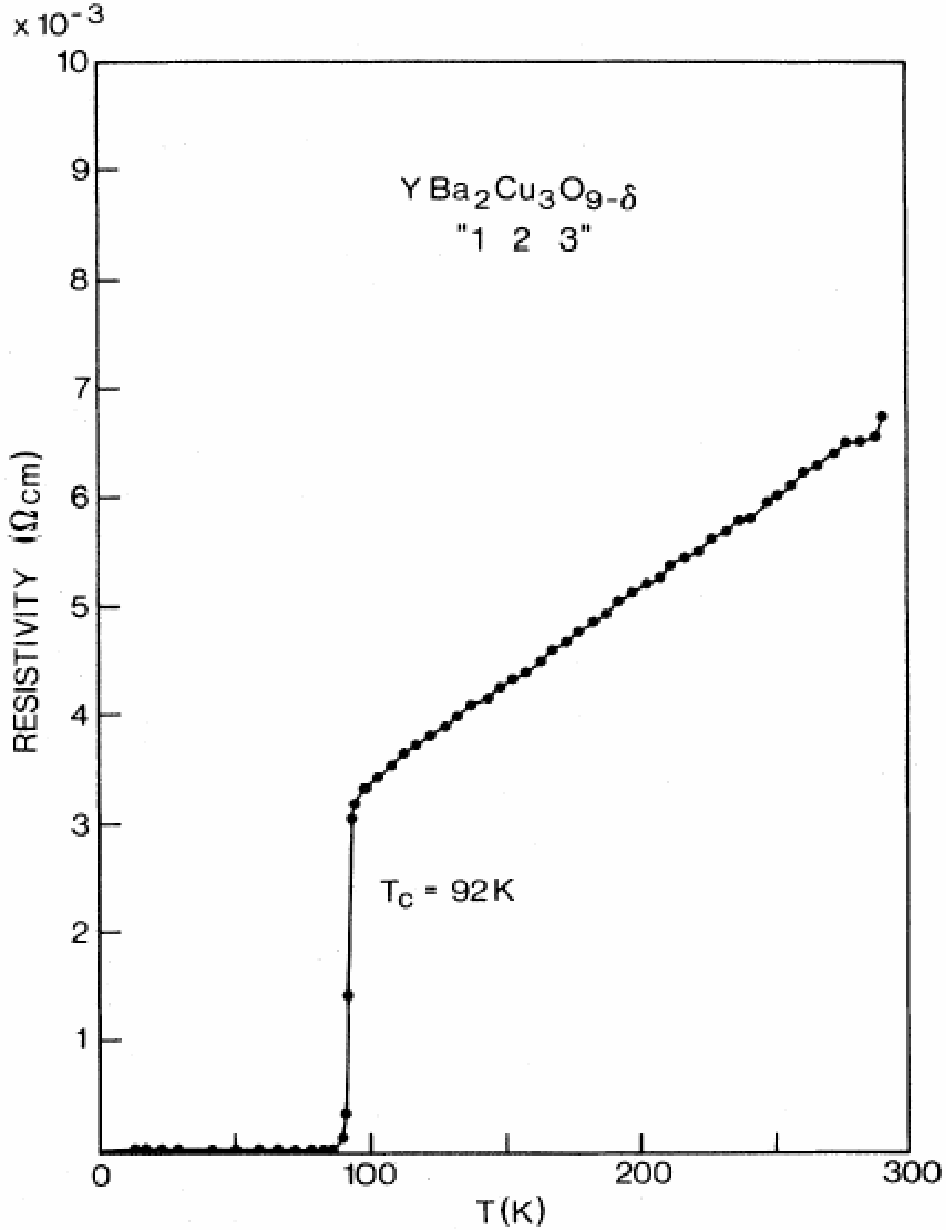


FIG. 14. Resistivity of a single-phase $YBa_2Cu_3O_7$ sample as a function of temperature.

Nobel Prizes in Superconductivity & Superfluidity

H. KAMERLINGH-ONNES investigations on the properties of matter at low temperatures which also led to the production of liquid helium (1913).

