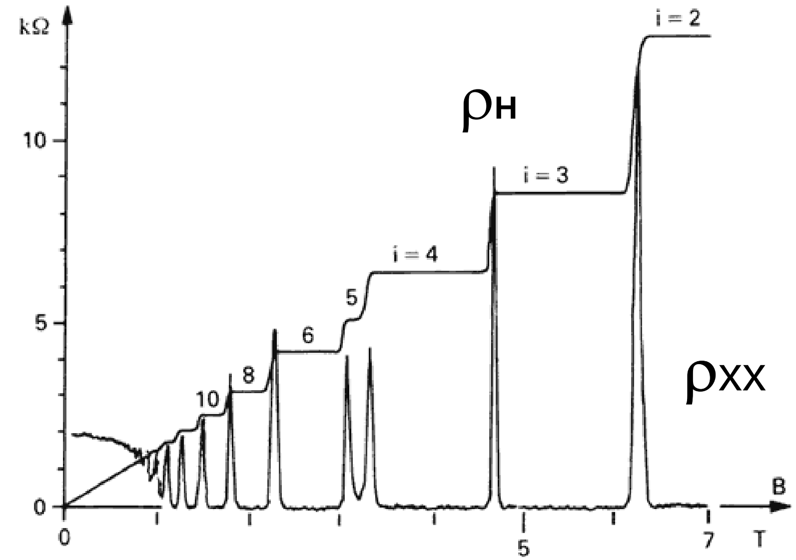
New Phases in Disordered Quantum Systems

DISORDER: yuch!! ∃

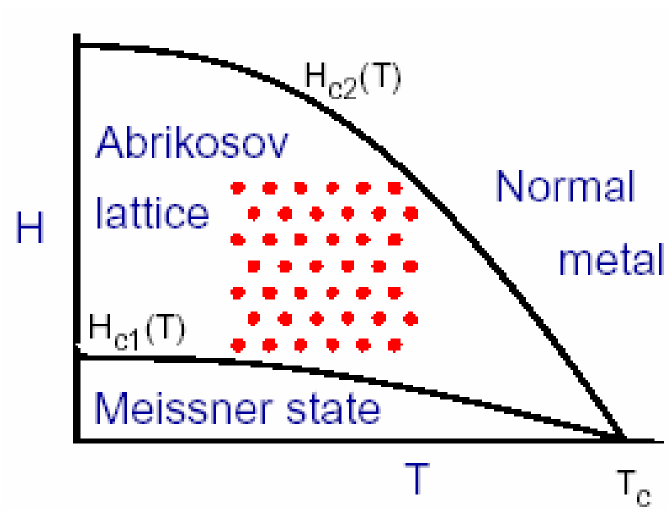
NEW PHENOMENA

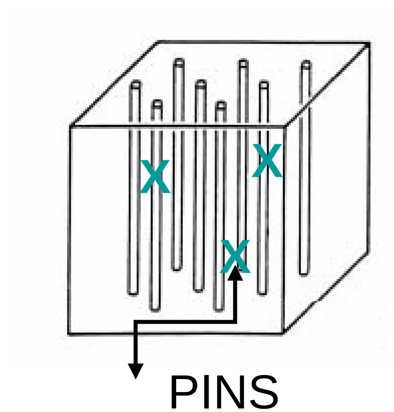
Quantum Hall Effect



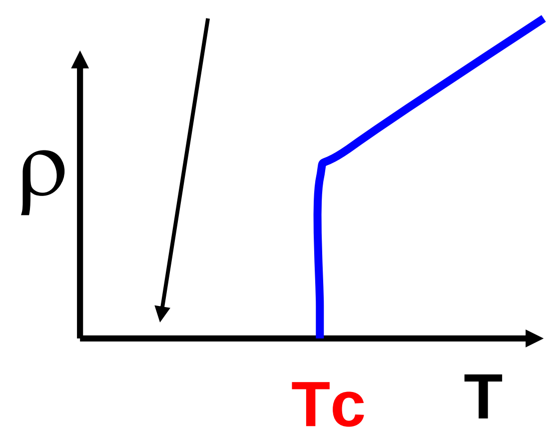
Quantization to 1 part in 10^8
ONLY if some disorder in sample

Superconductivity with vortices





ρ=0 only if vortices are pinned



T=0 Quantum Phase Transitions

Qualitative change of the ground state wave function by tuning a parameter:

Trapped atomic gases:

- -- density
- -- periodic potential
- -- interactions

Solid State Systems:

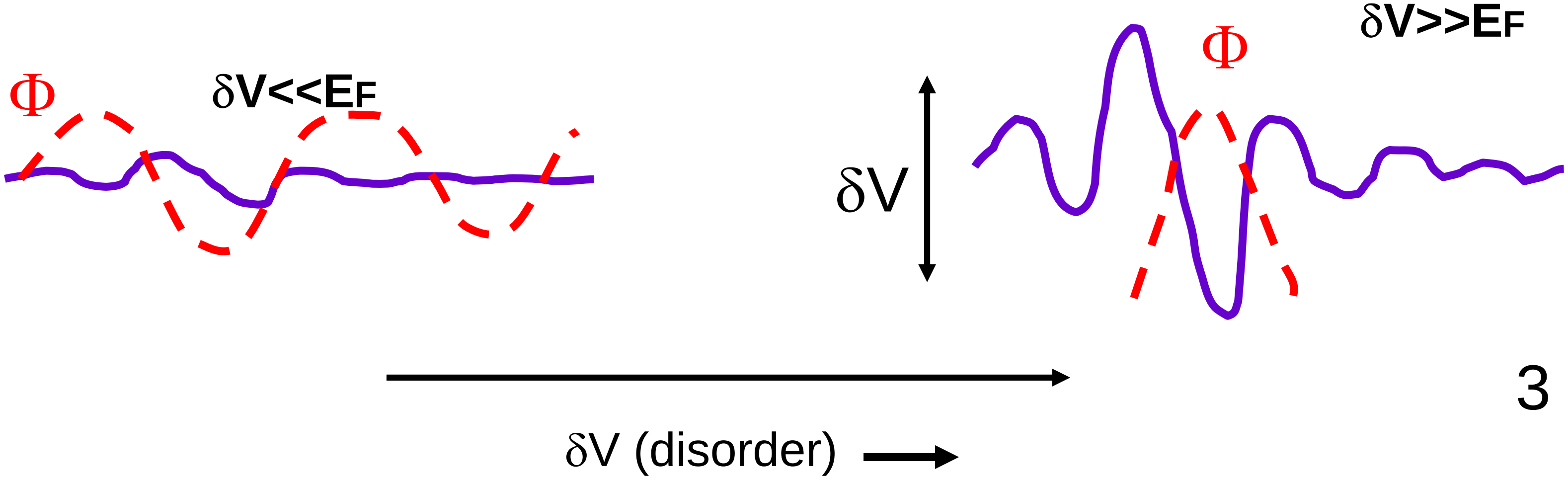
- -- density
- -- magnetic field
- -- pressure
- -- disorder

Main questions: nature of phases and excitations;
nature of phase transitions

Simplest disorder driven quantum phase transition
Anderson Localization (1958)

non interacting electrons
in a random potential

$$H = p^2 / 2m + V(r)$$



3 dim

CONDUCTOR

ANDERSON INSULATOR

Extended wave function
Sensitive to boundary conditions

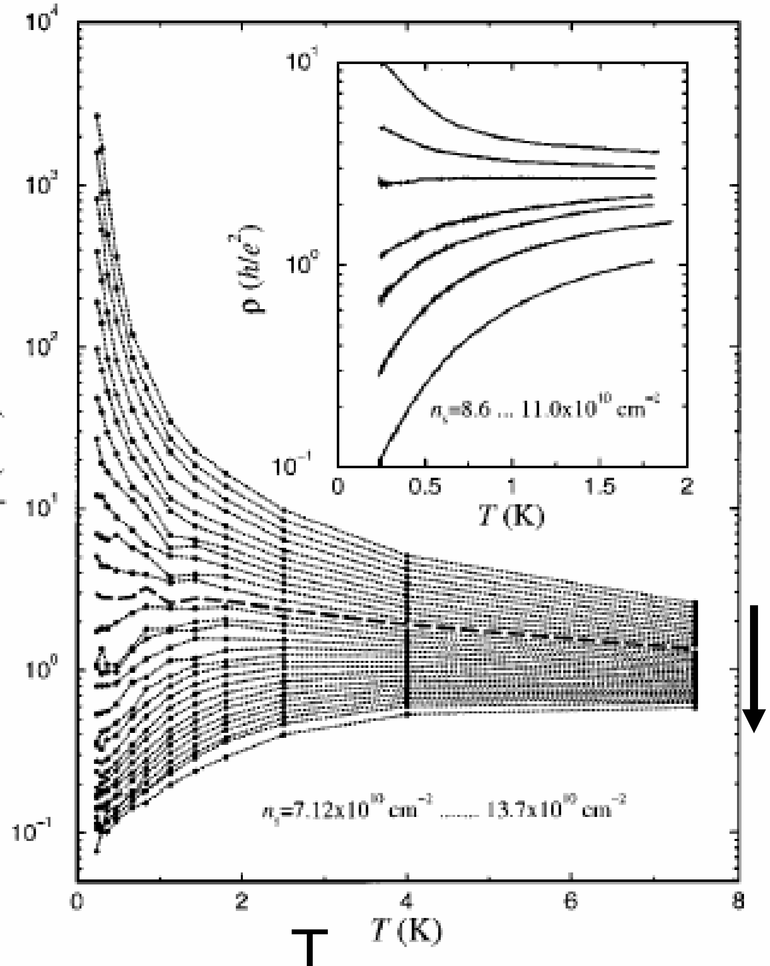
Localized wave function
Insensitive to boundaries

2d: All states are localized; No true metals in 2d

(Abrahams et.al PRL 1979)

METALS IN 2D ?

