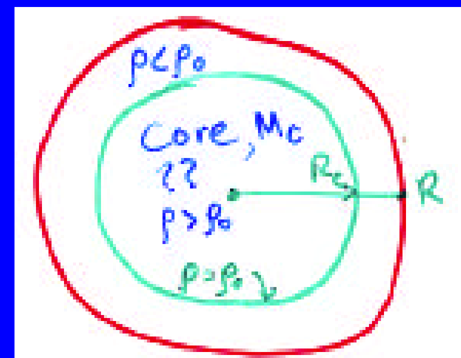
Maximum mass of a neutron star

Say that we believe equation of state up to mass density ρ_0 but e.o.s. is uncertain beyond



$\boxed{\rho(R_c) = \rho_0}$

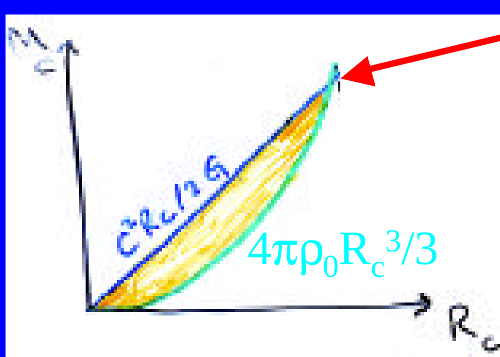
Weak bound:

a) core not black hole => $2M_cG/c^2 < R_c$

b) $M_c = \int_0^{R_c} d^3r\, \rho(r) \geq (4\pi/3)\, \rho_0 R_c^3$

=> $\boxed{c^2R_c/2G \geq M_c \geq (4\pi/3)\, \rho_0 R_c^3}$

$R_{s\odot} = 2M_\odot G/c^2 = 2.94$ km



$M_c^{max} = (3M_\odot/4\pi\rho_0 R_{s\odot}^3)^{1/2} M_\odot$

$\boxed{M_{max} \geq 13.7\, M_\odot \times (10^{14}\mathrm{g/cm^3}/\rho_0)^{1/2}}$

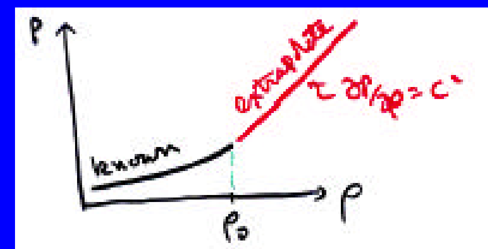
Outside material adds ~ 0.1 $M_\odot$

Strong bound: require speed of sound, c_s, in matter in core not to exceed speed of light:

$$\boxed{c_s^2 = \partial P/\partial\rho \leq c^2}$$

Maximum core mass when $c_s = c$

Rhodes and Ruffini (PRL 1974)



WFF (1988) eq. of state => $M_{max} = 6.7M–(10^{14} g/cm^3/\rho_0)^{1/2}$

V. Kalogera and G.B., Ap. J. 469 (1996) L61

$$\boxed{\begin{array}{lll} \rho_0 = 4\rho_{nm} & \Rightarrow M_{max} = & 2.2\ M– \\ \quad\ \ 2\rho_{nm} & \Rightarrow & 2.9\ M– \end{array}}$$

Neutron star mass vs. r $_0$

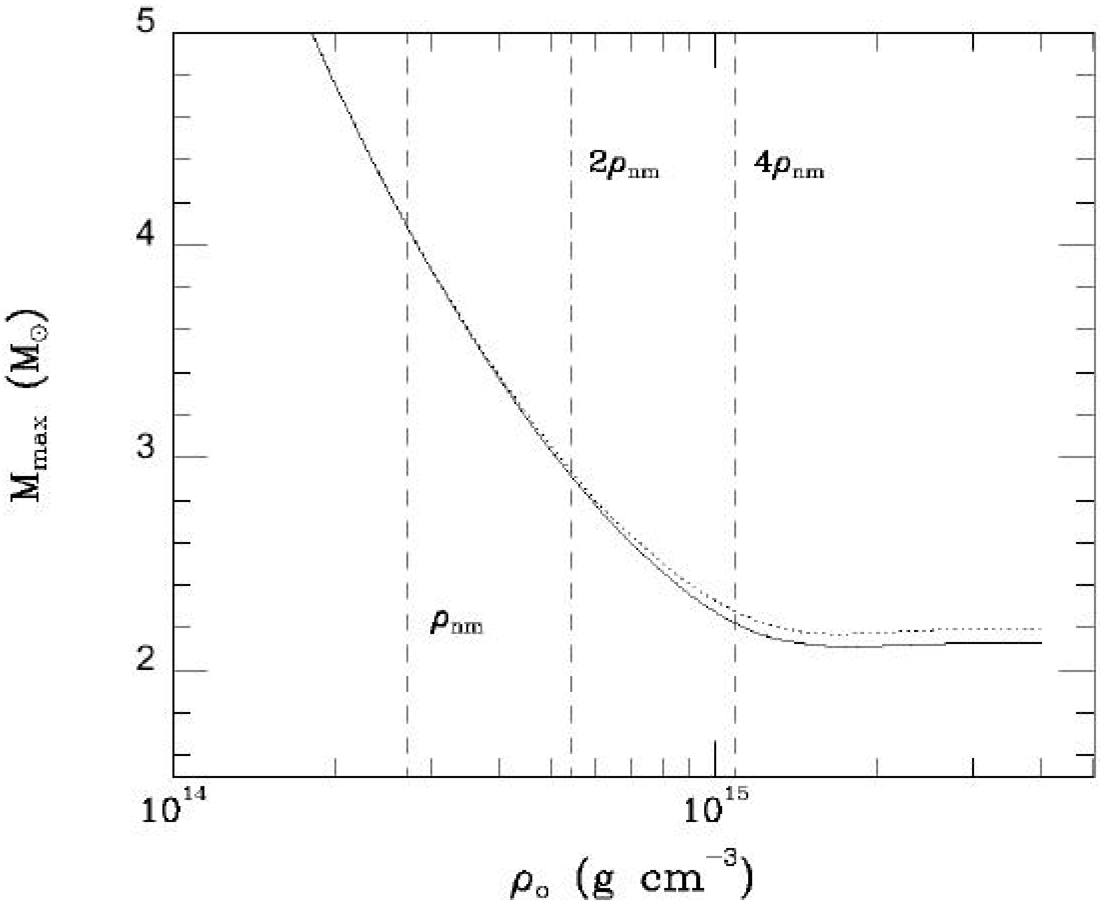
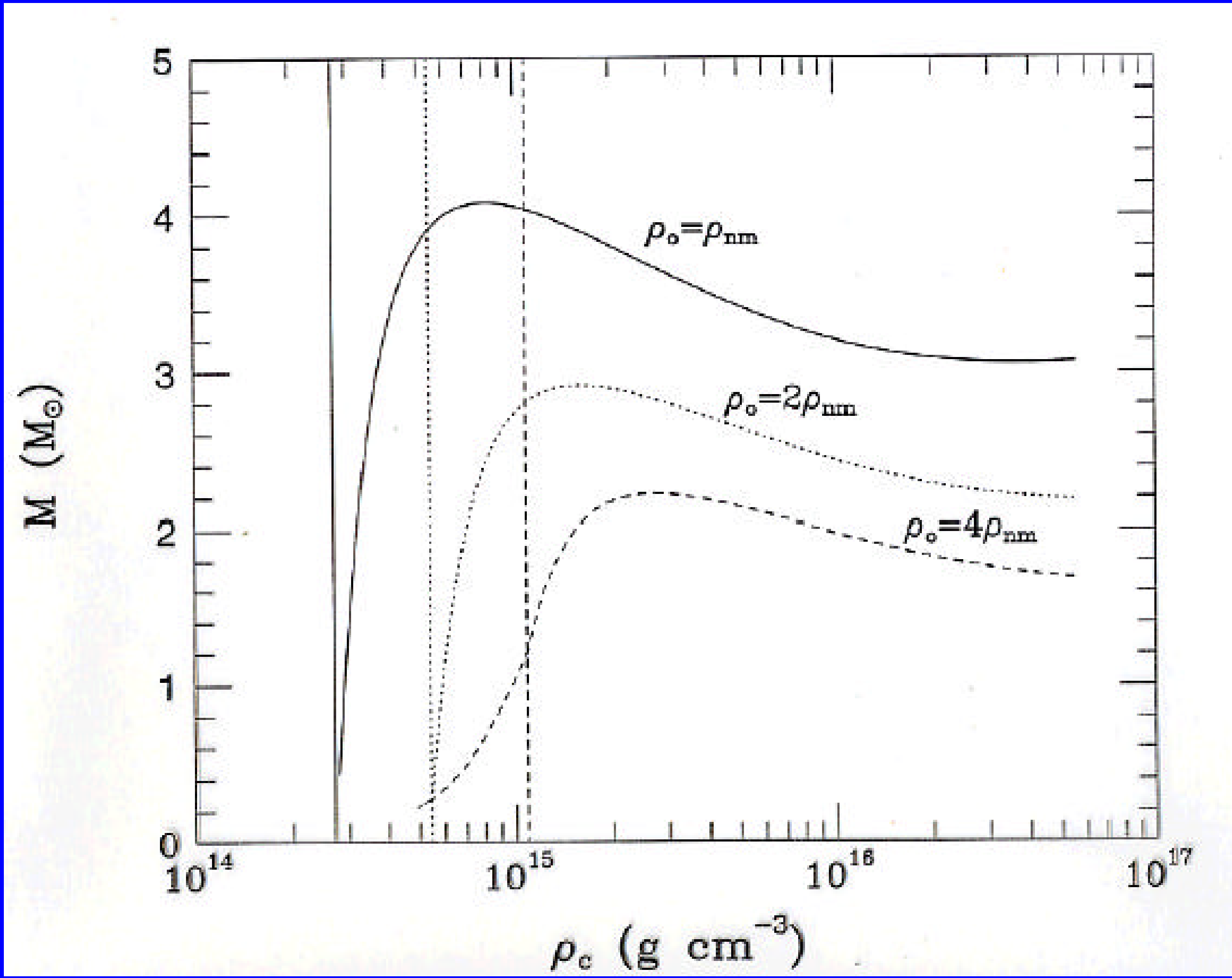