L. D. LANDAU pioneering theories for condensed matter, especially liquid helium (1962)

J. BARDEEN, **L. N. COOPER** and **J. R. SCHRIEFFER** BCS theory of superconductivity (1972)

B. D. JOSEPHSON prediction of Josephson effects (1973).

I. GIAEVER tunneling in superconductors (1973)

P. L. KAPITSA low-temperature physics (1978)

J. G. BEDNORZ and **K. A. MÜLLER** High Tc superconductors (1987).

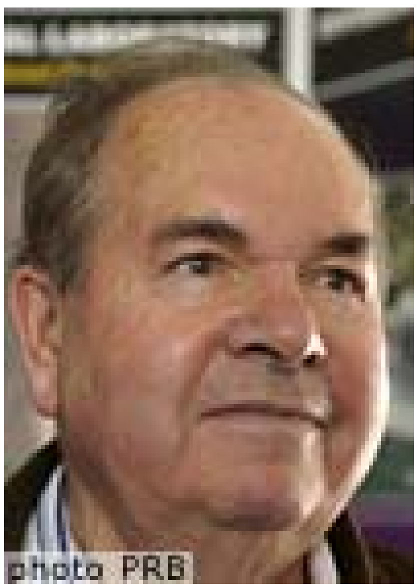
D. M. LEE, **D. D. OSHEROFF** and **R. C. RICHARDSON** discovery of superfluidity in helium-3 (1996).

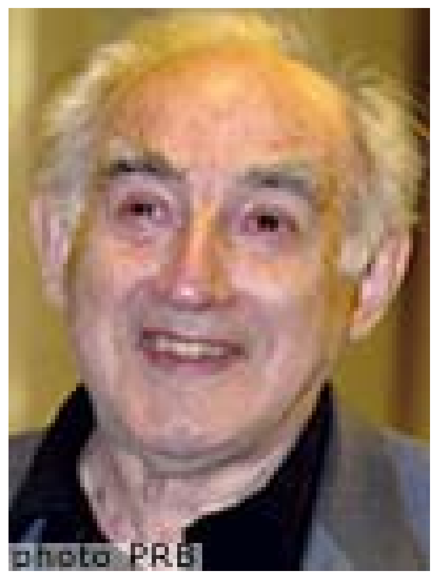
E. A. CORNELL, **W. KETTERLE** and **C. E. WIEMAN** Bose-Einstein condensation in dilute gases of alkali atoms (2001).

L. Onsager, R. P. Feynman, C. N. Yang, P. W. Anderson, P. G. deGennes

The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"





Alexei A. Abrikosov

Vitaly L. Ginzburg

Anthony J. Leggett

to levitated trains!