INSULATOR

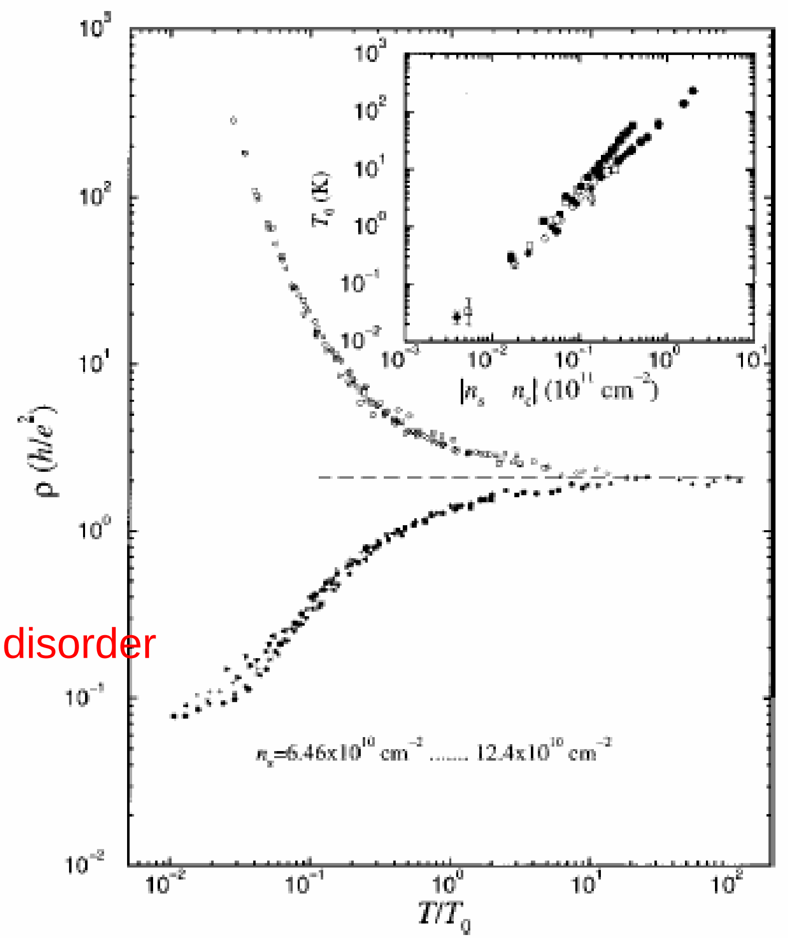


ρ

T

n

disorder



“METAL”

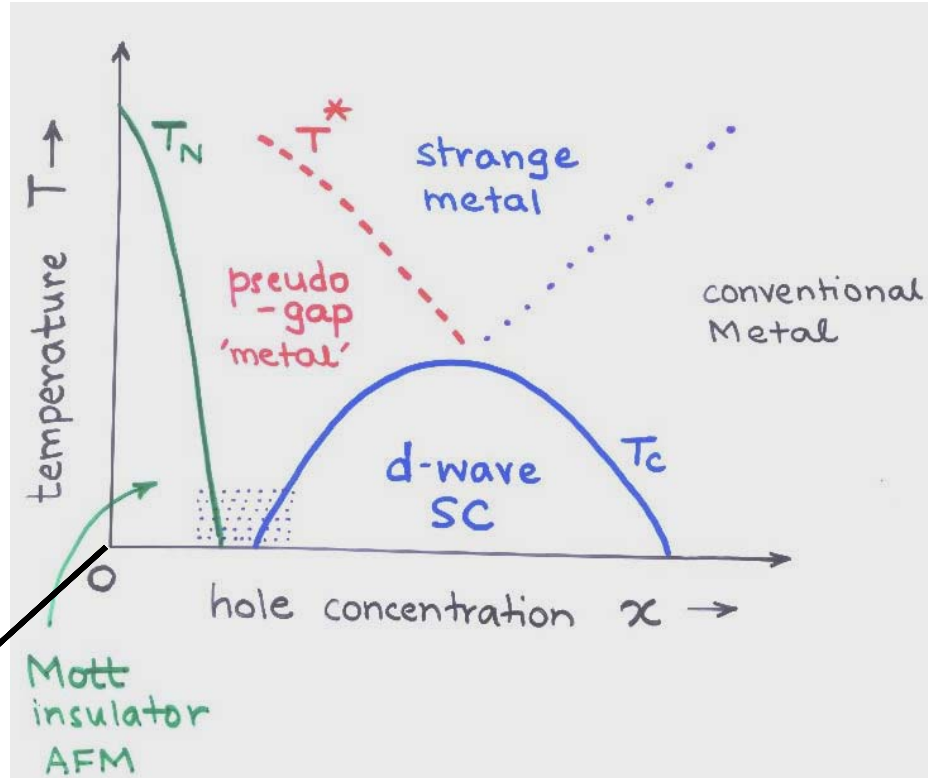
E. Abrahams, S. Kravchenko, M. Sarachik
Rev. Mod. Phys. 73, 251 (2001)

EXPERIMENTS

In condensed matter systems interactions cannot be ignored;
By localizing particles,
disorder only enhances the interactions

repulsive interactions: superfluids (bosons)
metals (fermions)
attractive interactions: superconductors (fermions)

Effect of Disorder on a Mott Insulator



MAIN QUESTION:

What is the effect of disorder on AFM long range order?
on charge gap?
Which is killed first?
Or are they destroyed together…

Model: Hubbard model + potential disorder

$H =$ Kinetic energy $$-t\sum_{<i,j>\sigma} c_{i\sigma}^{\;+} c_{j\sigma} + h.c.$$

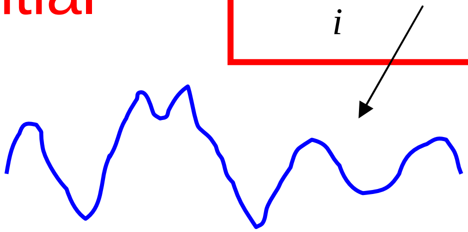
d=2

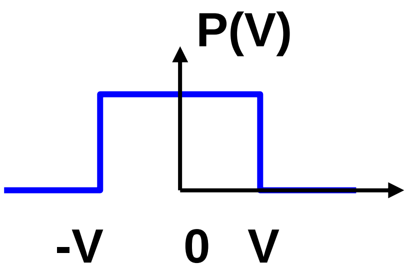
+

Repulsion $$+U\sum_{i} n_{i\uparrow} n_{i\downarrow}$$

+

Random potential $$+\sum_{i} (V_i - \mu) n_i$$





<n>=1

V=0   Mott insulator with finite charge gap and long range AFM order

U=0   localization problem of non-interacting electrons

Techniques:

No small parameter – need NONPERTURBATIVE methods

Field theoretic techniques for clean systems

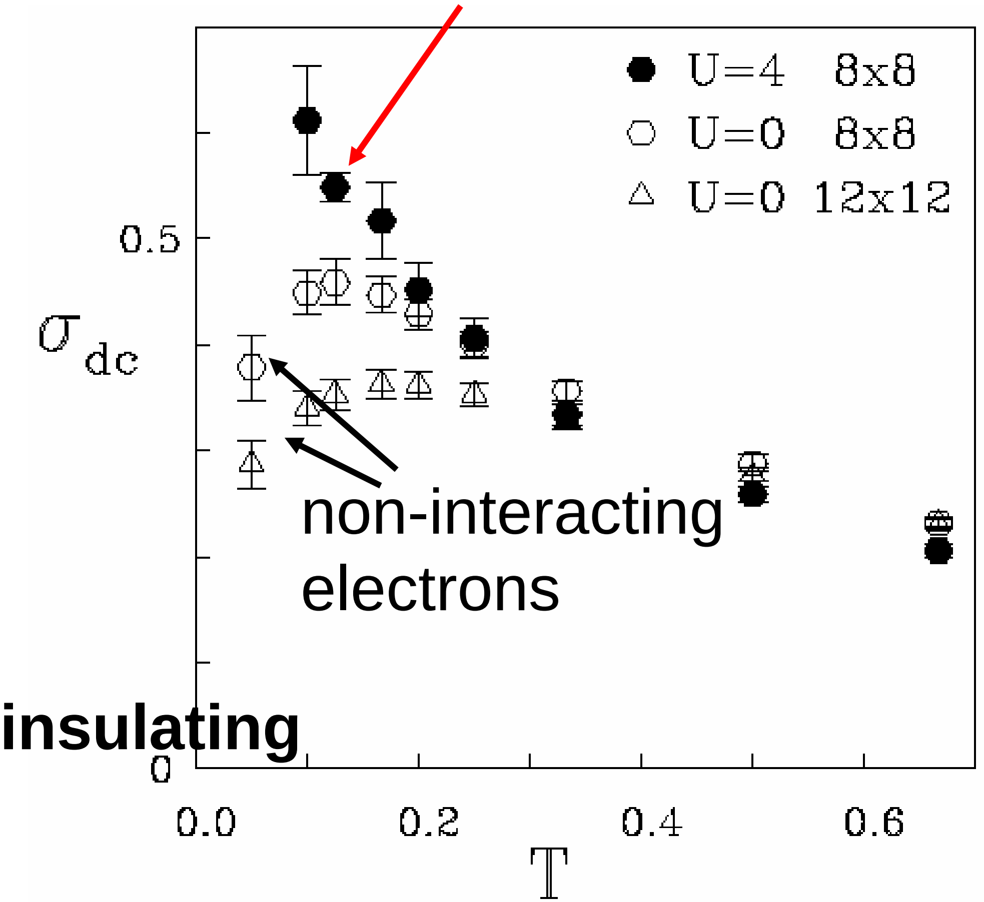
(Sachdev, "Quantum Phase Transitions")

Disordered Systems:

- numerical Quantum Monte Carlo techniques
- inhomogeneous mean field methods
- sum rules and bounds

INTERACTING ELECTRONS WITH DISORDER IN 2D

Interacting electrons



Interplay of interactions and disorder enhance conductivity

Can it drive the system metallic?

Technique:
Coherent state path integral for fermions

Sign problem at low T

P.J.H. Denteneer, R.T. Scalettar, N. Trivedi,
PRL **83**, 4610 (1999); PRL 87, 146401 (2001)

QMC SIMULATIONS

INHOMOGENEOUS MEAN FIELD THEORY

- STRENGTH OF INTERACTION U
- DENSITY OF ELECTRONS
- DISORDER PROFILE (RANDOM POTENTIAL)

} **INPUT**

MFT $h^{+}(r) = -|U|\langle c^{\dagger}_{r\uparrow} c_{r\downarrow}\rangle \quad h^{-}(r) = -|U|\langle c^{\dagger}_{r\downarrow} c_{r\uparrow}\rangle \quad \langle n_{i\sigma}\rangle$

2NX2N matrix

$$\begin{pmatrix} \hat{K}_{\uparrow} & \hat{h}^{-} \\ \hat{h}^{+} & \hat{K}_{\downarrow} \end{pmatrix} \begin{pmatrix} \psi_{n\uparrow} \\ \psi_{n\downarrow} \end{pmatrix} = \varepsilon_n \begin{pmatrix} \psi_{n\uparrow} \\ \psi_{n\downarrow} \end{pmatrix}$$

$$\hat{K}_{\sigma}\psi_{n\sigma}(r) = -t\sum_{\delta}\psi_{n\sigma}(r+\delta) + [V(r) - \tilde{\mu}_{\sigma}(r)]\psi_{n\sigma}(r) \qquad h^{\sigma}\psi_{n\sigma}(r) = h^{\sigma}(r)\psi_{n\sigma}(r)$$

- **pick some** $\{h^{\pm}(r)\}$ **and** $\{n_{\sigma}(r)\}$ $\qquad \tilde{\mu}_{\sigma}(r) = U(1/2 - n_{\bar{\sigma}}(r))$
- **solve MF equations and get** $\varepsilon_n, \psi_{n\sigma}(r)$
- **determine** $h^{+}(r) = -U\sum_{n}\psi^{*}_{n\uparrow}(r)\psi_{n\downarrow}(r)$

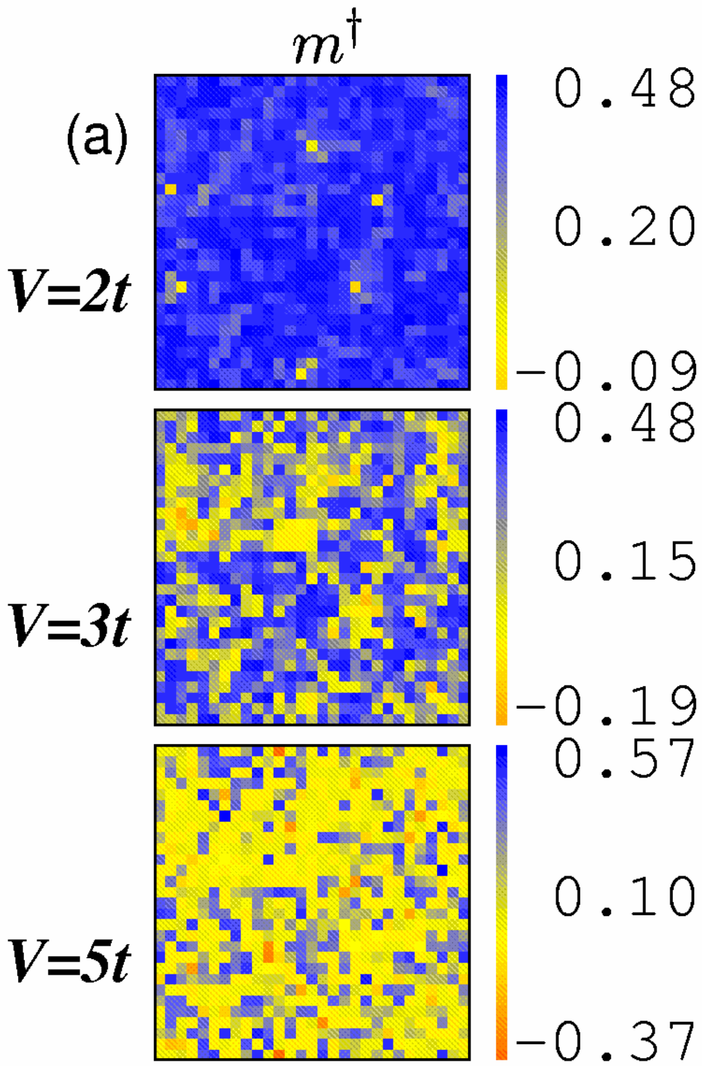
$$n_{\sigma}(r) = \sum_{n}|\psi_{n\sigma}(r)|^{2}$$

SELF CONSISTENCY

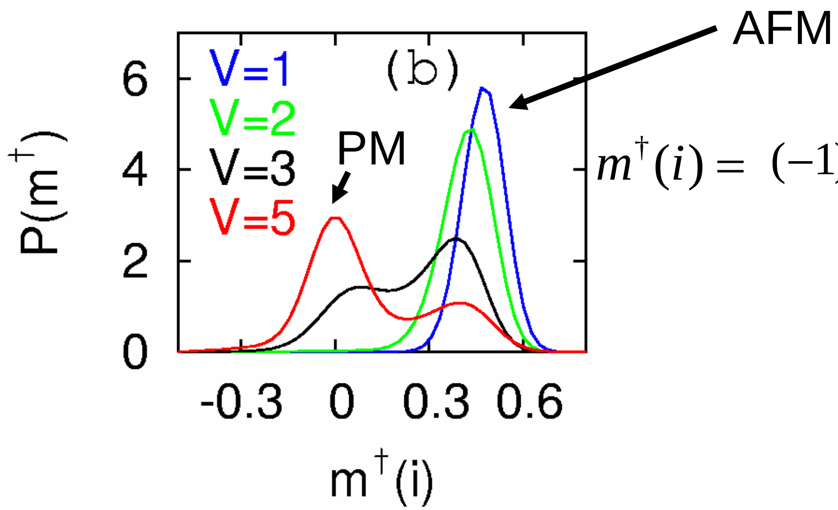
iterate until self consistency is achieved at EACH site

