

Tutorial on superfluidity and superconductivity

Three basic superfluids in nature:

Condensed Bose atoms, e.g., ^4He liquid at $T < 2.17\text{K}$,
atomic Bose condensates (^{23}Na , ^{87}Rb , ...)

Neutral BCS-paired Fermi atoms, e.g., ^3He liquid at $T < 1\text{mK}$,
atomic fermions (^6Li , ^{40}K), neutrons in neutron stars

Charged BCS-paired fermions -- **superconductors**
e.g., electrons in metal, protons in neutron stars



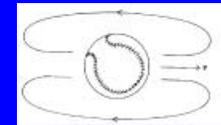
The many faces of superfluidity

(A.J. Leggett, RMP 71, S318 (1999))

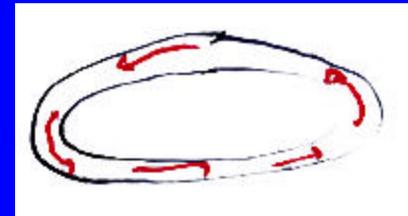
Flow through capillaries without friction
(viscosity $\eta < 0.0006 \eta_{\text{He I}}$)



Frictionless flow of object (e.g., ion) through system



Superfluid flow: metastable
flow around a closed pipe “forever”



Vortices



Hess-Fairbank effect: equilibrium



Collective excitations, e.g., second sound

Josephson effect

Early history of superfluidity

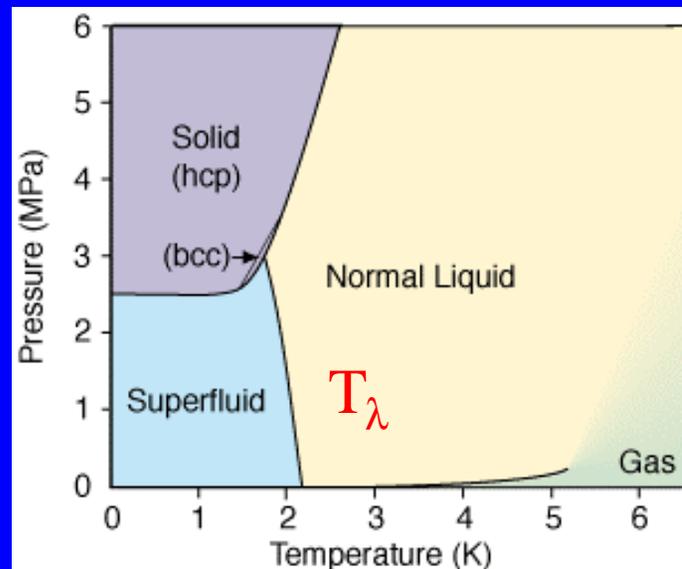
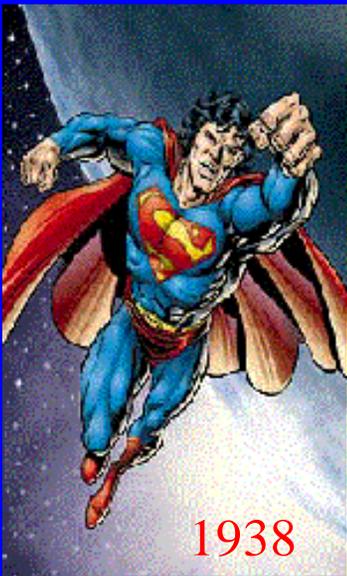
(Superfluidity and superconductivity developed on separate tracks)

1908: liquefaction of ^4He by *Kamerlingh Onnes*, Leiden ($T \gg 1.2\text{ K}$)

1911: discovery of superconductivity by *Kamerlingh Onnes*, Leiden

1933: Meissner effect -- superconductors expel magnetic fields

1937-38: discovery of superfluidity of ^4He : *Kapitza, Allen and Misener*;
 $T < T_\lambda = 2.17\text{ K}$ = lambda point. Called “superfluid” by Kapitza.



^4He phase diagram

Theory landmarks

1938: Connection of superfluidity and Bose-Einstein condensation (F. London)

1941: Landau two-fluid picture : superfluid (ρ_s) + normal (ρ_n)

1949: Quantization of vorticity: Onsager

1957: BCS theory of superconductivity

Experimental landmarks

1967: Hess-Fairbank experiment -- reduction of moment of inertia (analog of Meissner effect)

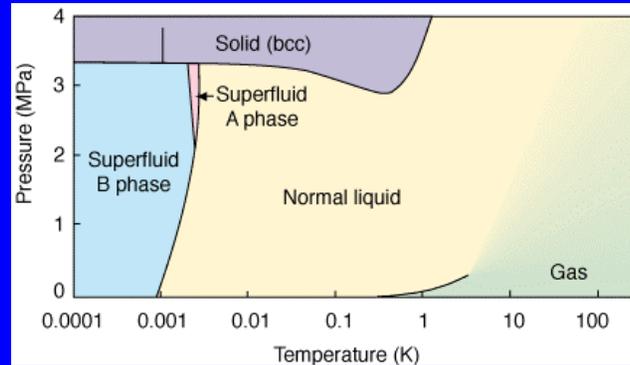
1973: superfluid ^3He

1995-2000: superfluidity of trapped atomic Bose-Einstein condensates

2003--now: superfluidity of trapped paired Fermi-Dirac atoms

New superfluids:

^3He : A and B phases



Dilute solutions of ^3He
in superfluid ^4He ($T_3 \gg \text{few } \mu\text{K}$)

Neutron and proton fluids in
neutron stars: pulsar speedups
Color superconductors in qcd

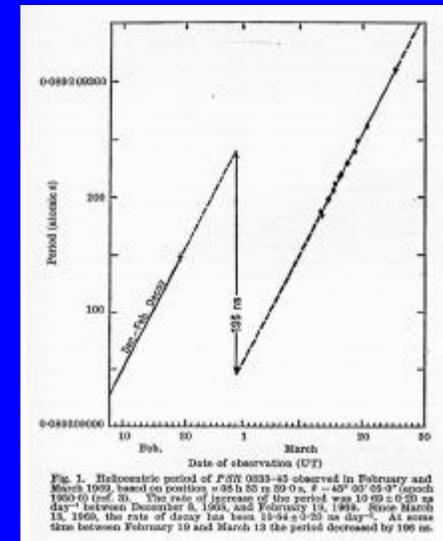
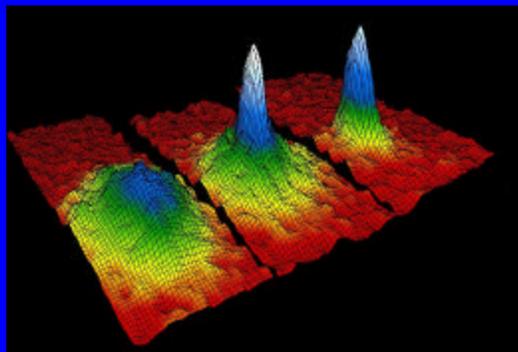
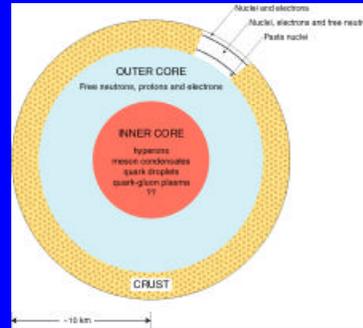


