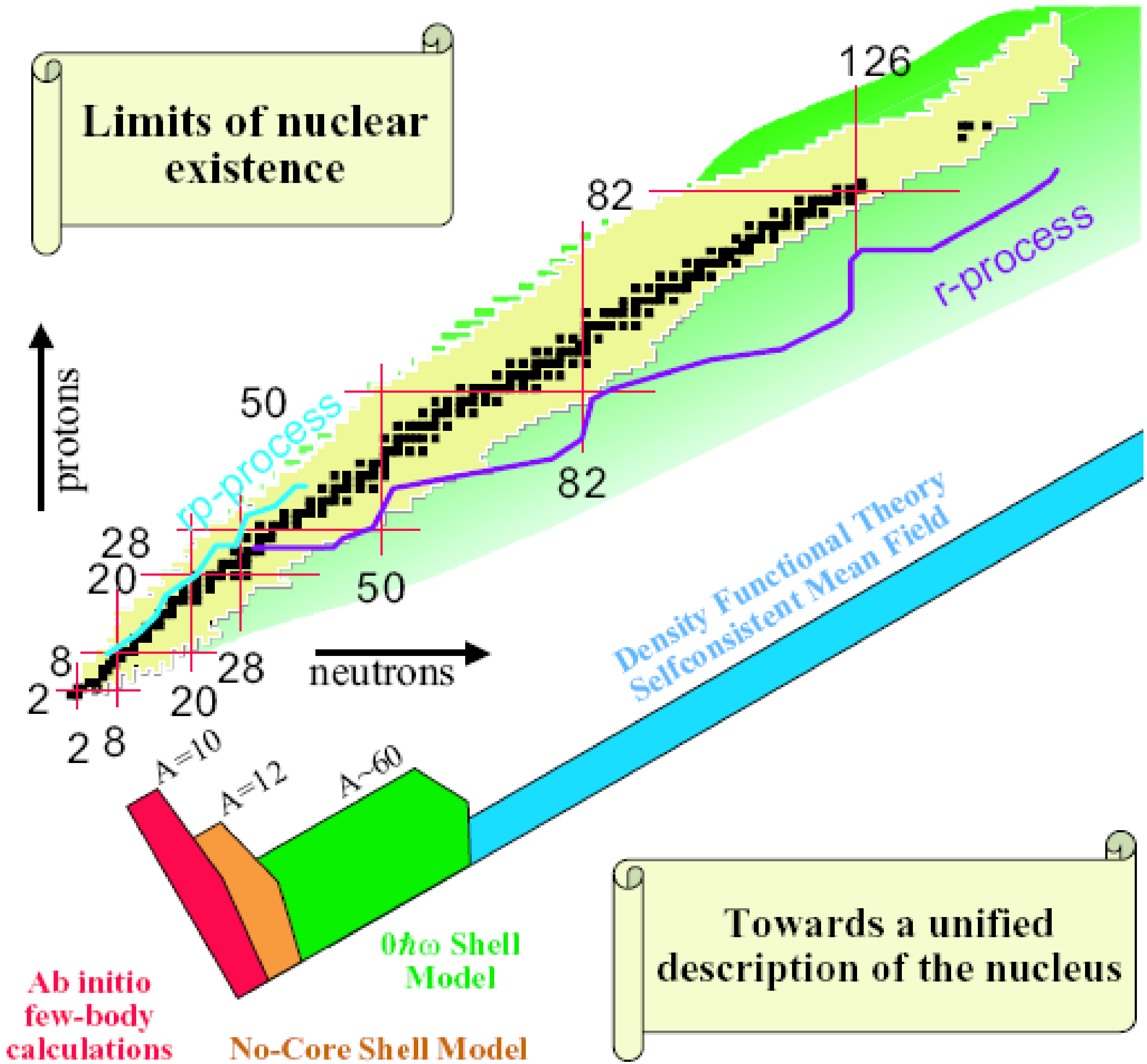
A Novel Approach to the Decay of Superdeformed Bands:

The Life and Death of Superdeformed Nuclei

Bruce R. Barrett
University of Arizona, Tucson

THE UNIVERSITY OF ARIZONA.
Arizona's First University.