Broyden method

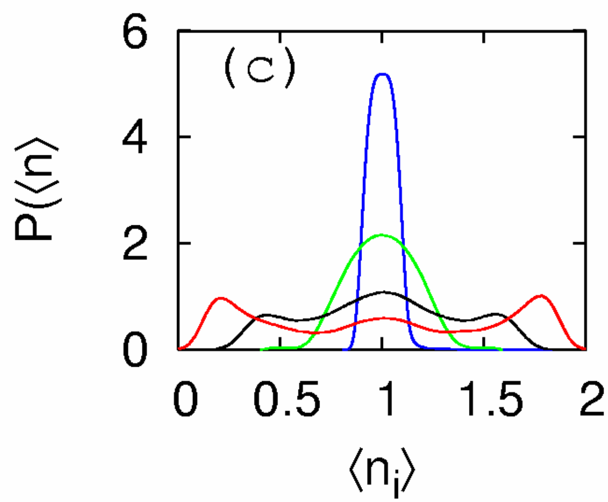
$$(-1)^{i_x+i_y}\langle S_z(i)\rangle$$



Probability distribution of local staggered magnetization



$$m^{\dagger}(i) = (-1)^{x+y} < S^z(i) >$$



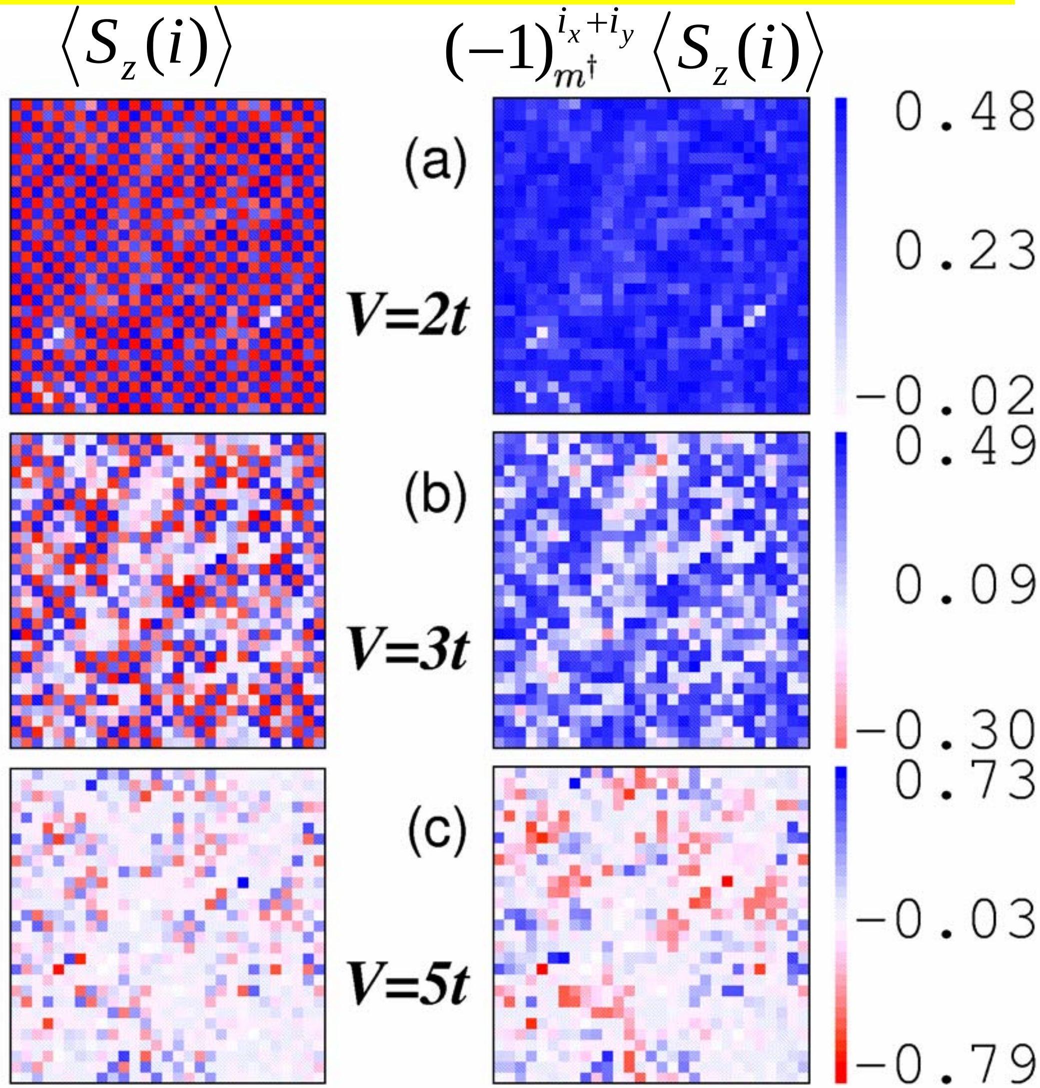
Probability distribution of local density <n(i)>

N=24x24 U=4t T=0 n=1
averaged over 10 realizations

DISORDERED HUBBARD MODEL AT HALF FILLING

Local magnetization

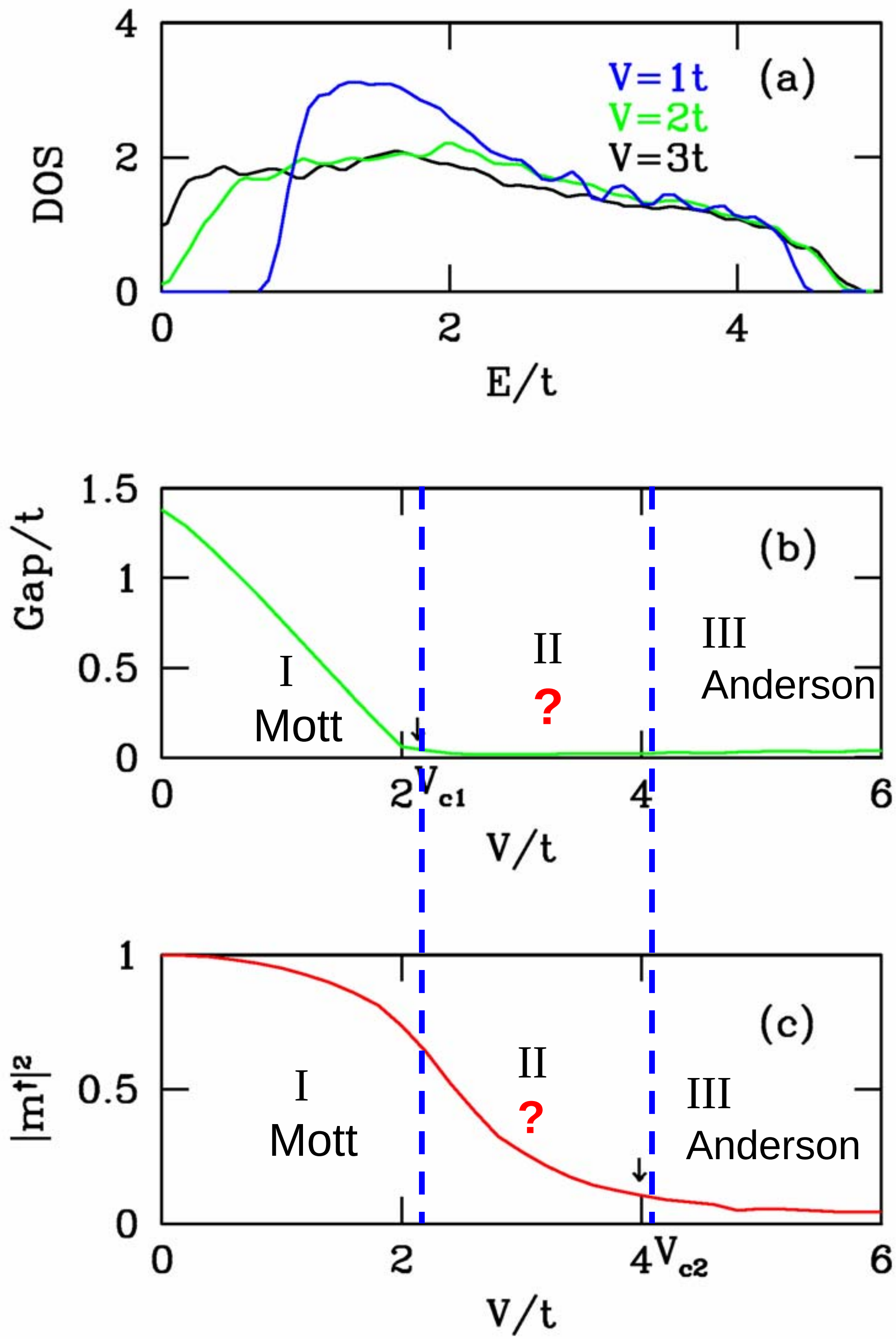
N=24x24
U=4t



Disorder V:
uniform distribution
couples to density

As disorder strength increases the defected regions i.e. regions with suppressed checker board pattern grows