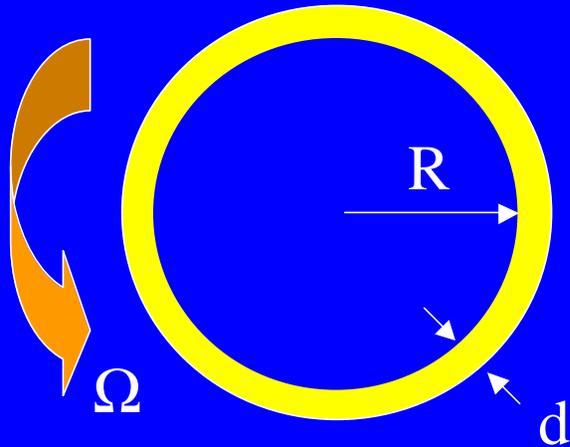
Fig. 1. Helioseismic period of PSR 0531-41 observed in February and March 1969, based on position $\alpha = 05^{\text{h}} 35^{\text{m}} 29.0^{\text{s}}$, $\delta = -45^{\circ} 03' 05.0''$ (epoch 1950.0) (ref. 3). The rate of increase of the period was $10.00 \pm 0.20 \text{ ns day}^{-1}$ between December 8, 1968, and February 13, 1969. Since March 13, 1969, the rate of decay has been $10.94 \pm 0.20 \text{ ns day}^{-1}$. At some time between February 19 and March 13 the period decreased by 190 ns.

Trapped atomic bosonic
& fermionic gases

Vela pulsar: Radhakrishnan & Manchester, Nature 1969

Hess-Fairbank experiment (Phys. Rev. Lett. 19, 216 (1967))

Rotate thin ($d \ll R$) annulus of liquid ${}^4\text{He}$ at Ω



1) Rotate slowly at $T > T_\lambda$: $\Omega < \Omega_c \gg 1/mR^2$
 liquid rotates classically with angular momentum $L = I_{\text{classical}} \Omega$.

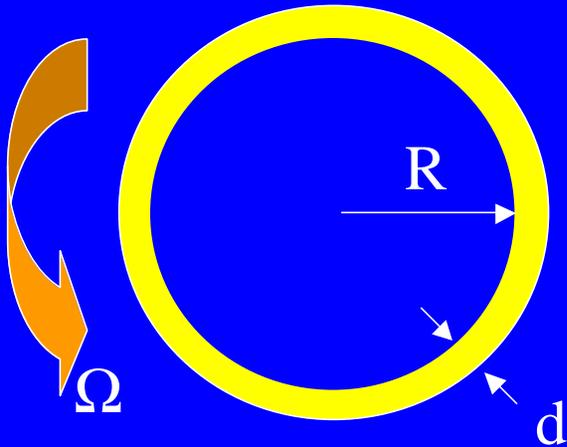
$$I_{\text{classical}} \approx NmR^2$$

2) **Cool to $T < T_\lambda$** : liquid rotates with reduced moment of inertia $I(T) < I_{\text{classical}}$. $I(T=0) = 0$.

Only the normal fluid rotates. $I(T) = (\rho_n/\rho) I_{\text{classical}}$
 The superfluid component remains stationary in the lab.

Reduction of moment of inertia is an equilibrium phenomenon.

Superfluid flow



1) Rotate rapidly at $T > T_\lambda$: $\Omega > \Omega_c$
liquid rotates classically with angular
momentum $L = I_{\text{classical}} \Omega$.

2) Continue rotating, cool to $T < T_\lambda$:
liquid rotates classically

3) Stop rotation of annulus. Liquid keeps
rotating with $L = I_s \Omega$,
where $I_s = (\rho_s / \rho) I_{\text{classical}}$.

Only the superfluid rotates. The normal component is stationary.

Superfluid flow is metastable (albeit with huge lifetime in
macroscopic system)

Landau Two-Fluid Model

Can picture superfluid ^4He as two interpenetrating fluids:

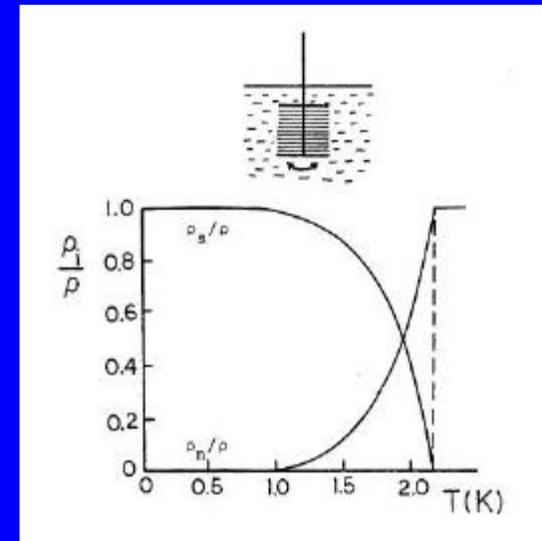
Normal: density $\rho_n(T)$, velocity v_n

Superfluid: density $\rho_s(T)$, velocity v_s

Mass current = $\rho_s v_s + \rho_n v_n$

Entropy current = $T s v_n$

:carried by normal fluid only

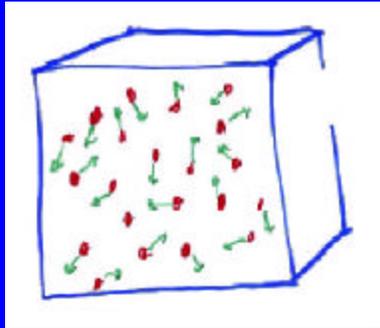


Second sound (collective mode) =

counter-oscillating normal and superfluids

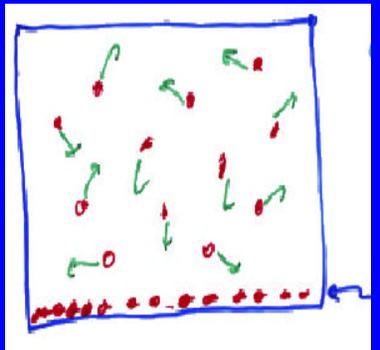
Landau critical velocity ($v_{Landau} = \partial\omega_k/\partial k$) neither necessary nor sufficient to destroy superfluidity. When violated, $\rho_s < \rho$.