FIG. 2.—Maximum neutron star mass, M_{max}, as a function of the fiducial density, ρ_0, for the two variations of the WFF88 equation of state: AV14 plus UVII potential (*solid line*) and UV14 plus UVII potential (*dotted line*). Vertical dashed lines lie at constant $\rho_0 = \rho_{nm}$, $2\rho_{nm}$, and $4\rho_{nm}$.

Neutron star mass vs. central density for given r $_0$



Black hole candidates

Conventional cut on black hole candidates: $M_{max} = 3.2M$–

$$M_{max} = 2.9\ M\text{–}\ \ (\rho_0 = 2\rho_{nm})$$

$$2.2\ M\text{–}\ \ (\rho_0 = 4\rho_{nm})$$

many new small mass black hole candidates

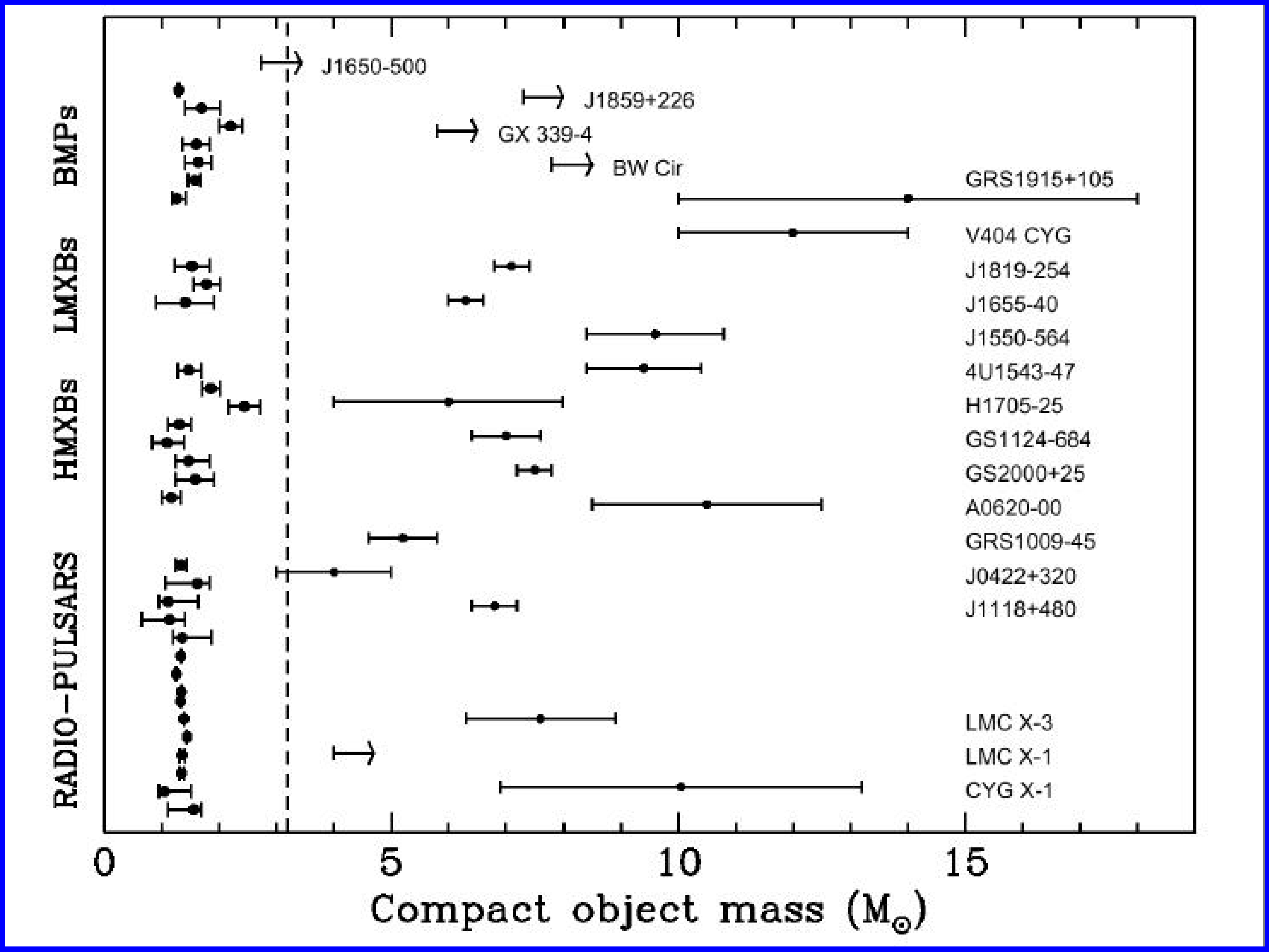
Mass function: $f_{opt} = (M_x{}^3 \sin i)\ /(M_x+M_{optical})^2 < M_x$