20th Chris Engelbrecht Summer School in Theoretical Physics, January 19-28, 2009



Limits of nuclear existence
protons
neutrons
126
82
50
28
20
8
2
2 8
20
28
50
82
rp-process
r-process
Density Functional Theory
Selfconsistent Mean Field
A=10
A=12
A~60
Ab initio few-body calculations
No-Core Shell Model
0ħω Shell Model
Towards a unified description of the nucleus

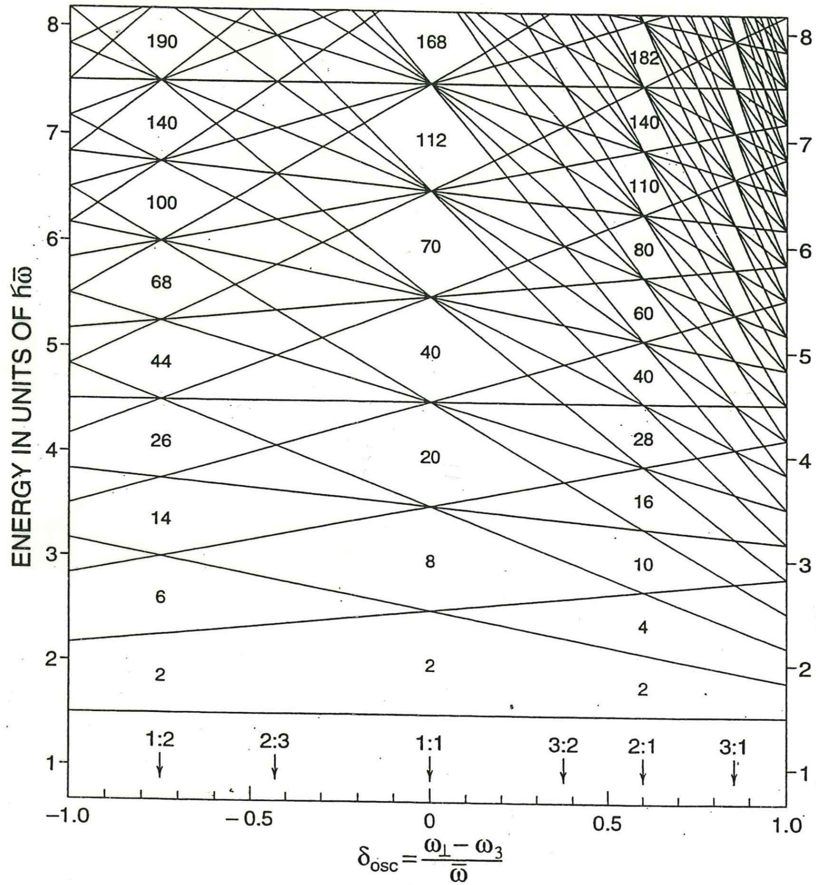
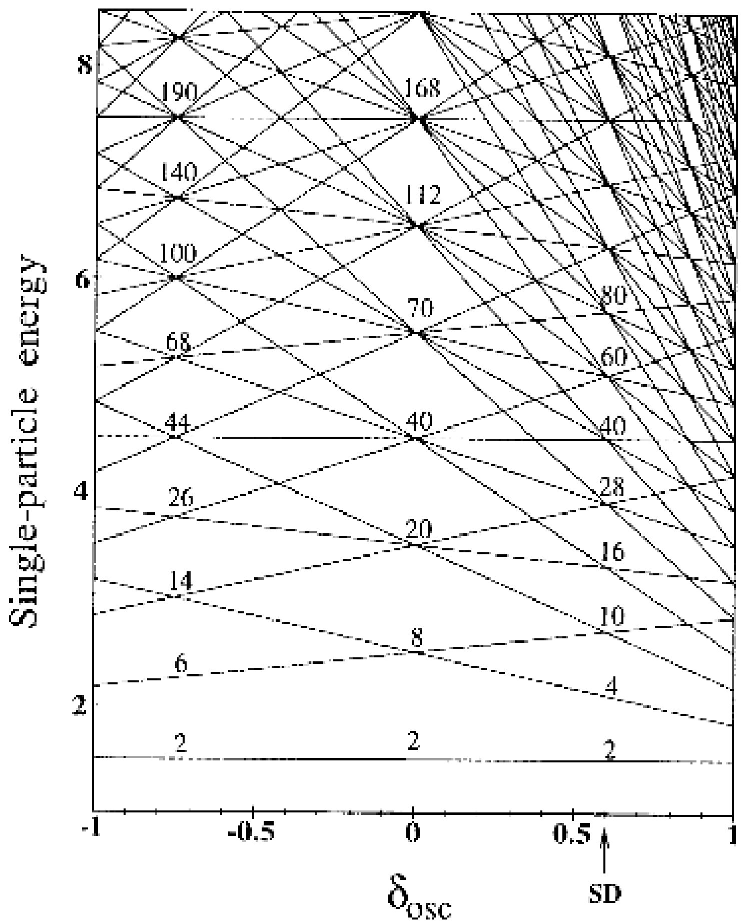