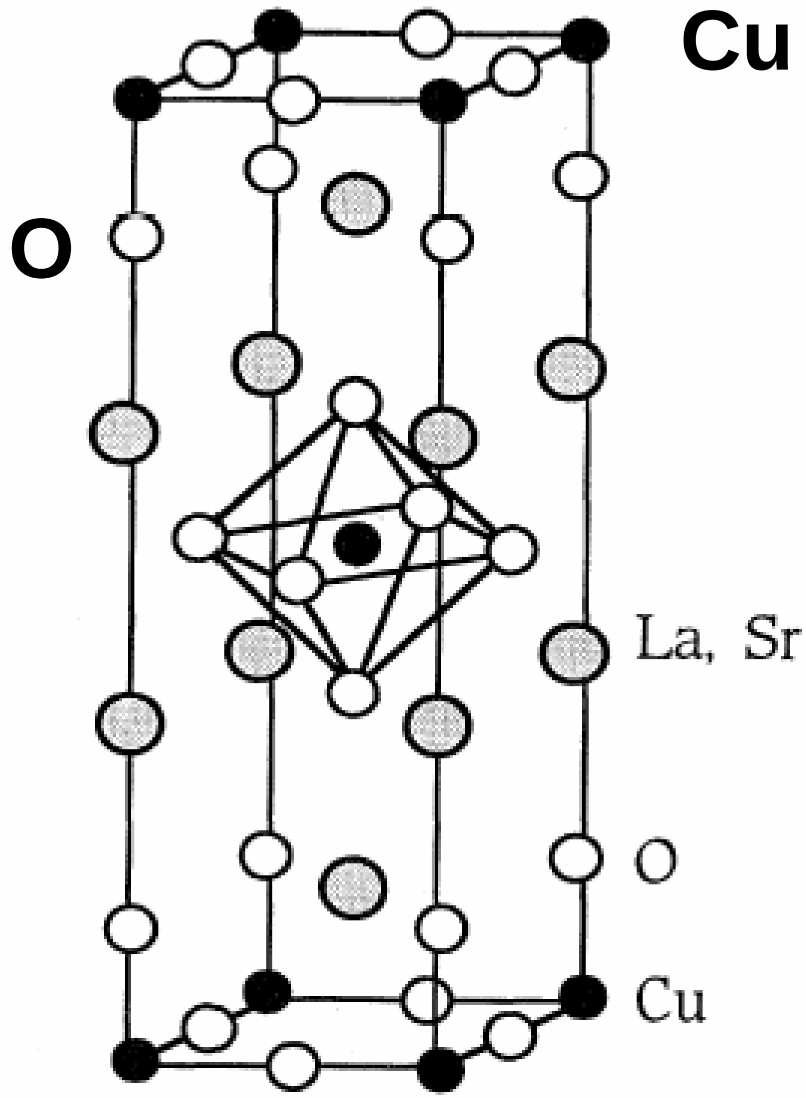


FIG. 1. Crystal structure of $La_{2-x}Sr_xCuO_4$ (T phase). Taken from Almasan and Maple (1991).

Material	T_c (K)
$H_g Ba_2Ca_2Cu_3O_{8+\delta}$	133
$Tl_2Ca_2Ba_2Cu_3O_{10}$	125
$YBa_2Cu_3O_7$	92
$Bi_2Sr_2CaCu_2O_8$	89
$La_{1.85}Sr_{0.15}CuO_4$	39
$Nd_{1.85}Ce_{0.15}CuO_4$	24
$RbCs_2C_{60}$	33
Nb_3Ge	23.2
Nb	9.25
Pb	7.20
UPt_3	0.54

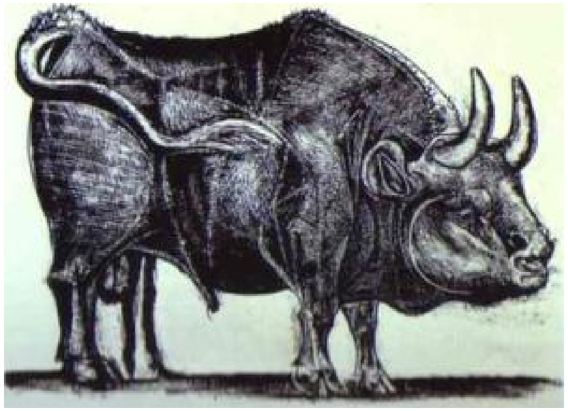
Extracting a Model

Anisotropic structure & electronic structure

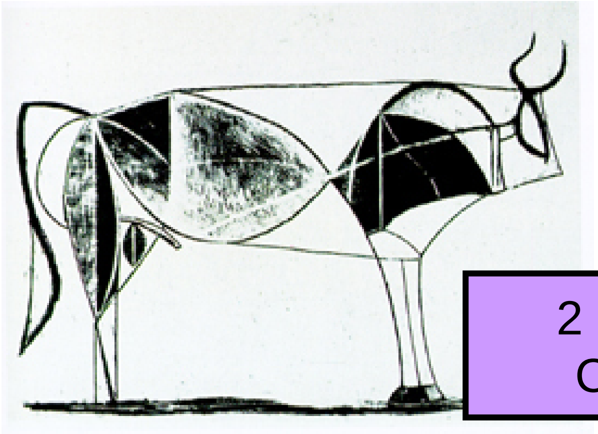
main action on 2 dimensional Cu-O planes