in low mass x-ray binaries

ex. Nova muscae, $f_{opt} = 3.1 \pm 0.4$

GRO J1655-40, $f_{opt} = 3.16 \pm 0.15$

Compact black hole masses



J. Caceres, astro-ph/0503071

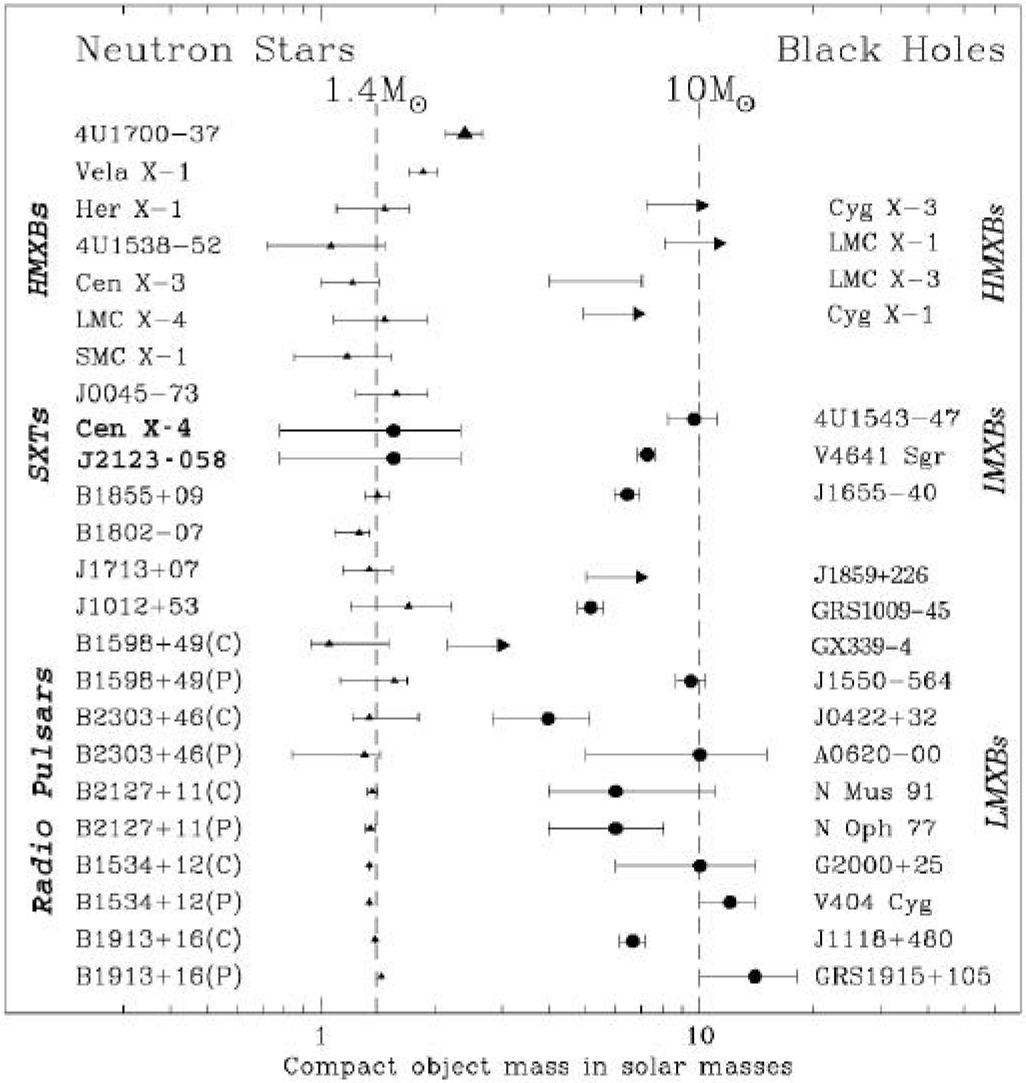


Fig. 5.21. The mass distribution of neutron stars and black holes. Note the remarkably narrow spread of NS masses, and the large factor by which the BH masses exceed the (canonical) maximum NS mass of $3.2 M_{\odot}$.

Charles & Coe, astro-ph/0302080

Stability of neutron stars vs. eqn. of state (Newtonian)

$$E = E_{internal} + E_{grav}$$

$$E_{grav} \sim -\frac{3}{5}\frac{M^2 G}{R} \propto V^{-\frac{1}{3}}$$

$$\frac{dE_{int}}{dV} = -\bar{P}$$

average pressure

$$\delta E \approx \left(-\bar{P}V + \frac{1}{5}\frac{M^2 G}{R}\right)\frac{\delta V}{V}$$

$$\int P d^3 r = -\frac{1}{3}E_{grav}$$

Newtonian virial theorem

Second variation:

$$\delta^2 E \approx -\frac{1}{2}\frac{d\bar{P}}{dV}(\delta V)^2 - \frac{2}{15}\frac{M^2 G}{R}\left(\frac{\delta V}{V}\right)^2$$

$$\Gamma \equiv -\frac{V}{P}\frac{dP}{dV} = \frac{n_b}{P}\frac{dP}{dn_b}$$

$$\delta^2 E = \frac{1}{2}\bar{P}V(\bar{\Gamma} - \frac{4}{3})\left(\frac{\delta V}{V}\right)^2$$