D. Heidarian and NT
PRL 93, 126401 (2004)

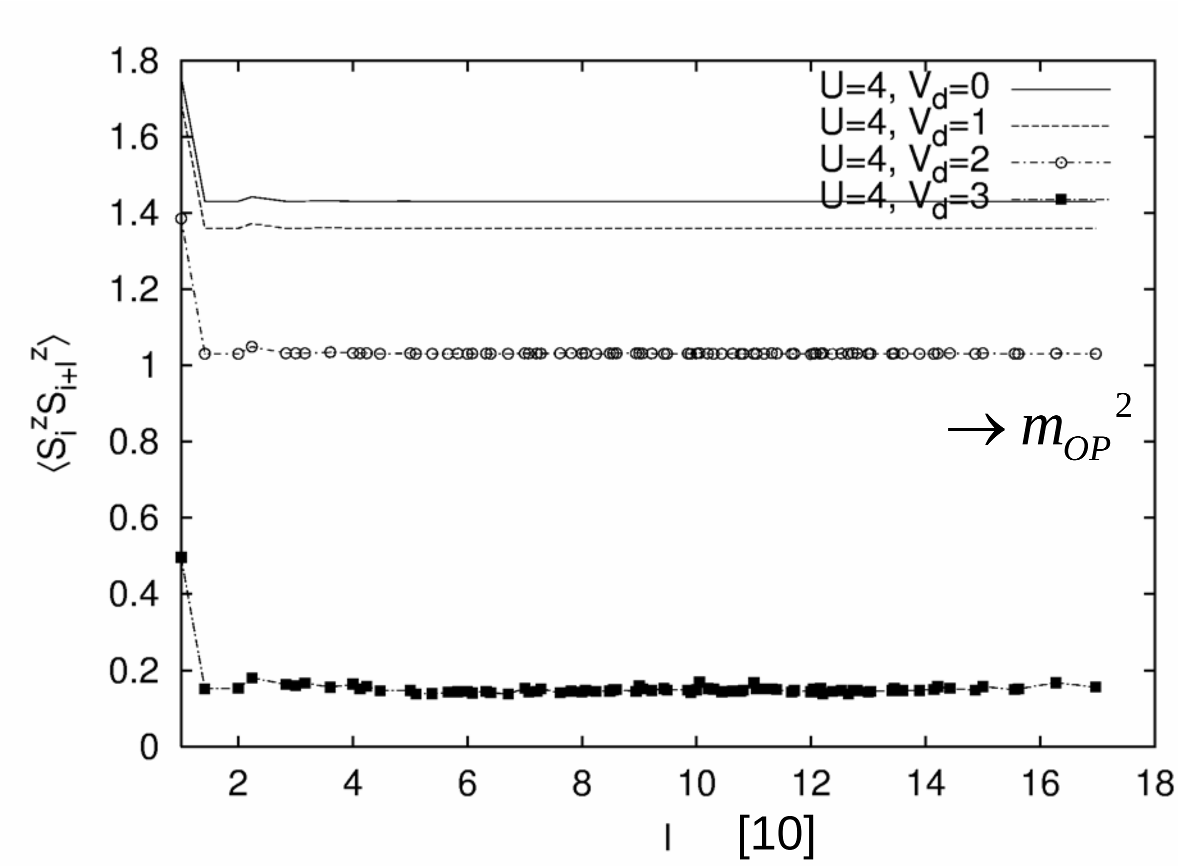


Gap ~ U

AFM J ~ $t^2/U \ll U$

- **Why is the gap killed first?**
- **What is the ? phase?**

spin-spin correlation function

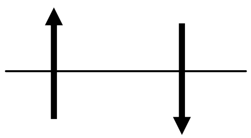


disorder

m_{OP} =order parameter

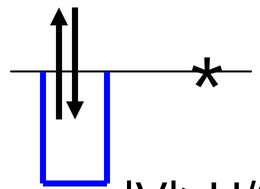
CONNECTIONS WITH PERCOLATION (t=0)

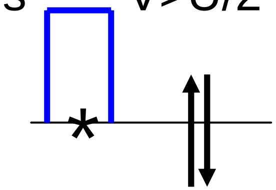
singly occupied sites



Contribute to AFLRO

doubly occupied and empty sites

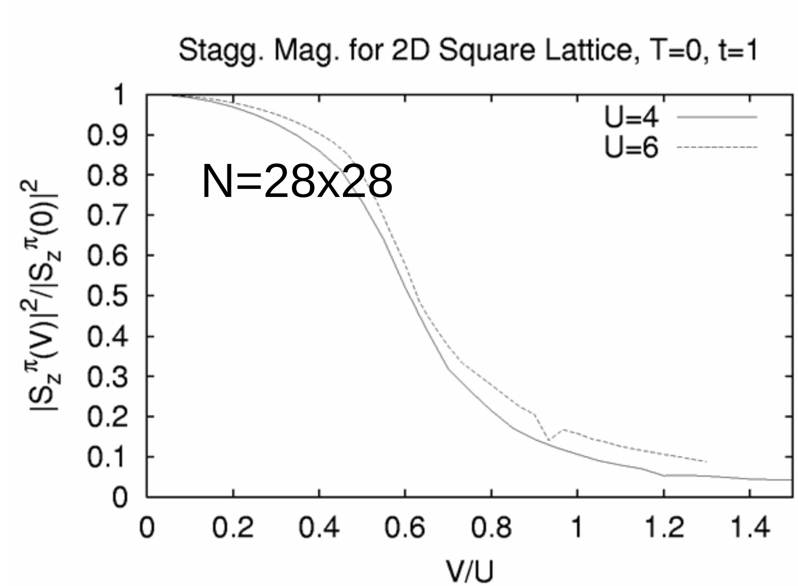




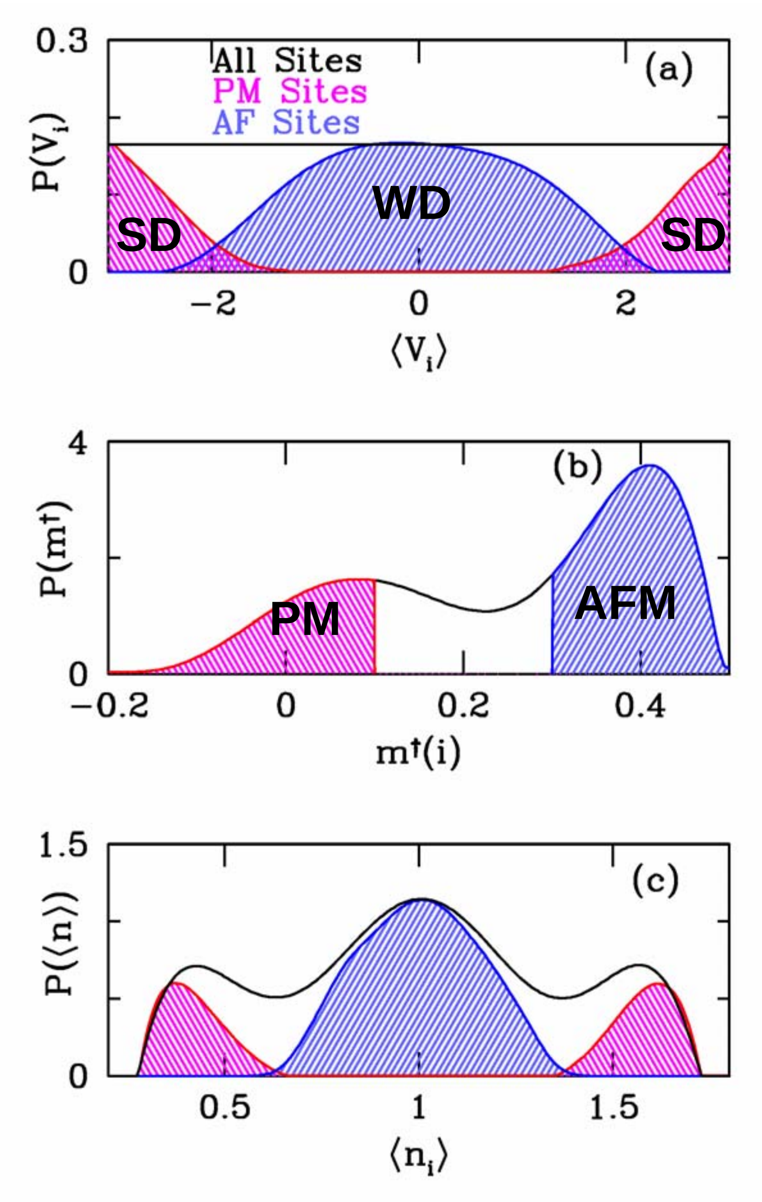
Destroy AFLRO

Classical percolation of vacancies on square lattice $p_{crit} = 0.41$

Probability of 'bad sites' $\Rightarrow 1 - \frac{U/2}{V} \equiv p_{crit}$ $\Rightarrow \left(\frac{V}{U}\right)_{crit} = \frac{1}{2(1-p_{crit})} = 0.85$