2D square lattice

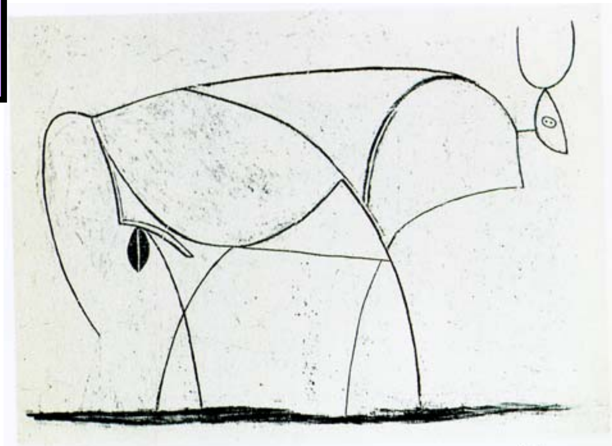
One band Hubbard model



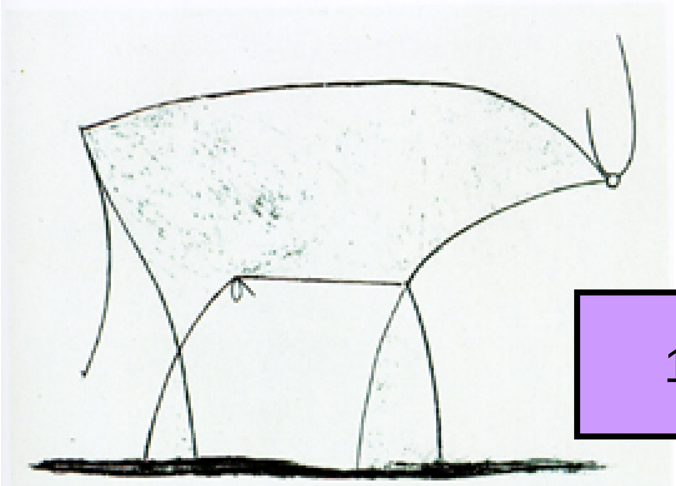
High tc problem



2 dimensions
Cu-O plane

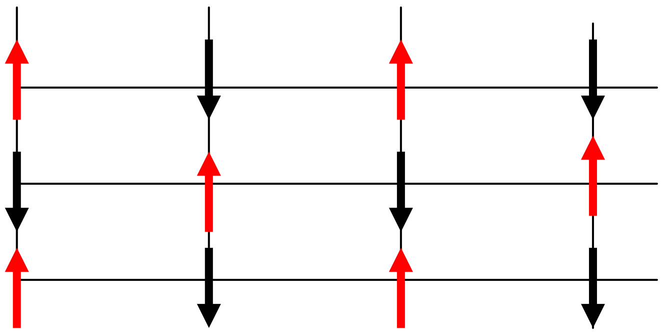


3 band model



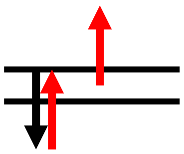
1 band model

HUBBARD MODEL

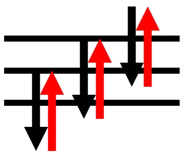


<n>=1

La_2CuO_4



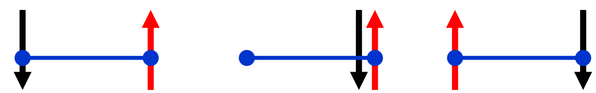
$d_{x^2-y^2}$



$$H = -t \sum_{<i,j>\sigma} c^{+}{}_{i\sigma} c_{j\sigma} + U \sum_i n_{i_\uparrow} n_{i_\downarrow}$$

MOTT insulator: Finite gap in spectrum

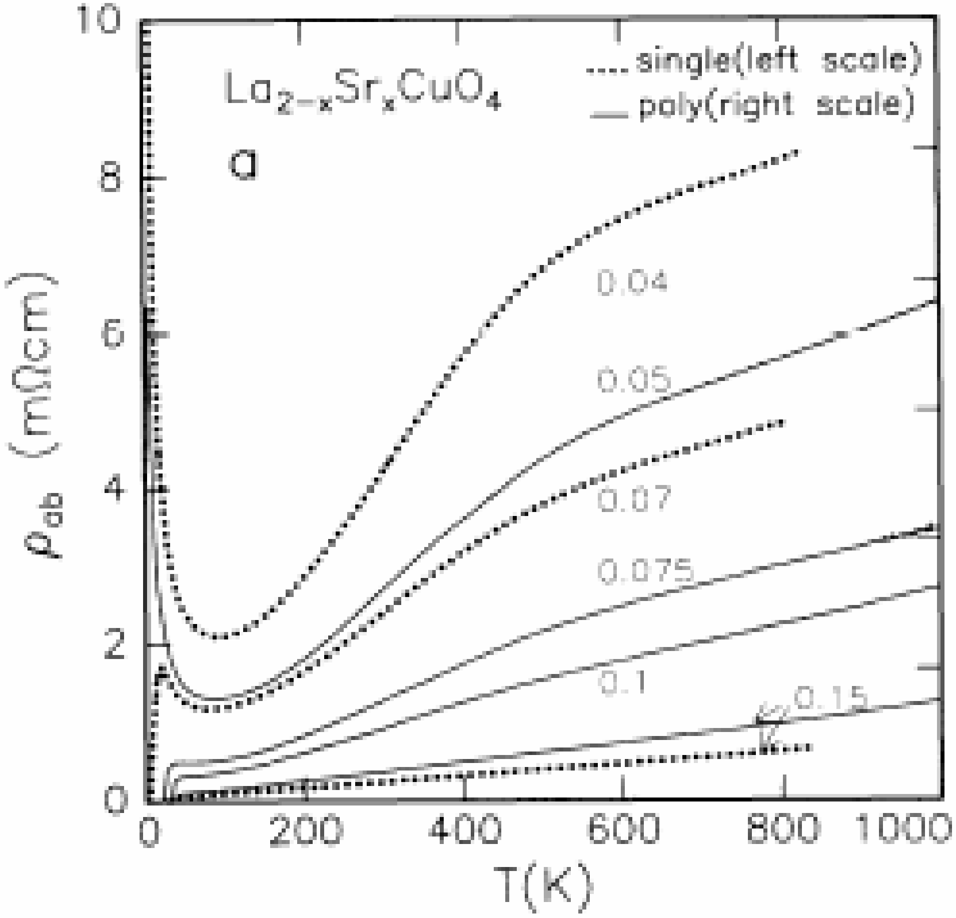
$$\lambda \equiv t/U << 1 \qquad H_{\lambda<<1} = J \sum_{<i,j>} \vec{S}_i \cdot \vec{S}_j \qquad J \sim t^2/U$$