Newtonian stability requires Γ > 4/3 on average

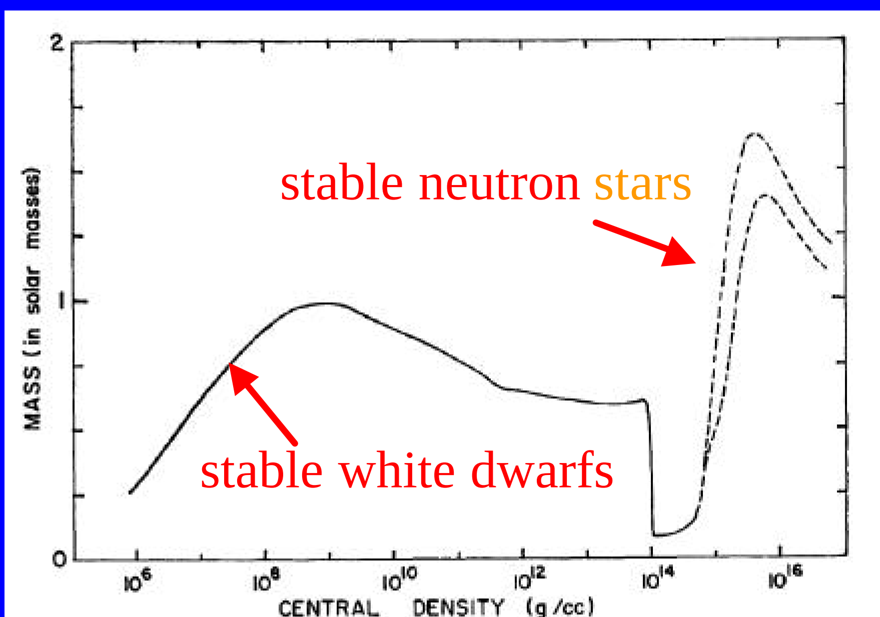
General relativistic corrections:

$$\boxed{\Gamma > (4/3)(1 + \mathbf{O}(R_s/R))}$$

$R_s = 2MG/c^2$ = Schwarzschild radius = 2.94 (M/M–) km

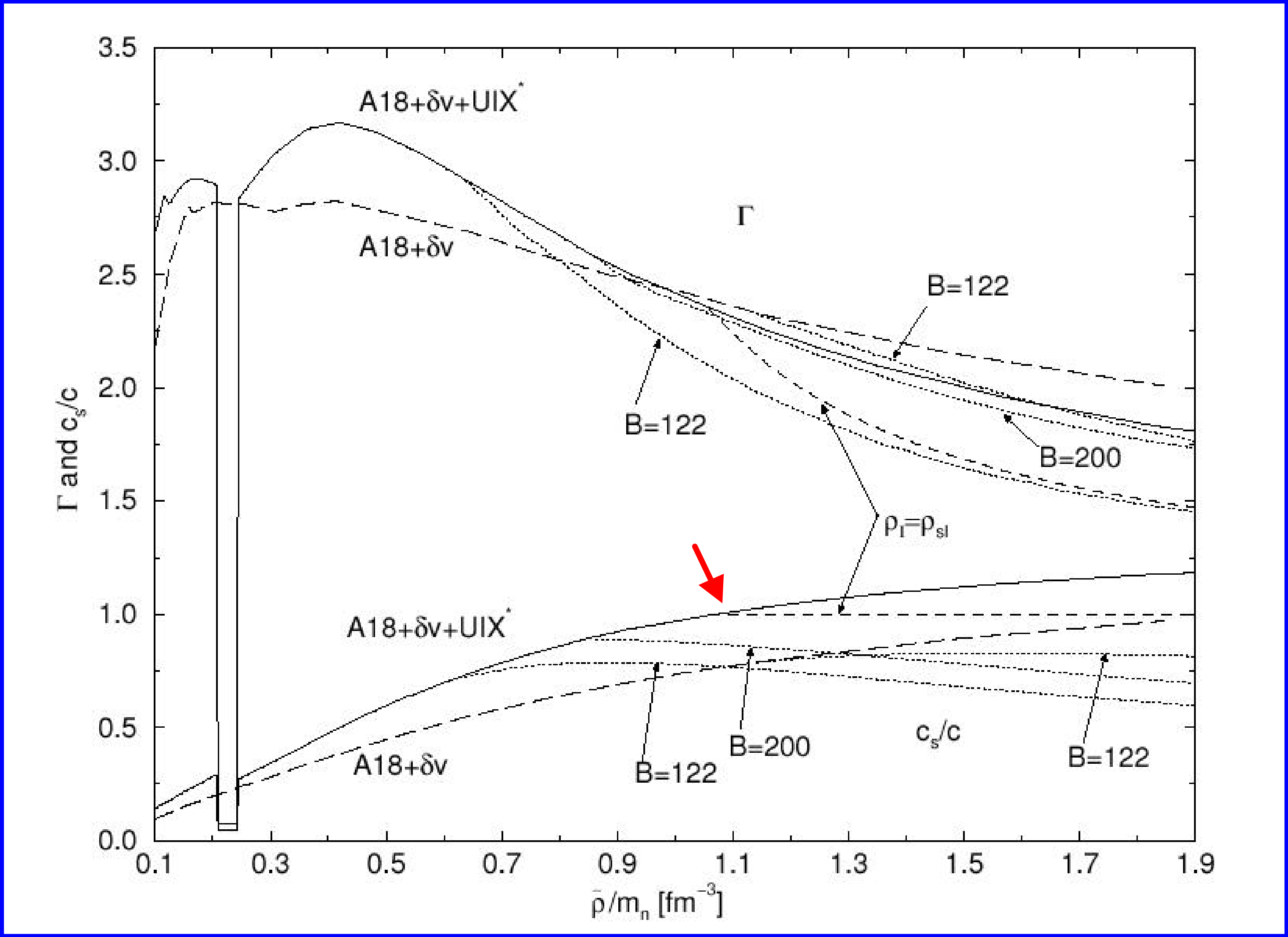
Numerically small (but crucial) correction for white dwarfs, where for relativistic electrons, $\Gamma = 4/3$

Large (» 50%) correction in neutron stars.