Bose-Einstein Condensation



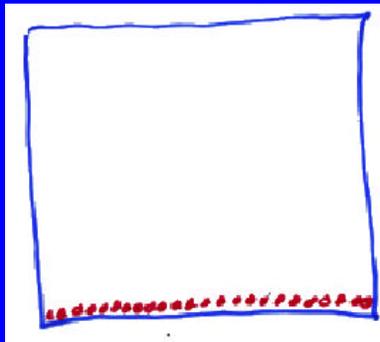
Hot atoms (bosons) in a box

↓
Gravity



Cool below Bose-Einstein transition temperature

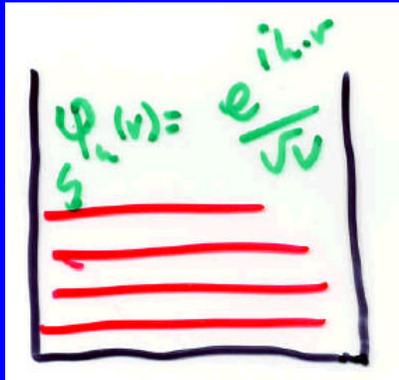
Bose-Einstein condensate



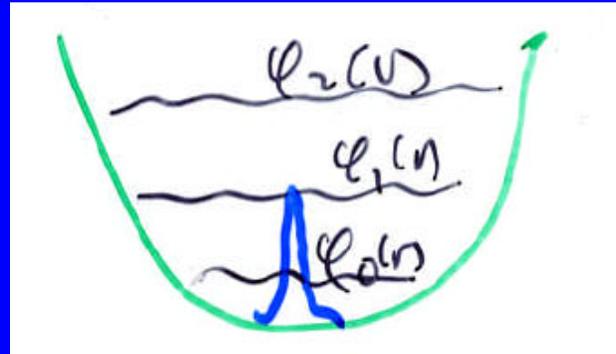
At absolute zero temperature motion “ceases”

Free Bose gas

$(t_0 = 1)$



Box



Potential well (trap)

In condensed system have **macroscopic occupation** of single (generally lowest) mode

$$\psi_0(r) = \frac{e^{i\vec{0}\cdot\vec{r}}}{\sqrt{V}} = \frac{1}{\sqrt{V}}$$

: ground state

$$\psi_m(r) = \frac{e^{im\varphi}}{\sqrt{V}}$$

: flow state (vortex)

Order parameter of condensate $\Psi(\vec{r}) = |\psi|e^{i\phi(\vec{r})}$

- ' wave function of mode into which particles condense

Defined more rigorously by eigenfunction of

largest eigenvalue of density matrix

$$\langle \psi(\vec{r})\psi^\dagger(\vec{r}') \rangle \rightarrow \Psi(\vec{r})\Psi(\vec{r}')^*$$

Superfluid velocity:

$$\vec{v}_s(\vec{r}) = \frac{\hbar}{m} \nabla \phi$$

Chemical potential:

$$\mu = -\partial\phi/\partial t$$

Superfluid acceleration eqn.:

$$\frac{\partial \vec{v}_s}{\partial t} + \nabla \mu = 0$$

Order parameter of BCS paired fermions

Paired seen in amplitude to remove a pair of fermions (" , +) then add pair back, and come back to same state:

$$\langle \psi_{\uparrow}^{\dagger}(1)\psi_{\downarrow}^{\dagger}(2)\psi_{\downarrow}(3)\psi_{\uparrow}(4) \rangle \simeq \langle \psi_{\uparrow}^{\dagger}(1)\psi_{\downarrow}^{\dagger}(2) \rangle \langle \psi_{\downarrow}(3)\psi_{\uparrow}(4) \rangle$$

[Cf., $\langle \psi(\vec{r})\psi^{\dagger}(\vec{r}') \rangle \rightarrow \Psi(\vec{r})\Psi(\vec{r}')^*$ in Bose system]

Order parameter $\langle \psi_{\downarrow}(r)\psi_{\uparrow}(r) \rangle \rightarrow \Psi(r)$, as in Bose system

Similar physics as in Bose system

$$\Psi(\vec{r}) = |\psi|e^{i\phi(\vec{r})}$$

Supercurrent velocity:

$$\vec{v}_s(\vec{r}) = \frac{\hbar}{m} \nabla \phi$$

Chemical potential:

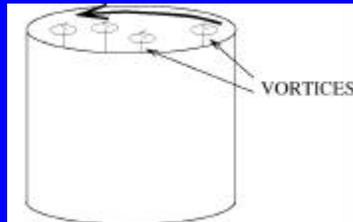
$$\mu = -\partial\phi/\partial t$$

Vortices in superfluids

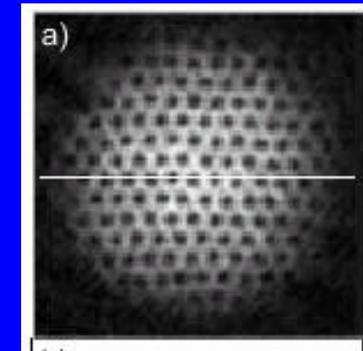
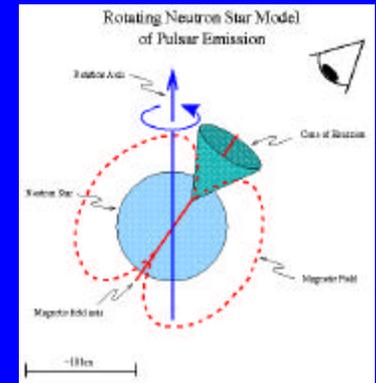


Rotating superfluid neutrons

Rotating superfluid threaded by triangular lattice of vortices parallel to stellar rotation axis



Bose-condensed ^{87}Rb atoms
Schweikhard et al., PRL92 040404 (2004)



Circulation of superfluid velocity about a vortex is quantized:

$$\oint_C \mathbf{v}_s \cdot d\mathbf{l} = \frac{2\pi\hbar}{2m_n}$$

Vortex core $\gg 10$ fm

Vortex separation $\gg 0.01P(\text{s})^{1/2}\text{cm}$; Vela contains $\gg 10^{17}$ vortices

Angular momentum of vortex $=N\sim(1-r^2/R^2)$ decreases as vortex moves outwards \Rightarrow **to spin down must move vortices outwards**

Superfluid spindown controlled by rate at which vortices can move against barriers, under dissipation