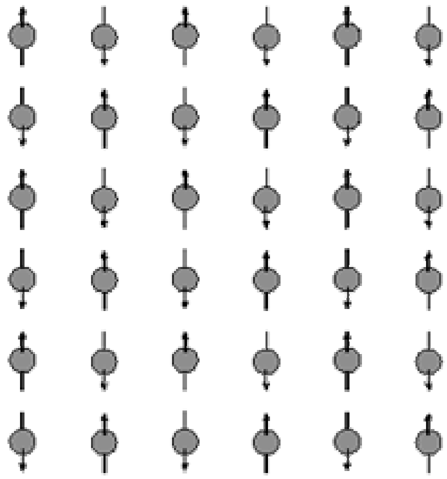
Heisenberg Model

Antiferromagnetic long range order

Mott Insulator



Ignoring Interactions
metal
Experiment: La**2**CuO**4**
Insulator!

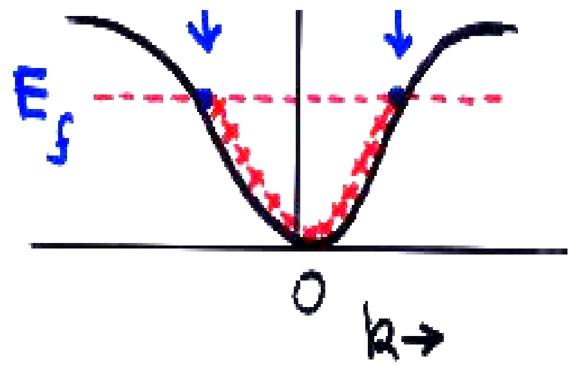


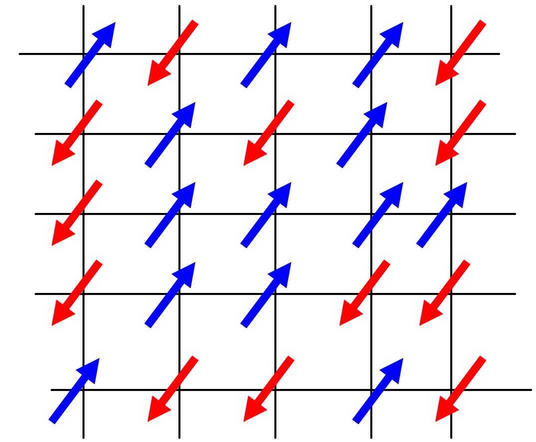
Strong Coulomb Interaction U
Half-filled in **r**-space: one el./site