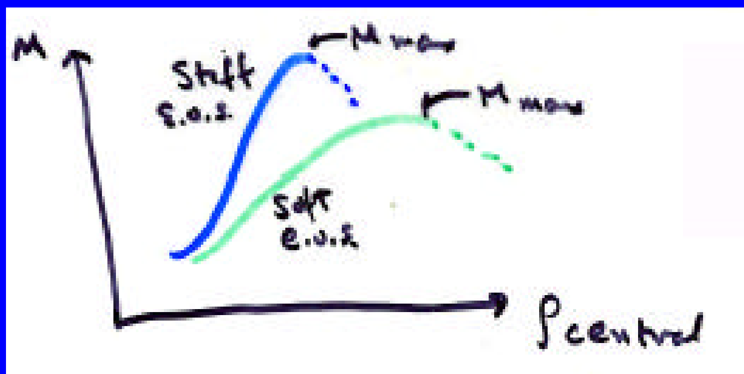


Speed of sound and adiabatic index vs. mass density

Akmal, Pandharipande and Ravenhall, Phys. Rev. C58 (1998) 1804



Learning about dense matter from neutron star mass determinations



Softer equation of state => lower maximum mass and higher central density

Binary neutron stars » 1.4 M–: consistent with soft e.o.s

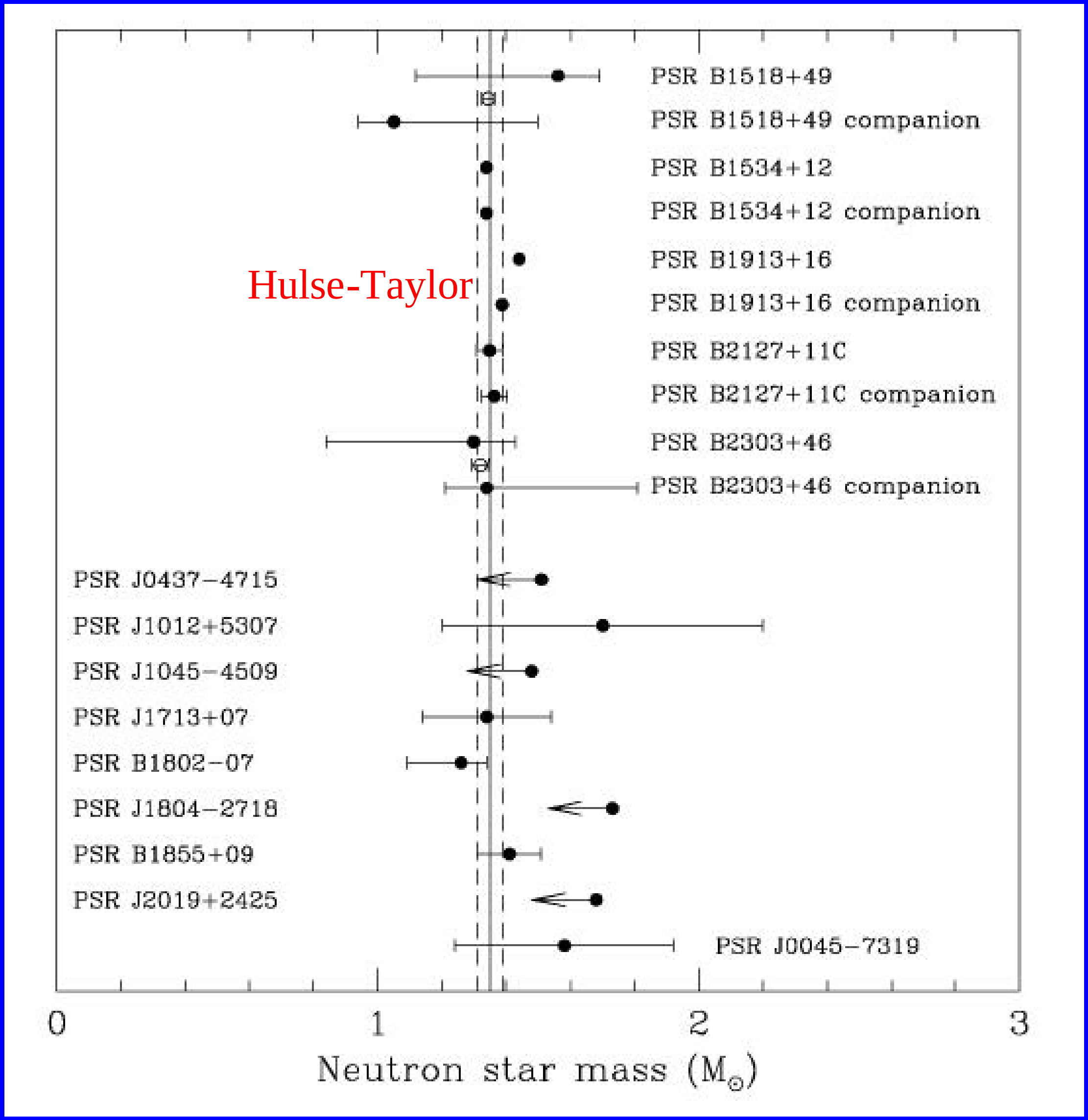
M_{max} ~ 1.56M_{sun} (Bethe-Brown, no neutron star observed in SN1987A)

Cyg X-2: M=1.78 ± 0.23M–	
Vela X-1: M=1.86 ± 0.15M–	allow some softening

PSR J0751+1807: M » 2.1 M–	no softening
QPO 4U1820-30: M » 2.2-2.3 M–	challenge microscopic e.o.s.

Measured neutron star masses in radio pulsars

Thorsett and Chakrabarty, Ap. J. 1998

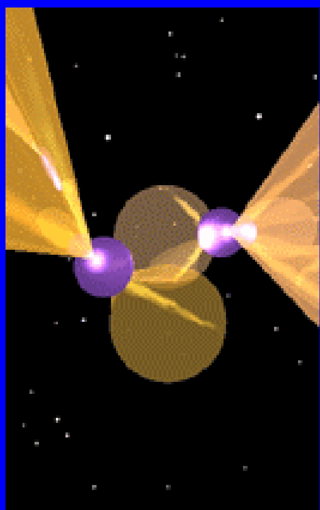


neutron star
- neutron star
binaries

M=1.35±0.04M–

NEW BINARY PULSAR SYSTEM

Lyne et al., Science 303, 1153 (2004)



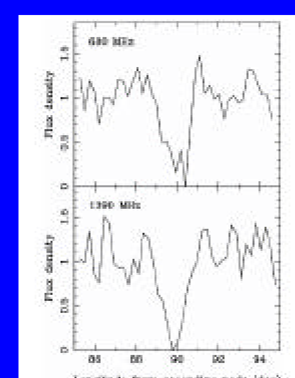
22-ms pulsar J0737–3039A

+2.7-sec pulsar J0737–3039B companion

orbital period = 2.4 hours!