Vortices in superfluids: quantized circulation

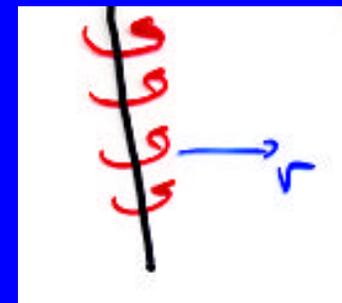
Order parameter: $\Psi(\vec{r}) = |\psi| e^{i\phi(\vec{r})}$

Superfluid velocity: $\vec{v}(\vec{r}) = \frac{\hbar}{m} \nabla \phi$

Quantized circulation: $\oint_C \vec{v} \cdot d\vec{\ell} = \frac{2\pi n \hbar}{m}$ $n = \text{integer} (! \quad 1)$

Singly quantized ($n=1$) vortex flow:

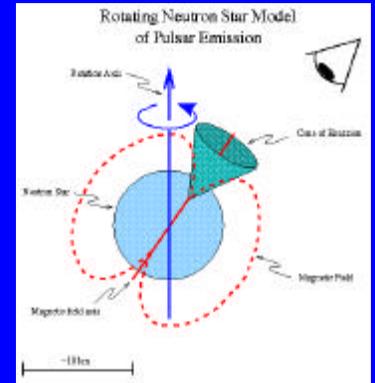
$$v_\phi(r) = \frac{\hbar}{mr}$$



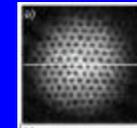
Superconducting protons in magnetic field

Even though superconductors expel magnetic flux, for magnetic field below critical value, flux diffusion times in neutron stars are \gg age of universe.

Proton superconductivity forms with field present.



Proton fluid threaded by triangular (Abrikosov) lattice of vortices parallel to magnetic field (for Type II superconductor)



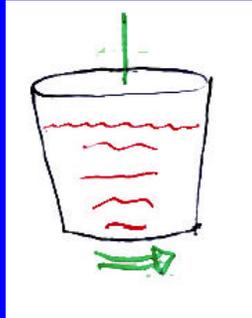
Magnetic flux associated with each vortex is quantized:

$$\oint_C \mathbf{B} \cdot d\mathbf{l} = \frac{2\pi\hbar c}{2e} = \phi_0 = 2\pi \times 10^{-7} \text{G}.$$

Vortex core \gg 10 fm,

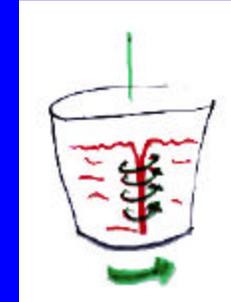
$$n_{\text{vort}} = B/\phi_0 \Rightarrow \text{spacing} \sim 5 \times 10^{-10} \text{ cm } (B / 10^{12} \text{G})^{-1/2}$$

Critical velocity for vortex formation in rotating superfluid



$$\Omega < \Omega_{c1}$$

$\Omega > \Omega_{c1}$:
form vortices



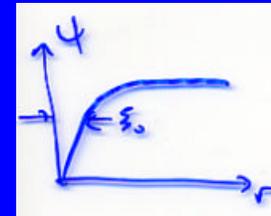
Rotate system at Ω . Minimize energy in rotating frame: $E^0 = E - \Omega L$,
 $L =$ angular momentum of the system.

First vortex appears when $E_0 + E_{1\text{vortex}} - \Omega L_{1\text{vortex}} = E_0$

$$E_{1\text{vortex}} = \int d^3r n \frac{1}{2} m v(\vec{r})^2 = \int d^3r n \frac{\hbar^2}{2m r_{\perp}^2} = N \frac{\hbar^2}{m R^2} \ln \frac{R}{\xi_0}$$