Mott Insulator:
Antiferromagnet
Gap ~U

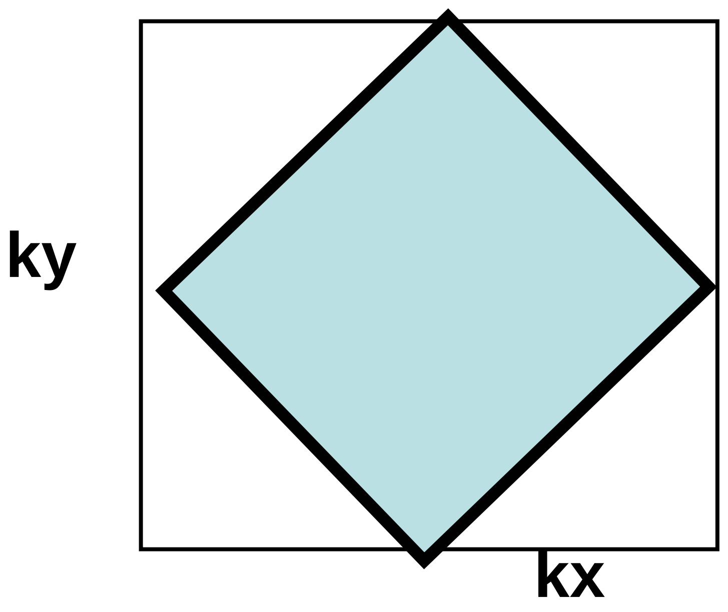
Failure of Band Theory: Mott Insulators

Band theory
La_2CuO_4:
Half-filled
in **k**-space

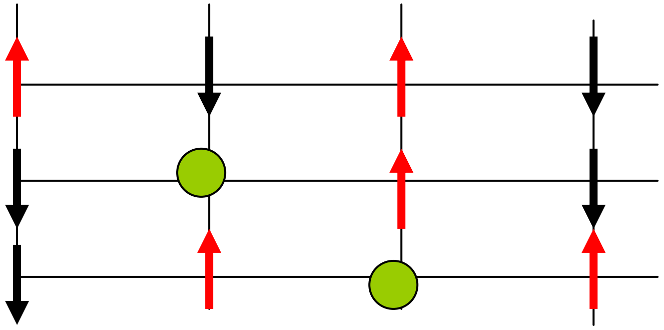




r-space

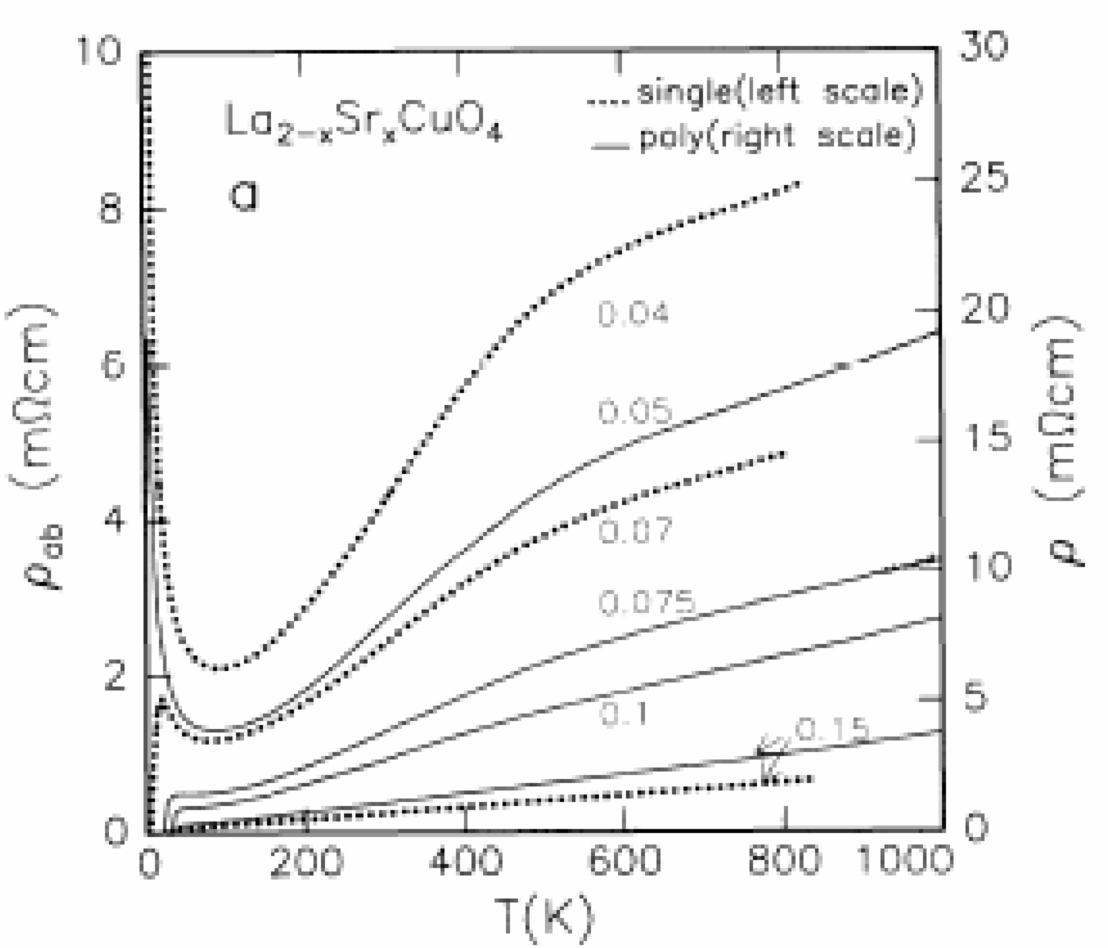


$$La_{2-x}Sr_xCuO_4$$



What happens when there are holes?