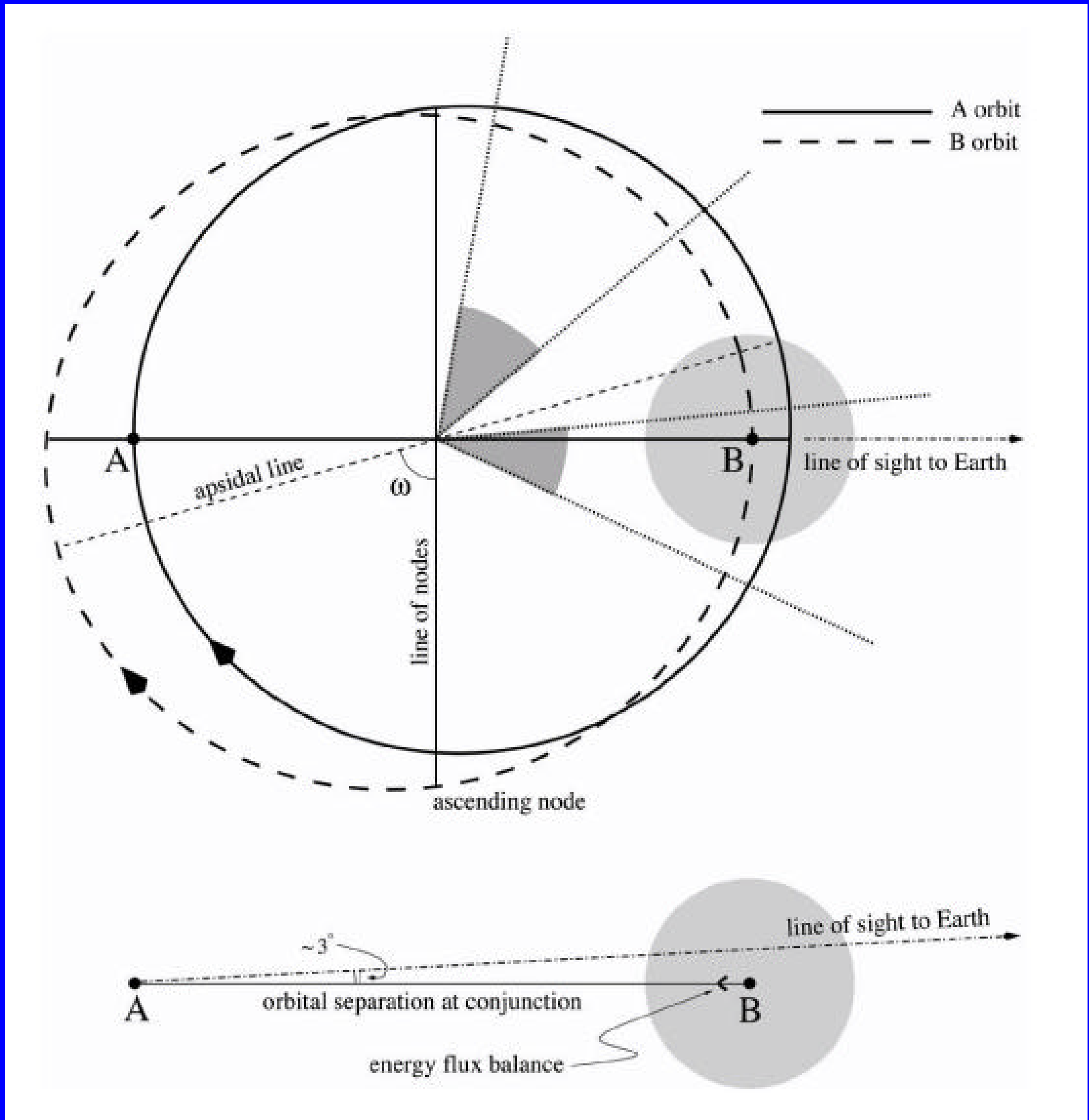
Highly-relativistic double-neutron-star system

See eclipsing of A by B

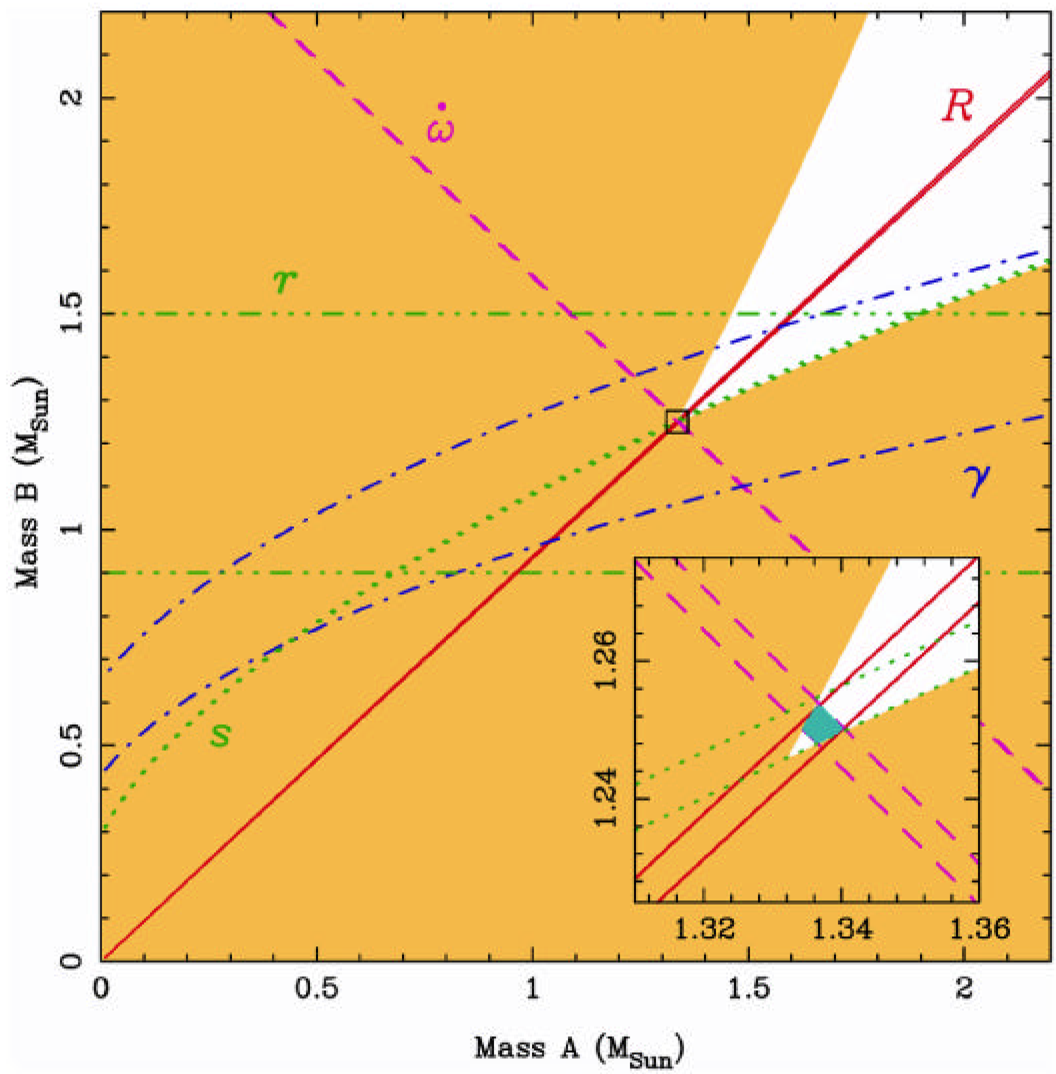


Laboratory for gravitational physics!