$$L_{1\text{vortex}} = N \hbar$$

$$\Omega_{c1} = \frac{\hbar}{m R^2} \ln \frac{R}{\xi_0}$$



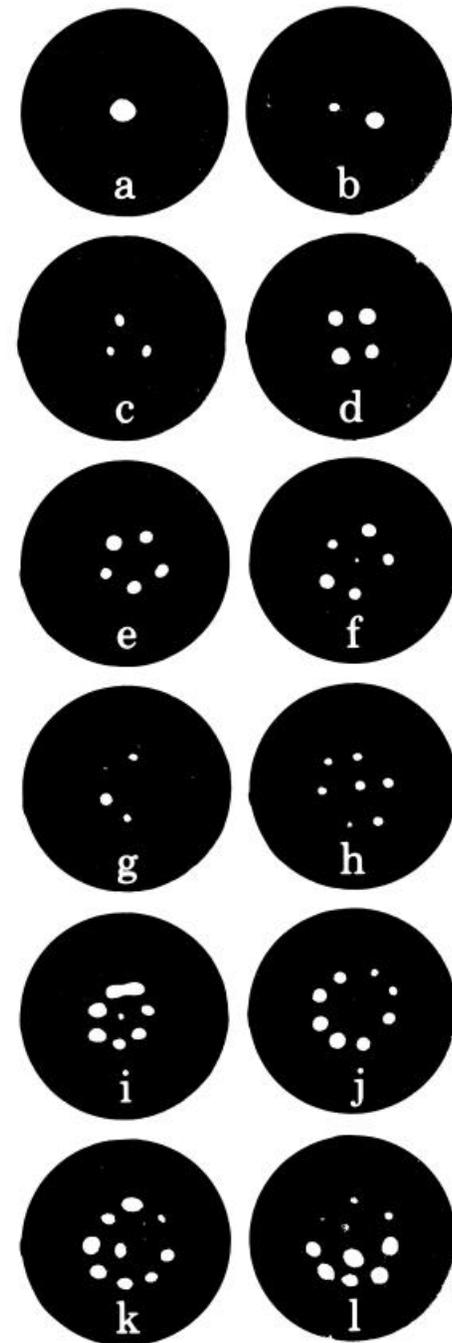
$R =$ system
radius

$\xi_0 =$ vortex
core radius

Vortices in superfluid ^4He

View along rotation axis. Image
by trapping electrons in cores

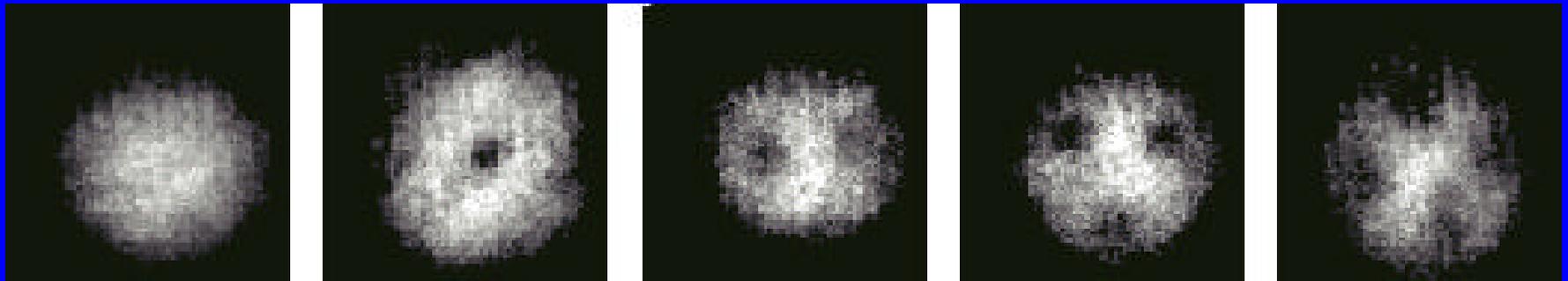
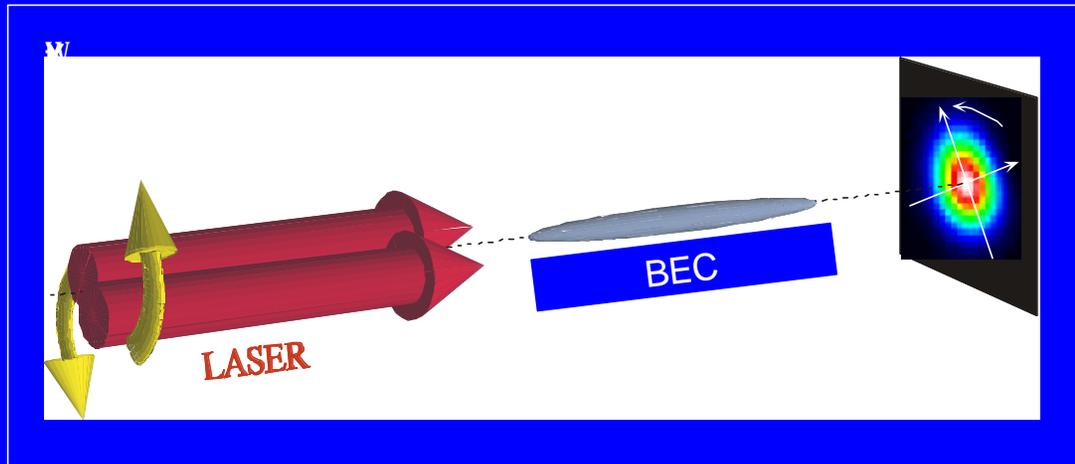
*Yarmchuk, Gordon, & Packard
PRL43, 214 (1979)*



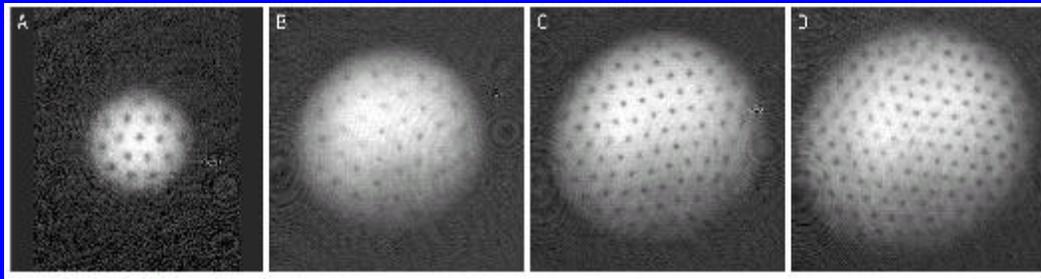
Vortices in Bose-Einstein condensates

Bose condensed ^{87}Rb (ENS)

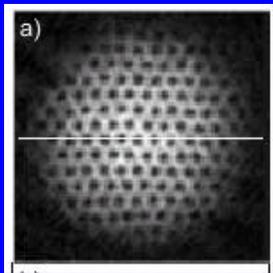
K. W. Madison, F. Chevy, W. Wohlleben, J. Dalibard 1999



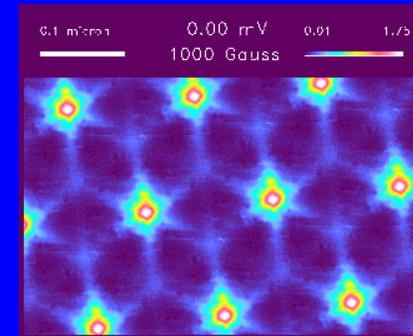
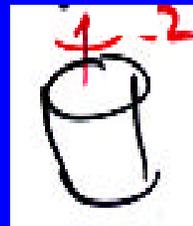
Rapidly rotating superfluid contains (nearly) triangular lattice of vortices



*Abo-Shaeer et al.
(MIT) 2001*



*Engels et al.
(JILA) 2002*



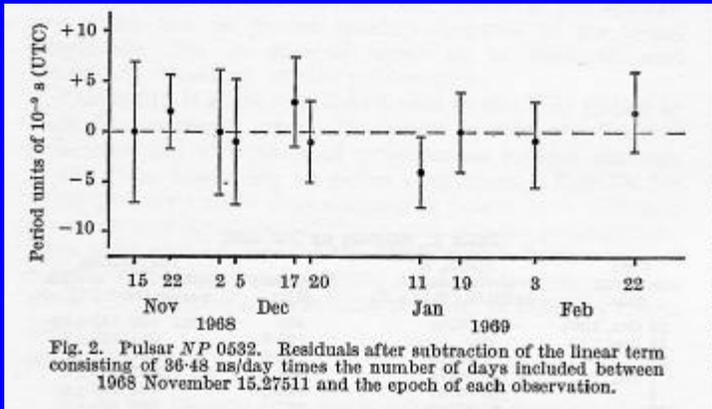
Type II superconductor

As Ω grows, what happens to vortex lattice?
How does a Bose gas carry large quantities of angular momentum, $L/N \gg (10^2 - N)\hbar$? End of superfluidity!