SW



WD: weakly disordered
SD: strongly disordered

Correlation of AFM regions with WD and <n>~1

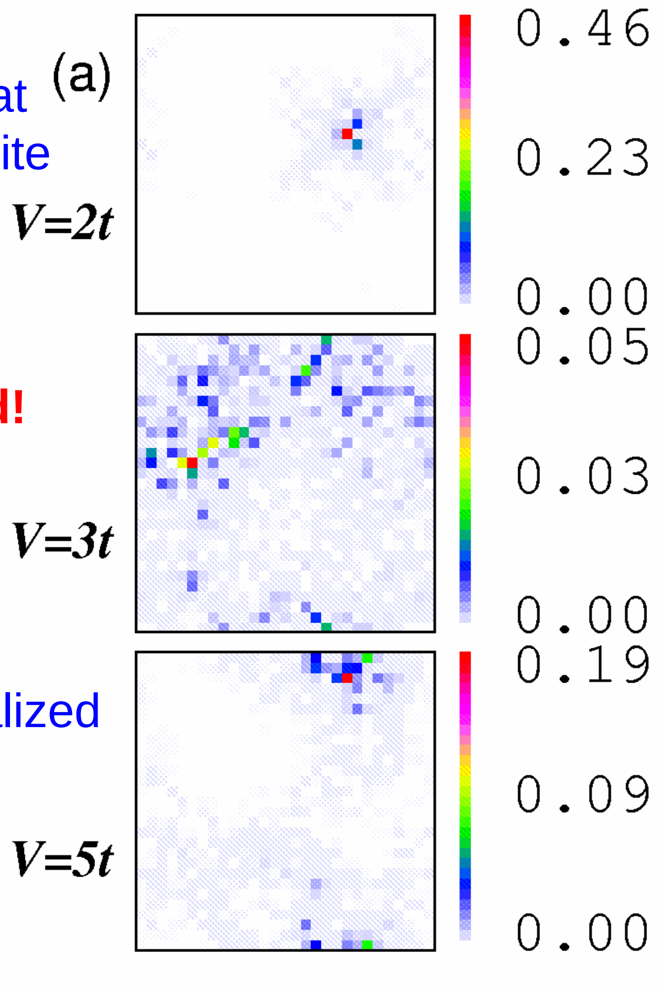
PM regions with SD and bimodal <n>

NATURE OF EIGENFUNCTIONS AROUND THE FERMI ENERGY

Localized at defective site

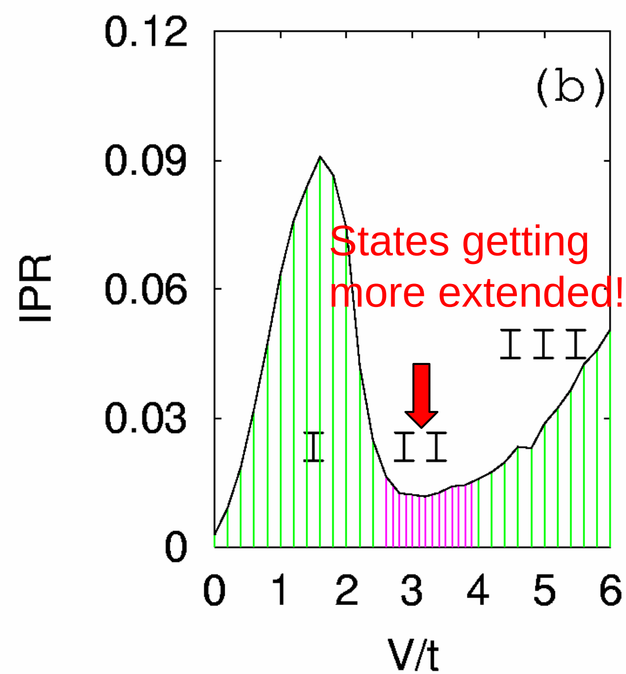
extended!

again localized



Inverse participation ratio

$$IPR = \sum_{i=1}^{N} |\psi(i)|^4 \quad \sim 1/\xi^2$$

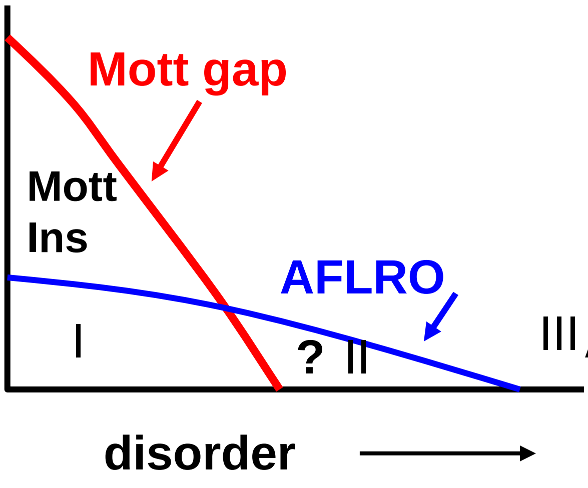


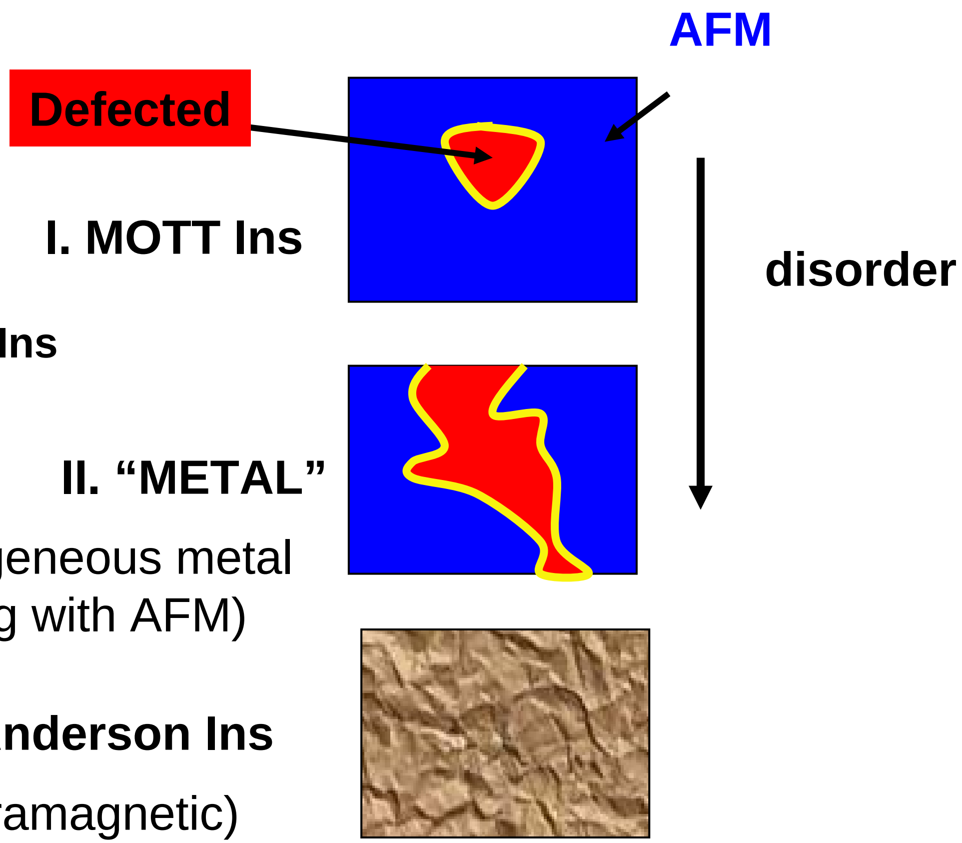
$$\sum_{i=1}^{N} |\psi(i)|^2 = 1$$

Extended state: $\psi(i) \sim 1/\sqrt{N}$ $\quad IPR \sim 1/N$

Localized state: $\psi(i_0) \sim 1$ $\quad IPR \sim 1$

CONCLUSIONS





I. MOTT Ins

II. “METAL”

(inhomogeneous metal
coexisting with AFM)

III. Anderson Ins

(paramagnetic)

II. Anomalous metallic state:

- **states live in the defected region**
- **get more extended with increasing disorder**
- **percolating metallic regions coexist with AFM regions…**

GENERAL CONCLUSIONS

NANOSCALE inhomogeneities induced by disorder

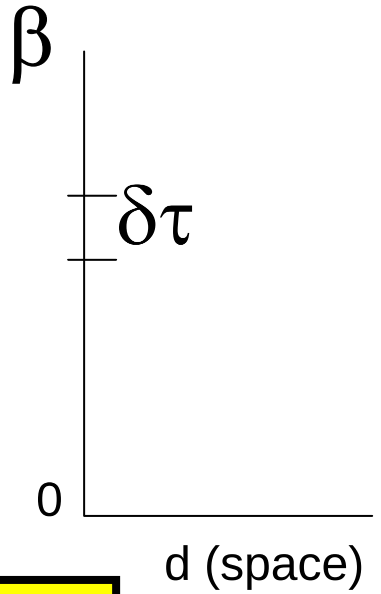
SELF ORGANISATION of system into regions of relatively high disorder and regions of low disorder