See orbit almost edge-on:



Mass determinations:

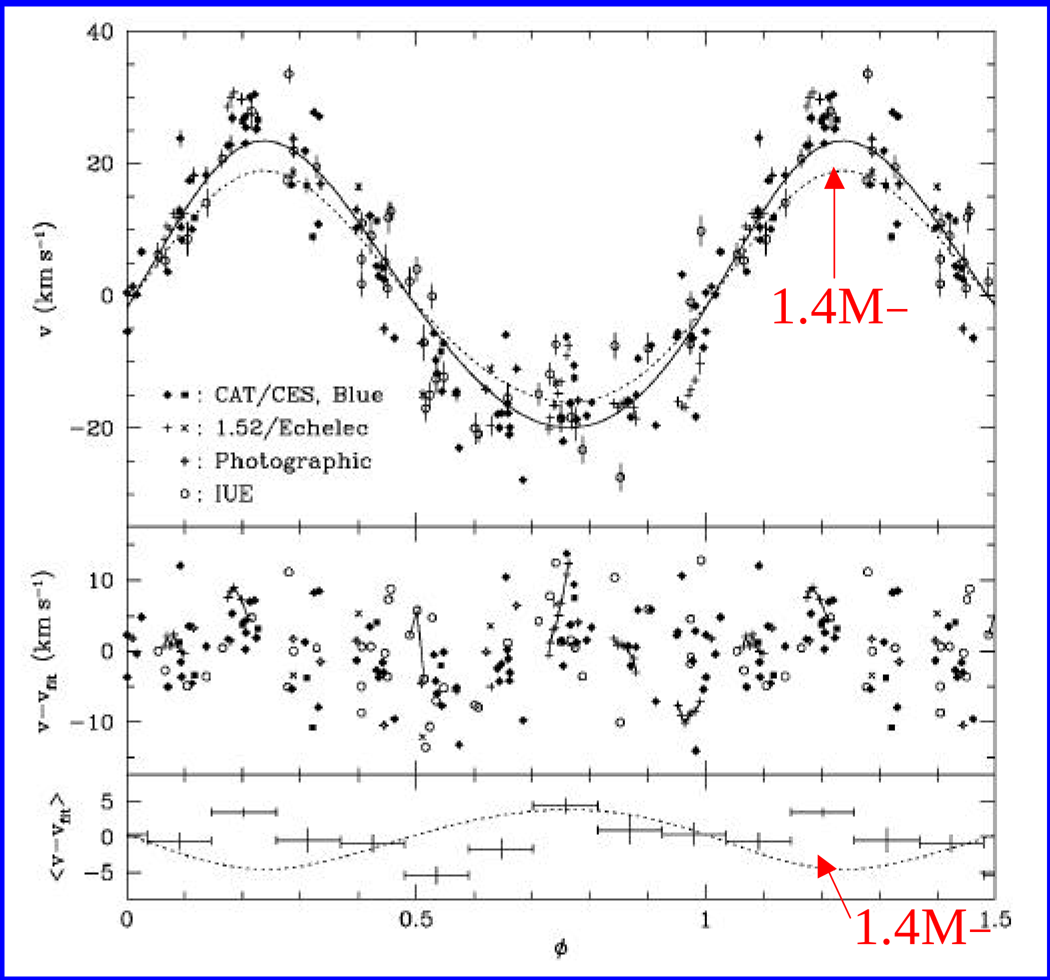


Stellar masses A=1.337(5)M– , B=1.250(5)M–

Vela X-1 (LMXB) light curves

Serious deviation from Keplerian radial velocity

Excitation of (supergiant) companion atmosphere?



$\boxed{\text{M=1.86} \pm \text{0.33 (2}\sigma\text{)M–}}$

M. H. van Kerkwijk, astro-ph/0403489

1.75M–<M<2.44M–

Quaintrell et al., A&A 401, 313 (2003)

PSR J0751+1807

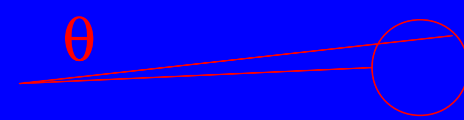
3.4 ms. pulsar in circular 6h binary w. He white dwarf

D. Nice et al., astro-ph/0508050

Pulsar slowing down due to gravitational radiation: dP/dt = 6.4£10^{-14}

Shapiro delay of signal due to gravitational field of companion:

$$\Delta t = -(2Gm_2/c^3)\ \ln(1-\cos\theta)$$



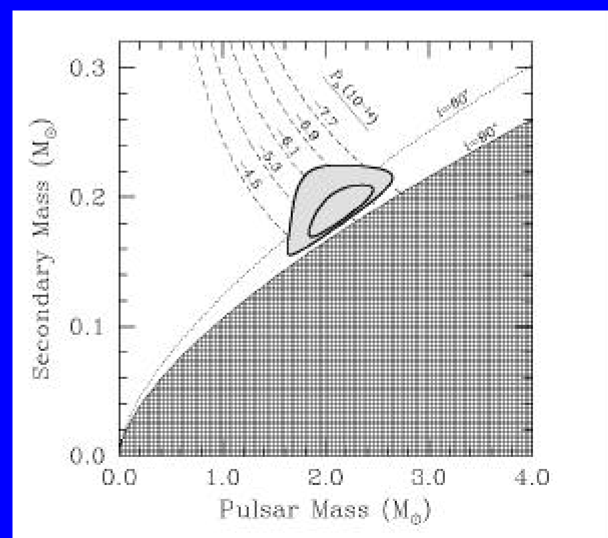
θ = angle between ns and wd seen by observer