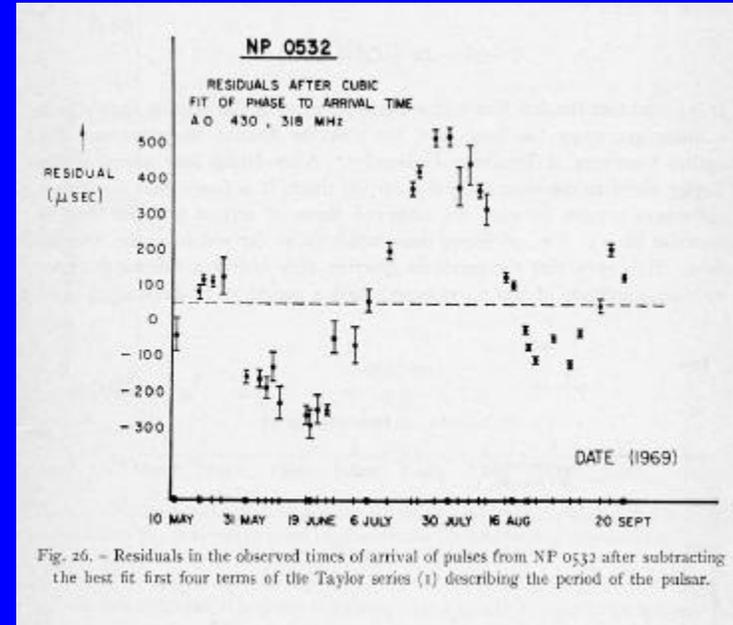
Early timing (1969) of the Crab pulsar, NP0532

Period of pulsar

$$= P_0 + P_1 t + \text{interesting residuals}$$

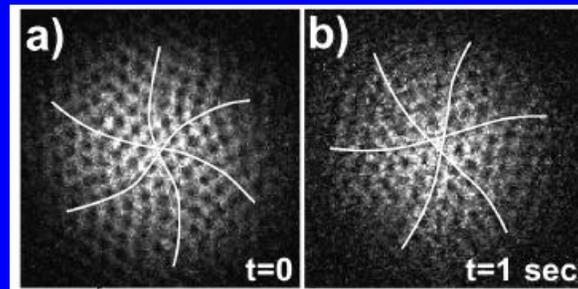


Richards and Comella, Nature 1969



Drake et al., Nature 1969

Ruderman (*Nature 1970*) proposed Tkachenko modes



Schweikhard et al., PRL92 040404 (2004)

Fundamental period $\sim (\pi / 2 R_{\text{star}}) (\Omega / m_n)^{1/2}$ sec. ~ 4 months

BCS paired fermions: a new superfluid

Production of trapped degenerate Fermi gases: ${}^6\text{Li}$, ${}^{40}\text{K}$

Feshbach resonances to increase attractive interaction

At resonance have “unitary regime”: no length scale

Experiments: JILA, MIT, Duke, Innsbruck, ...

Detecting pairing and superfluidity

${}^7\text{Li}$

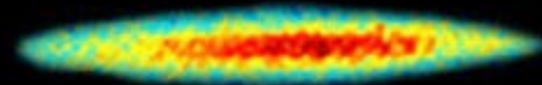
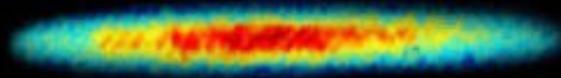
${}^6\text{Li}$

Observing Statistics

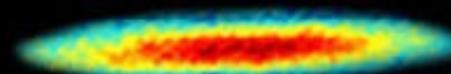
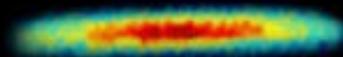
Bosons

Fermions

High T:
Boltzmann
distribution

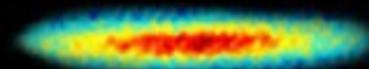
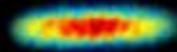


810 nK



510 nK

Low T:
Degenerate gas



240 nK

Hulet

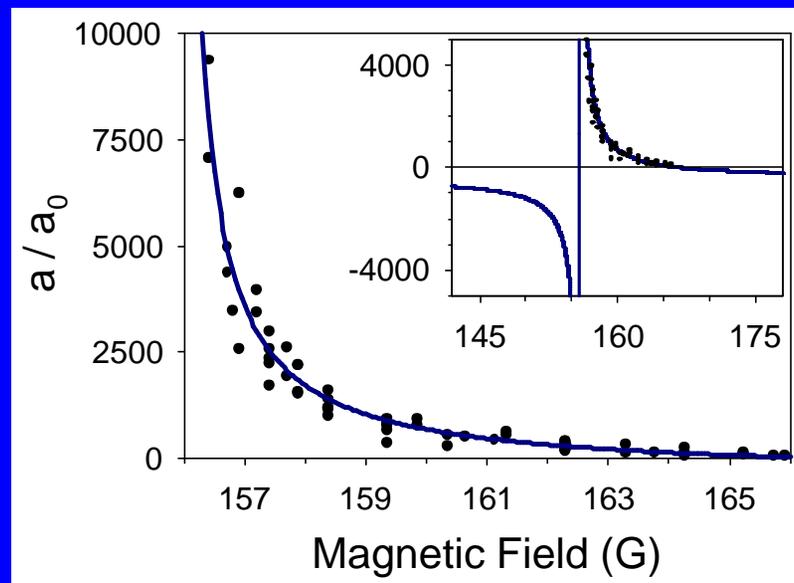
Controlling the interparticle interaction

Effective interparticle interaction is short range and s-wave:

$$V(\mathbf{r}_1-\mathbf{r}_2) = (4\pi\hbar^2 a/m) \delta(\mathbf{r}_1-\mathbf{r}_2)$$

a = s-wave atom-atom scattering length. $\sigma=8\pi a^2$

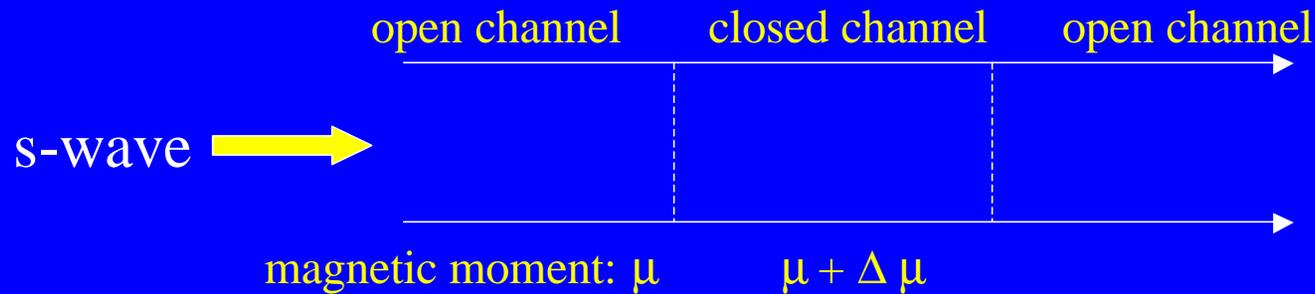
Via **Feshbach resonance** in magnetic field can vary particle interaction strength -- and even reverse sign of interaction



^{85}Rb

Cornish *et al*, (JILA) PRL **85**, 1795 (2000)