Mott Insulator



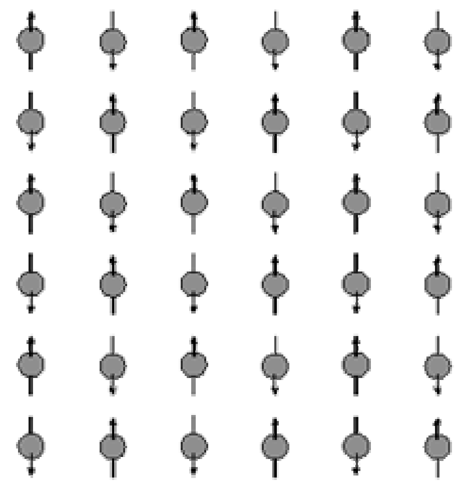
Strong Coulomb Interaction U

Half-filled in **r**-space: one el./site

Ignoring Interactions metal

Experiment: $La_{\mathbf{2}}CuO_{\mathbf{4}}$

Insulator!



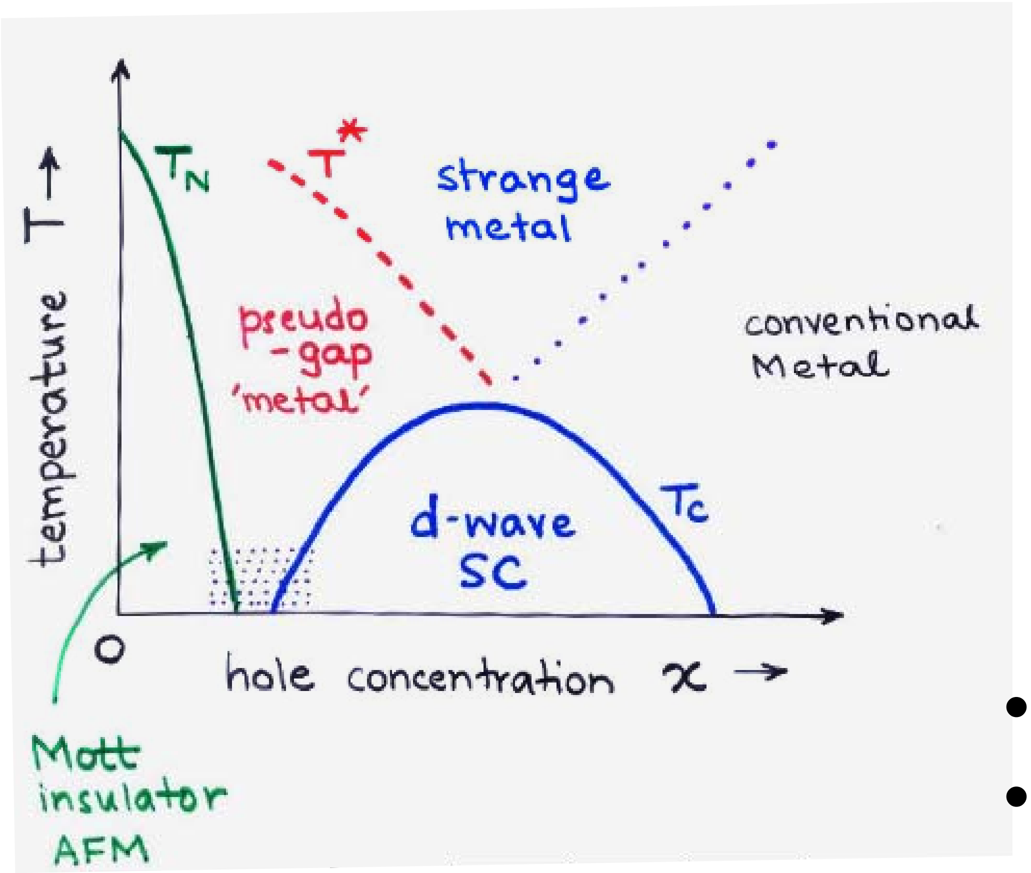
Mott Insulator:

Antiferromagnet

Gap ~U

High Temperature Superconductors:

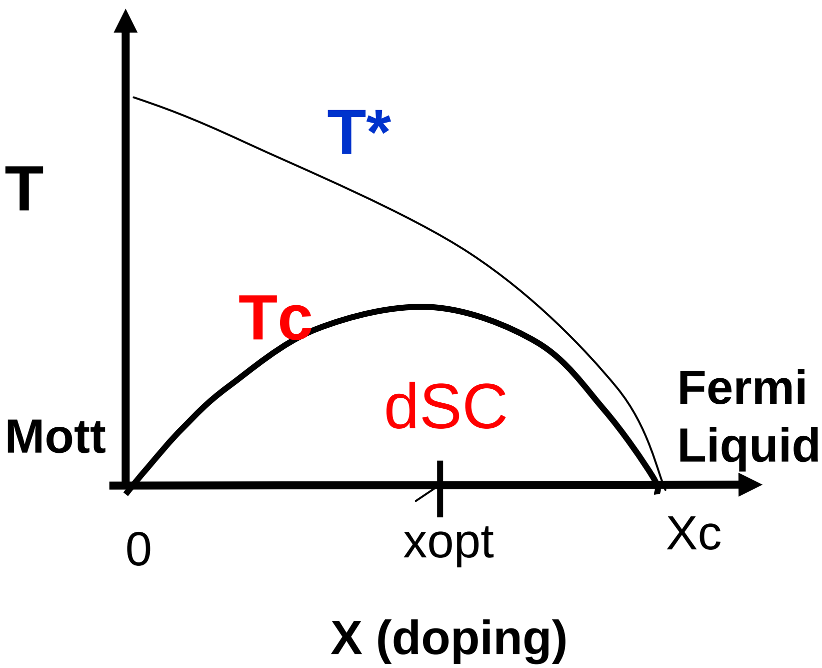
schematic phase diagram



- **novel phases**
- **unusual phase transitions**
- **unusual crossovers**

Our Philosophy

- Look at the strongly correlated SC state by itself; not as an instability from another state
- Look at instabilities out of the SC state
- Minimal model to understand
- Systematically build up to get entire complexity of the cuprates



INPUTS strong correlations between electrons

APPROACH Variational Wave Functions

No obvious small parameter

success stories:

BCS : SC
Laughlin: FQHE
Feynman: He4

Anderson suggested projected BCS wave functions in 1987 for hitc

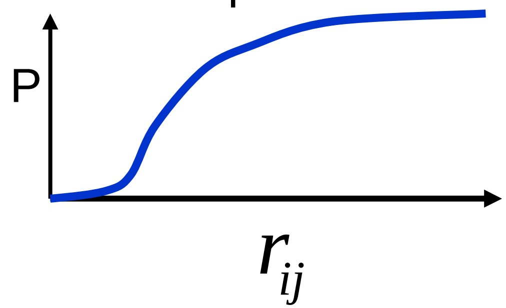
Anderson, Science 235, 1196 (1987)

how do we construct wave functions for correlated systems?

$$\left|\phi_{bose}\right\rangle = \left(a^{+}_{k=0}\right)^{N}\left|0\right\rangle$$ = uniformly spread out in real space

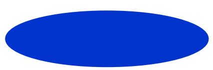
What is the w.f for bosons with repulsive interactions?

$$\left|\psi_{\text{int}\,bosons}\right\rangle = \prod_{i<j}$$

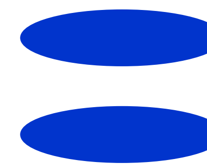


Correlation physics:
Jastrow factor

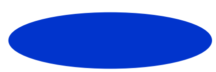
$$\left|\phi_{1}^{IQHE}\right\rangle =$$



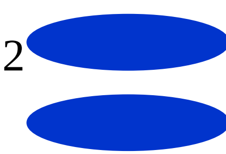
$$\left|\phi_{2}^{IQHE}\right\rangle =$$



$$\left|\phi_{1/3}^{FQHE}\right\rangle = \prod_{i<j}\left(z_i - z_j\right)^2$$



$$\left|\phi_{2/5}^{FQHE}\right\rangle = \prod_{i<j}\left(z_i - z_j\right)^2$$



Jastrow correlation factor
Keeps electrons further apart

how do we construct wave functions for correlated systems?

$$\left|\psi_{BCS}\right\rangle = \sum_k \left(\phi(k) c^{+}_{k\uparrow} c^{+}_{-k\downarrow}\right)^{N/2} \left|0\right\rangle$$

$$\left|\psi_0\right\rangle = P\left|\psi_{BCS}\right\rangle$$

Explains the phenomenology of correlated SC in hitc

THE PROPERTIES OF $\left|\psi_0\right\rangle$

ARE COMPLETELY DIFFERENT FROM THOSE OF $\left|\psi_{BCS}\right\rangle$