Figure 1. Single-particle level energies calculated for an axially symmetric harmonic oscillator (from reference 2).

$$H = \frac{\vec{P}^2}{2m} + \frac{1}{2} m \left(\omega_1^2 x^2 + \omega_2^2 y^2 + \omega_3^2 z^2\right)$$

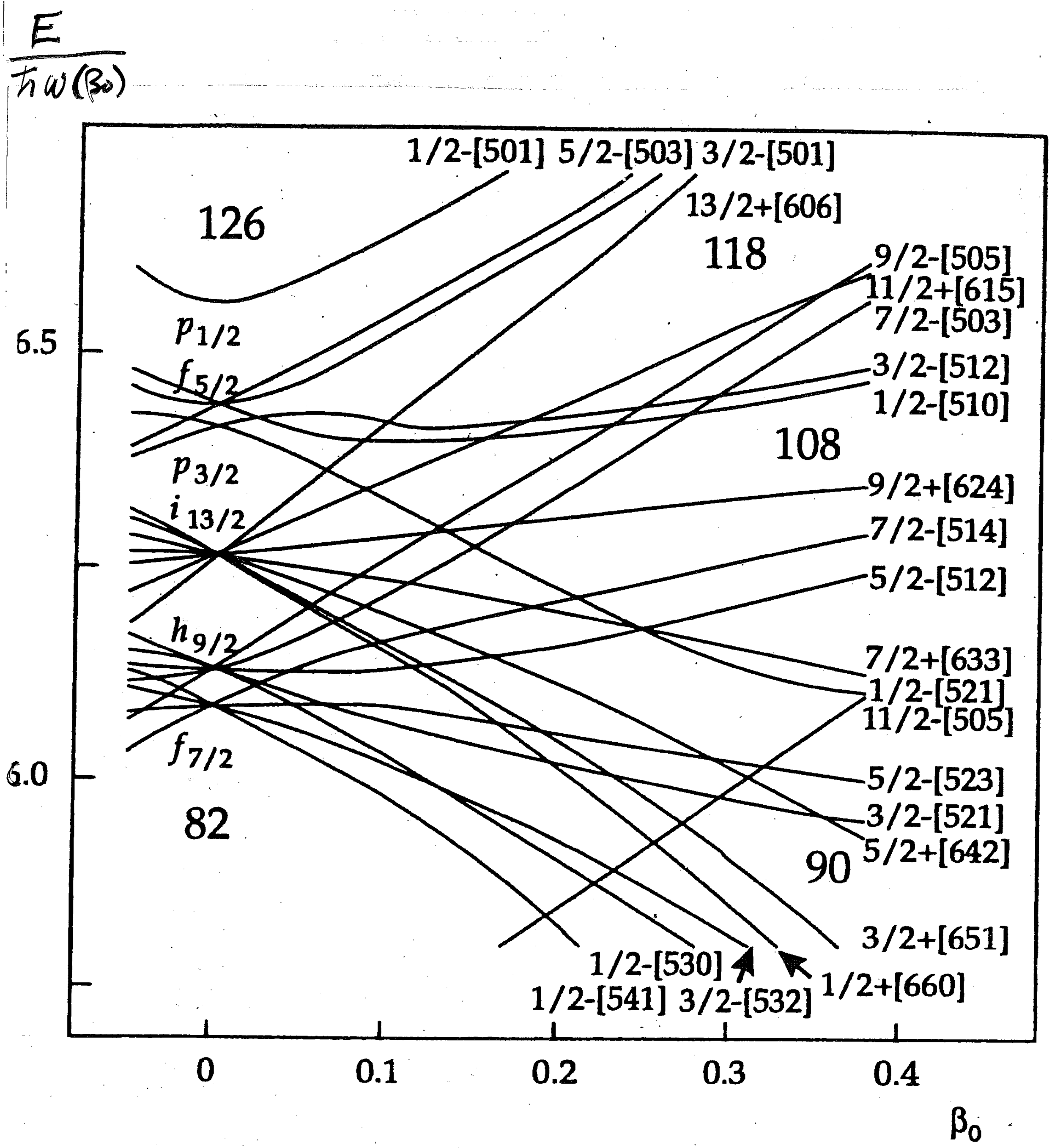
$$\omega_1 = \omega_2 \equiv \omega_\perp$$