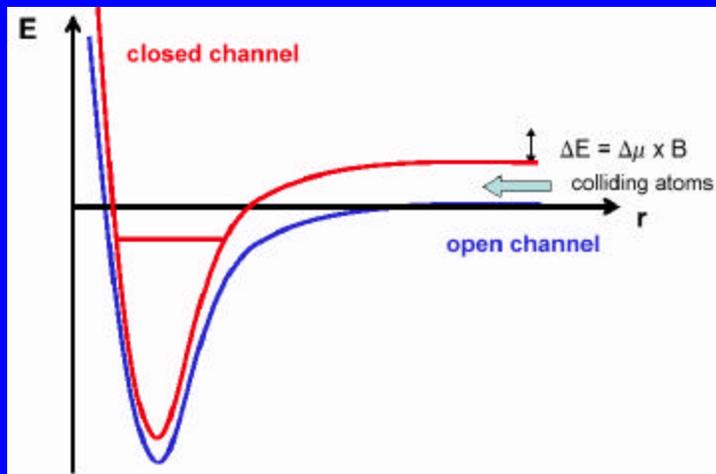
Feshbach resonance in atom-atom scattering



$$\text{Scattering amplitude} \propto \frac{|M|^2}{E_c - E_0}$$

$$E_c - E_0 \gg \Delta\mu B + \dots$$

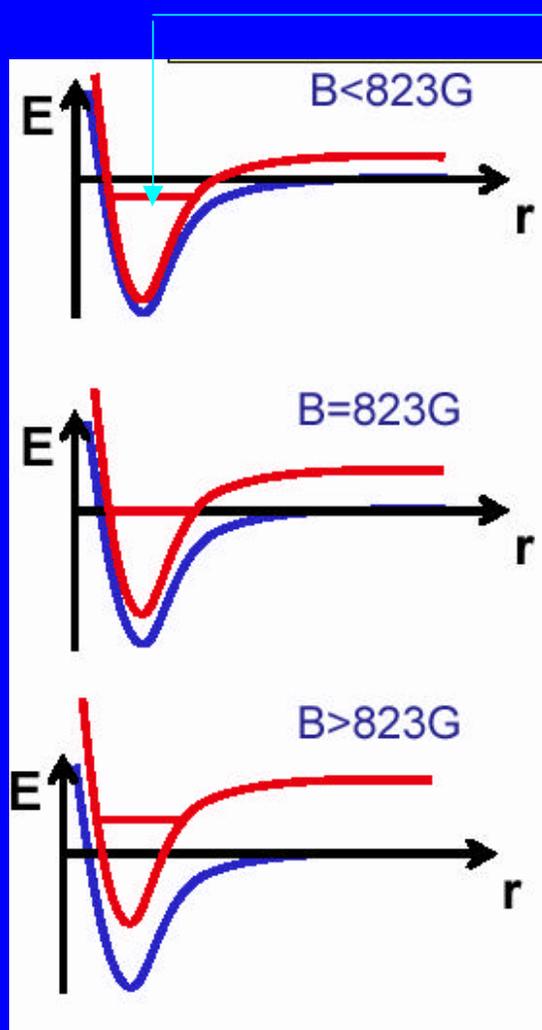
Low energy scattering dominated by bound state closest to threshold



Adjusting magnetic field, B , causes level crossing and resonance, seen as divergence of s-wave scattering length, a

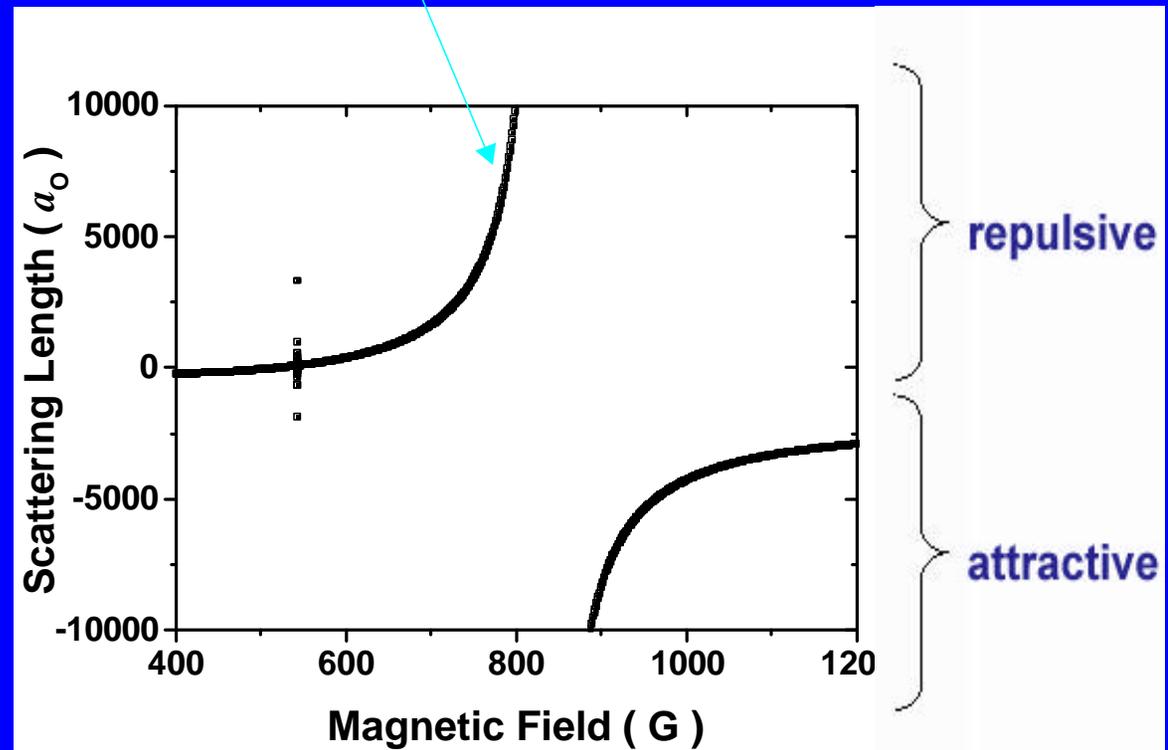
$$a(B) = a_{bg} \left(1 - \frac{\Delta}{B - B_{Feshbach}} \right)$$

${}^6\text{Li} - {}^6\text{Li}$ Feshbach resonance



weakly bound molecule
in closed channel

Effective
interaction



Broad resonance around 820-830 Gauss

Increasing magnetic field through resonance changes interactions from repulsive to attractive; very strong in neighborhood of resonance