Measurements free of uncertainties due to atmospheric distortion in companion

M=2.1M–

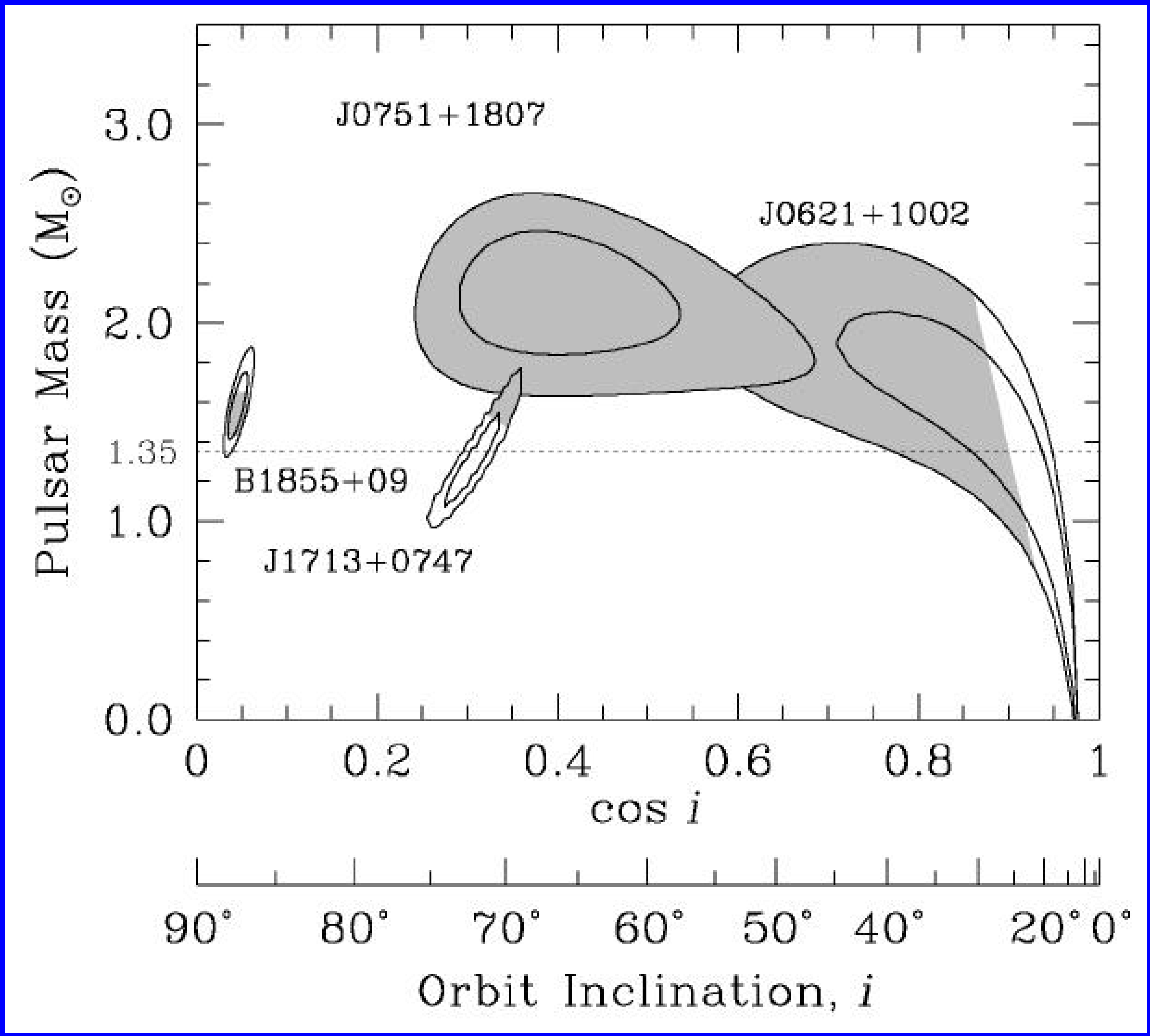
$$m_1 = \begin{cases} 2.1\pm0.2\,\mathrm{M}_\odot & (68\%\ \text{confidence}) \\ 2.1^{+0.4}_{-0.5}\ \mathrm{M}_\odot & (95\%\ \text{confidence}), \end{cases}$$

$$m_2 = \begin{cases} 0.191\pm0.015\,\mathrm{M}_\odot & (68\%\ \text{confidence}) \\ 0.191^{+0.033}_{-0.029}\ \mathrm{M}_\odot & (95\%\ \text{confidence}) \end{cases}$$



Neutron star (pulsar) - white dwarf binaries

Nice, Splaver, & Stairs, astro-ph/0411207



QPO 4U1820-30 (RXTE)

Lamb, Miller, Psaltis

innermost circular stable orbit

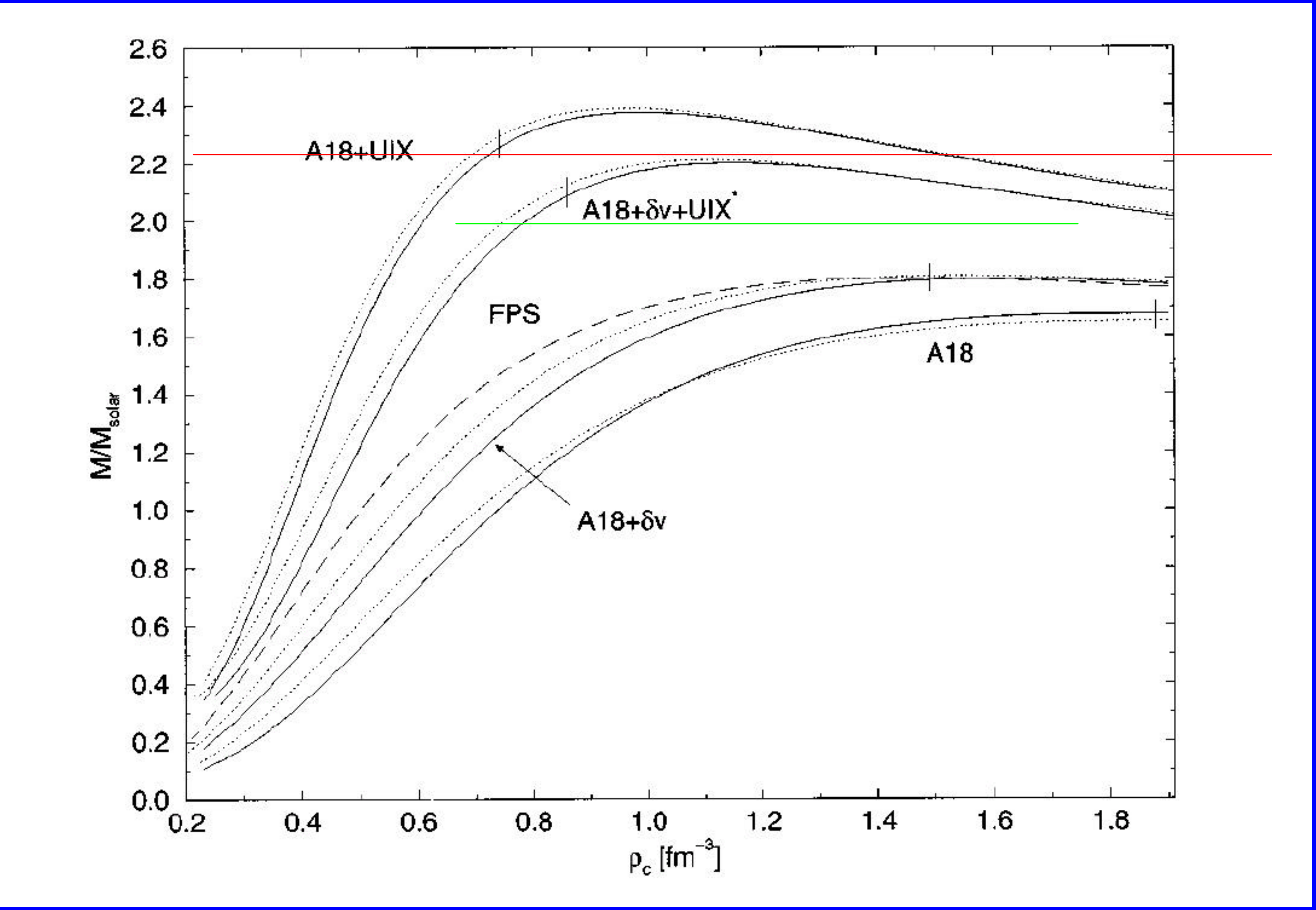
$R=6MG/c^2$

=> M~ 2.2-2.3 M–

Implies very stiff equation of state. No exotica likely.

Central density ~ 1.0 fm^{-3} ~ $6\rho_{nm}$

Observations beginning strongly to constrain microscopic nuclear physics



Akmal, Pandharipande and Ravenhall, 1998