Superdeformation



from Wong (1998).

- General prediction of shell models.
- Ellipsoidal and highly deformed: $\frac{\text{major}}{\text{minor}} \approx 2$.
- Clear experimental signature
  - Large electric quadrupole: $Q \approx .007ZA^{2/3}\text{eb}$.
  - Little centrifugal stretching: rigid rotor spectrum.
- For very high angular momenta, SD states can be yrast.



E / ħω(β0)
6.5
6.0
126
118
108
90
82
p1/2
f5/2
p3/2
i13/2
h9/2
f7/2
1/2-[501]
5/2-[503]
3/2-[501]
13/2+[606]
9/2-[505]
11/2+[615]
7/2-[503]
3/2-[512]
1/2-[510]
9/2+[624]
7/2-[514]
5/2-[512]
7/2+[633]
1/2-[521]
11/2-[505]
5/2-[523]
3/2-[521]
5/2+[642]
3/2+[651]
1/2+[660]
1/2-[530]
1/2-[541]
3/2-[532]
0
0.1
0.2
0.3
0.4
β0

harmonic oscillator potential, but the occurrence of shell gaps at large deformation persists. When the proton and neutron numbers are both favorable for the occurrence of shell gaps, superdeformation is found. This leads to SD nuclei, which congregate in local regions in the chart of nuclides, with mass numbers around 80, 130, 150, 190 and 240 (Fig. 3, from Ref. [8]).