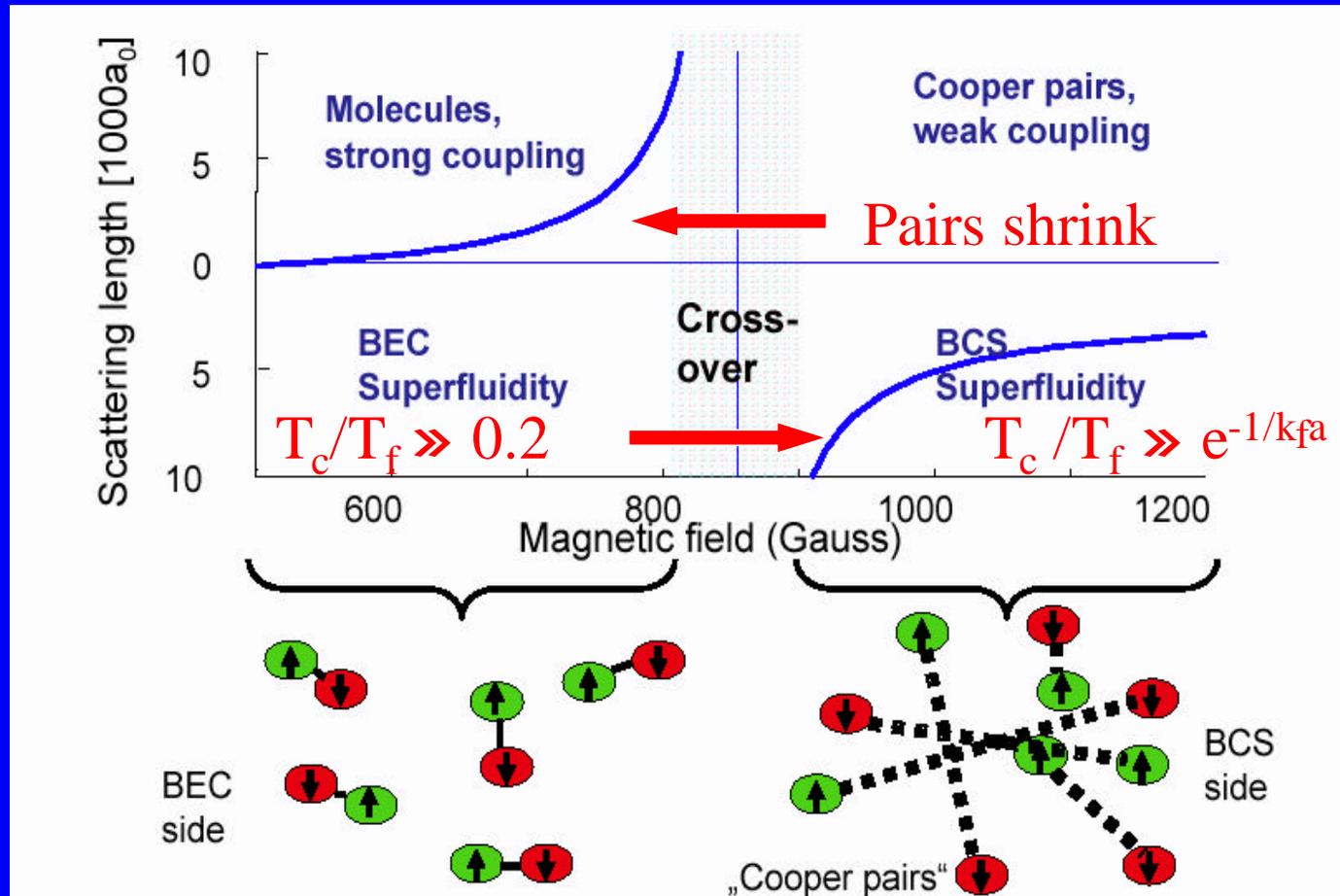
BEC-BCS crossover in Fermi systems

Continuously transform from molecules to Cooper pairs:

D.M. Eagles (1969)

A.J. Leggett, J. Phys. (Paris) C7, 19 (1980)

P. Nozières and S. Schmitt-Rink, J. Low Temp Phys. 59, 195 (1985)



⁶Li

Vortices in trapped Fermi gases

M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, and W. Ketterle, Nature 435, 1047 (2005)

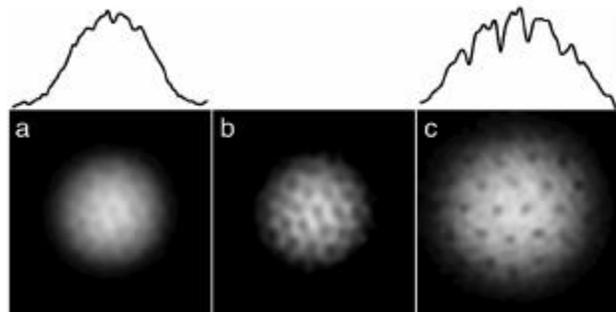


Fig. 1: Observation of a vortex lattice in a molecular condensate. (a) Fixed field. Stirring for 800 ms, followed by 400 ms of equilibration, and imaging after 12 ms time-of-flight all took place at 766 G. The vortex core depletion of the integrated density profile is barely 10%, as indicated by the 5- μm -wide cut on top. (b) Fourier-filter applied to (a) to accentuate the vortex contrast.

Resonance at $\gg 834\text{G}$

$B < 834\text{G} = \text{BEC}$

$B > 834\text{G} = \text{BCS}$

${}^6\text{Li}$

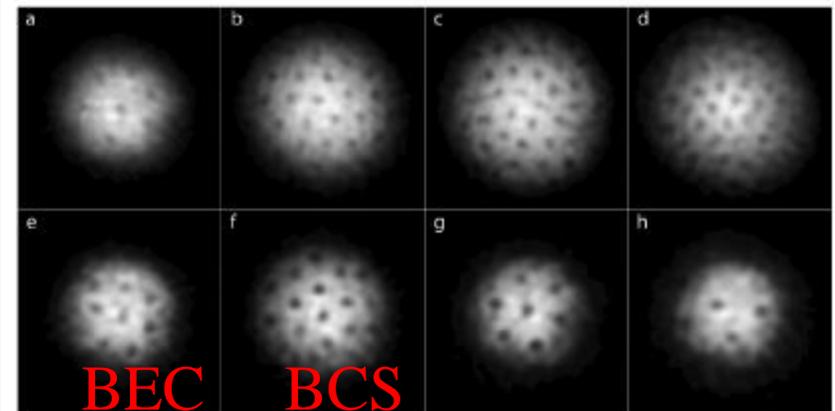


Fig. 2: Vortices in a strongly interacting gas of fermionic atoms on the BEC- and the BCS-side of the Feshbach resonance. At the given field, the cloud of lithium atoms was stirred for 300 ms (a) to 500 ms (b-h) followed by an equilibration time of 500 ms. After 2 ms of ballistic expansion, the magnetic field was ramped to 735 G for imaging (see text for details). The magnetic fields were (a) 740 G, (b) 766 G, (c) 792 G, (d) 812 G, (e) 833 G, (f) 843 G, (g) 853 G and (h) 863 G. The field of view of each image is $880\ \mu\text{m} \times 880\ \mu\text{m}$.