GENERIC BEHAVIOR seen in quantum Hall systems, manganites, superconductors and Mott insulators

LOCAL PROBES to see charge, spin and superconductivity

(STM, STS, spin polarized tunneling, Josephson tunneling…)

Coherent state path integral Monte Carlo: Basic idea



$$H_{Hubbard} = -t \sum_{<i,j>\sigma} c_{i\sigma}^{+} c_{j\sigma} + U_0 \sum_i n_{i\uparrow} n_{i\downarrow} \equiv K + U$$

$$e^{-\beta H} = \left[e^{-\delta\tau H} \right]^L \cong \left[e^{-\delta\tau K} e^{-\delta\tau U} \right]^L$$

$$U_0 n_{i\uparrow} n_{i\downarrow} \rightarrow S_{i\tau}(n_{i\uparrow} - n_{i\downarrow})$$

$$Z = Tr e^{-\beta(H-\mu N)} = \sum_{\{S\}} Tr \prod_{\tau\sigma} e^{-\delta\tau \sum_{i,j} c_{i\sigma}^{\dagger} h(\tau,\sigma) c_{j\sigma}}$$

$$Z = \sum_{\{S\}} \det M_{\uparrow}(S) M_{\downarrow}(S)$$

$$M_{\sigma}(\{S\}) = \left[I + \prod_{\tau} e^{-\delta\tau h(\tau,\sigma)} \right]$$

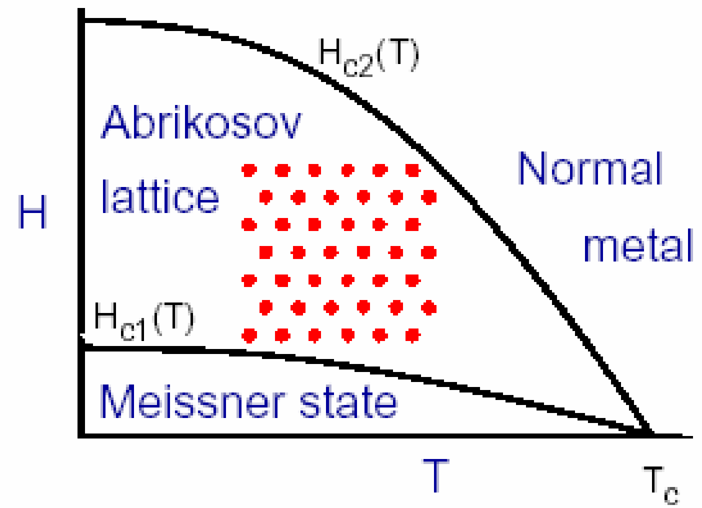
VMC variational	GFMC Green function	PIMC Path integral	DMC Determinantal
T=0	T=0	$T \neq 0$	$T \neq 0$
Ψ_{trial}	Ψ_{GS}	$Z = Tr\hat{\rho}(\beta)$	$Z = \sum Det(M_{\uparrow})Det(M_{\downarrow})$
Wave function dependent Easy to apply	“exact” for bosons; mixed estimator	“exact” for bosons	can be “exact” for some fermion problems also
$\mid \Psi \mid^2$sampled	SIGN PROBLEM FOR FERMIONS		
NO sign problem	Nodes in w.f --fixed node Release node	$(-1)^P$ Cancellation of amplitudes for fermions	sign absorbed in operator; divergence of <sign> at low T
Results only as good as input wave function BUT….	trial state must have non-zero overlap with true ground state	no aprior inputs required Disorder YES	No apriori inputs Disorder YES
		Analytic continuation to get finite frequency	

STUDYING CLASSICAL PHASE TRANSITIONS WITH QUANTUM MONTE CARLO TECHNIQUES!!

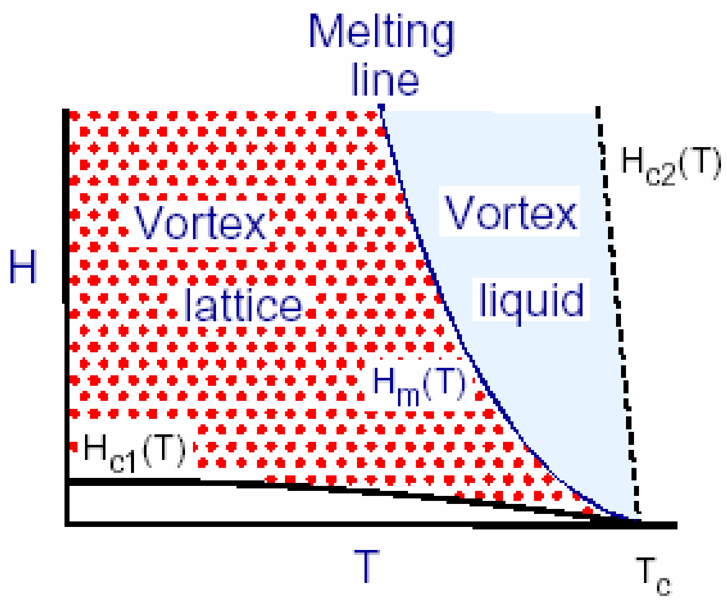
VORTICES

Conventional superconductors:

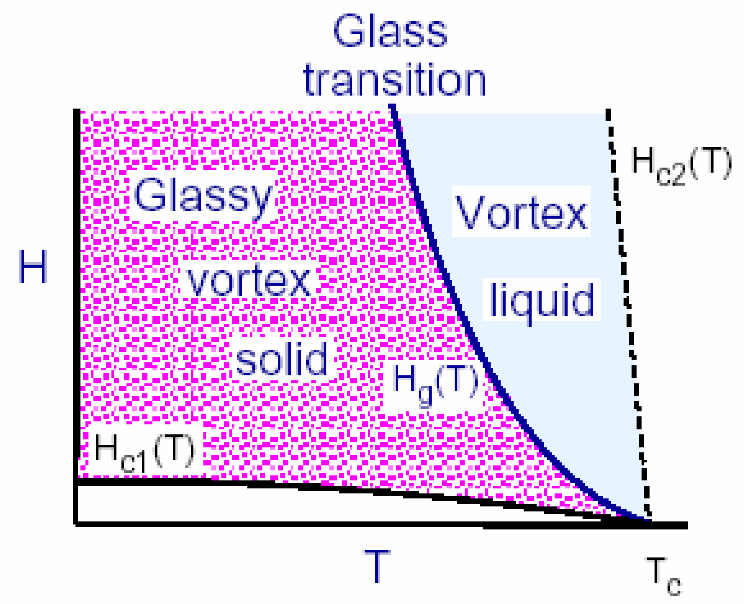
(type II)



High-temperature superconductors: (layered SC, high Tc, fluctuations)



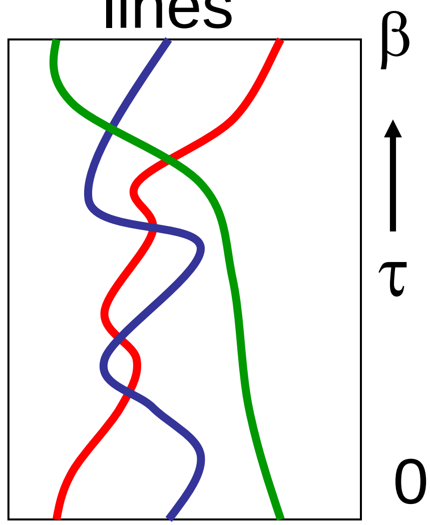
clean system



disordered system

boson world lines

quantum
d-space
1-time



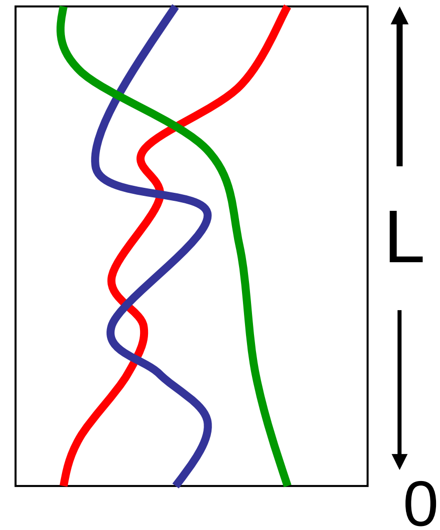
density n

$\hbar$
(quantum fluctuations)

Random potential

vortices

classical
d+1-space

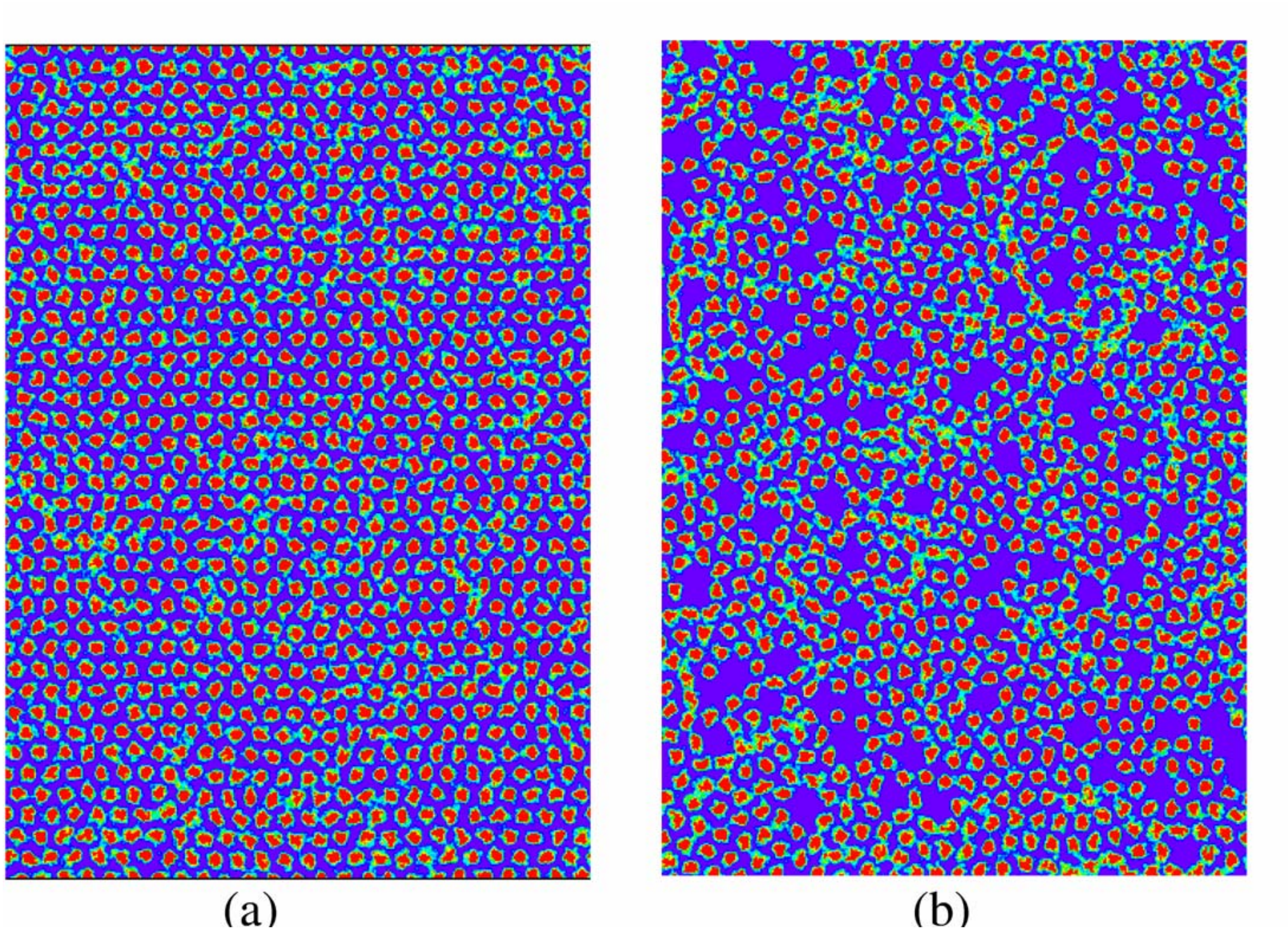


magnetic field B

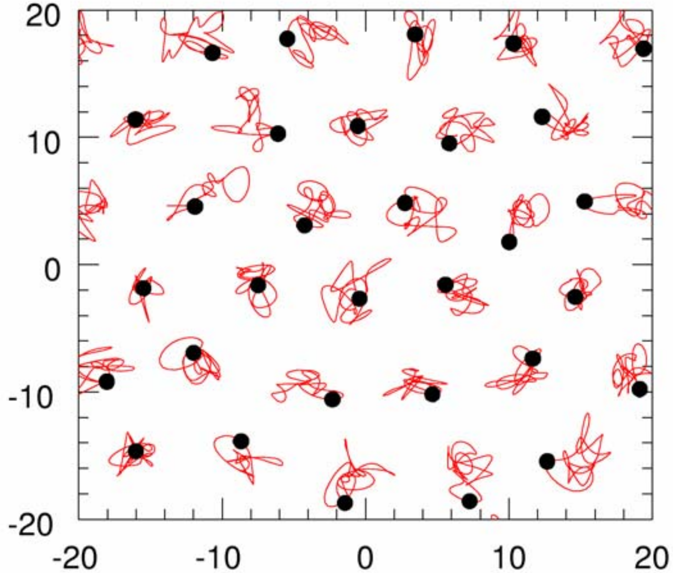
T
(thermal fluctuations)

Columnar pins

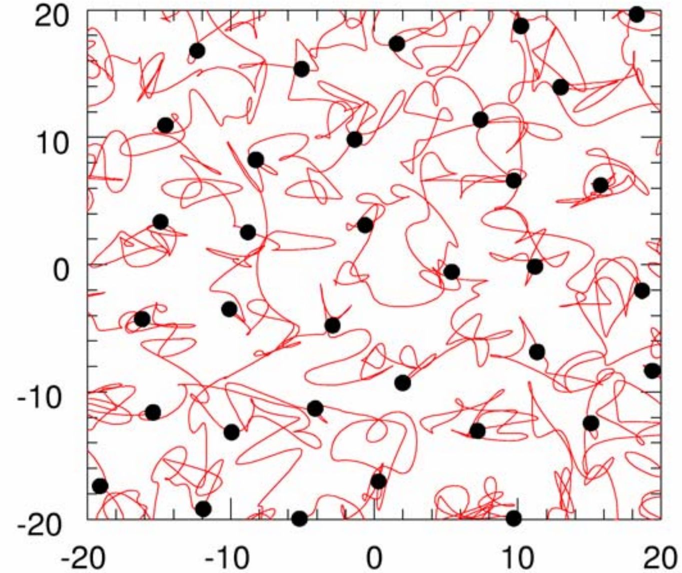
Nelson



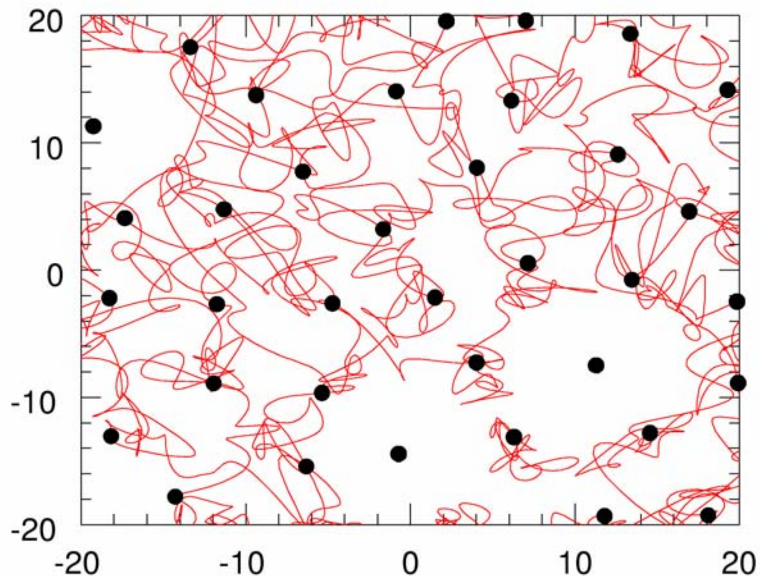
P. Sen, N. Trivedi, D.M. Ceperley, PRL **86**, 4092 (2001).



$\Lambda=0.045$(clean)



$\Lambda=0.065$(clean)



$\Lambda=0.065$(with pins)

Examples:

	Lattice models
Quantum magnetism:	**Heisenberg antiferromagnet**
Strongly interacting bosons: atoms in traps; optical lattices:	**+U Bose Hubbard model**
Feshbach resonance: BCS-BEC crossover:	**-U Fermion Hubbard Model**
High temperature superconductivity:	**+U Fermion Hubbard model**
Quantum Hall Effect:	

Disorder driven Quantum Phase transitions

Superfluid—Bose Glass transition: (Josephson Junction arrays; helium in aerogels)	**+U Bose Hubbard model + disorder**
Superconductor-Insulator Transition: (ultra thin films; high Tc SCs)	**-U Fermion Hubbard model + disorder**
Metal-Insulator transition: (disordered Mott insulators; 2D electron gases)	**+U Fermion model + disorder**