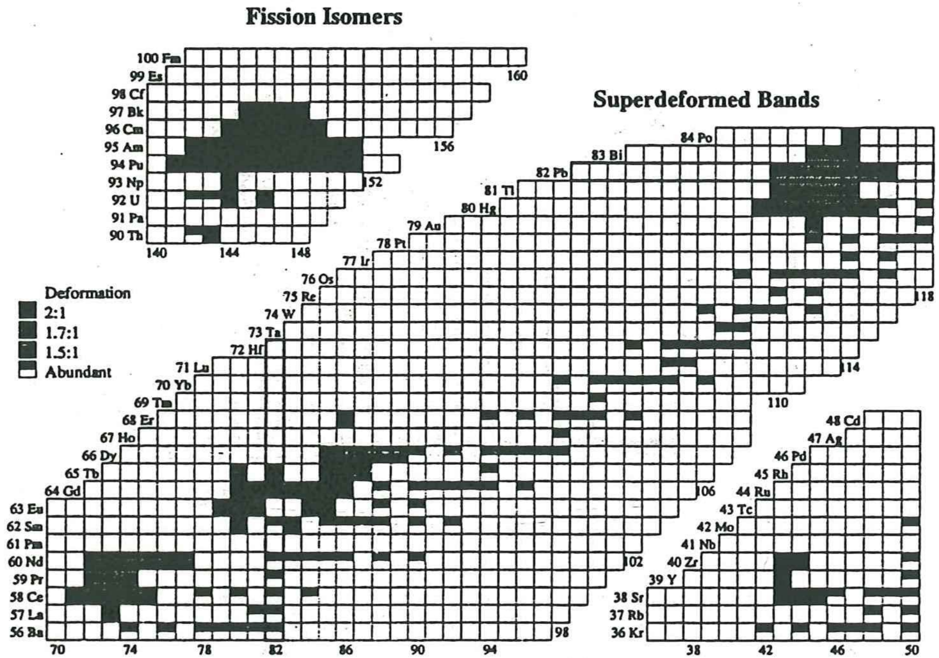


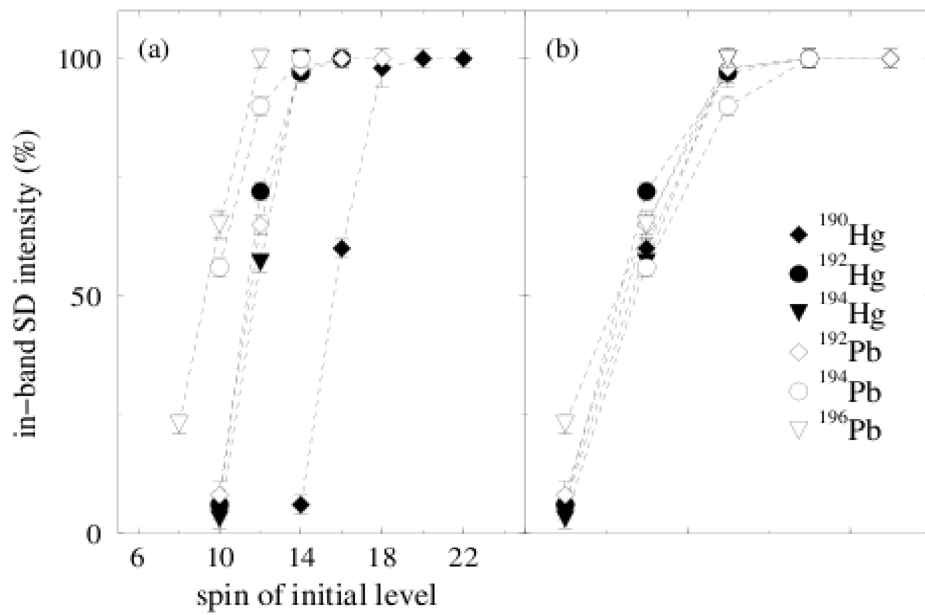
Figure 3: Chart of nuclides, showing local regions of superdeformation in the A ~ 80, 130, 150, 190 and 240 regions; from Ref. [8].

A fascinating feature of almost all SD bands is the sudden drop in intraband transition intensity at low spin, after a string of up to 20 consecutive transitions (Fig. 4). In other words, after a long sequence of transitions within the false vacuum, there is a sudden decay to the true vacuum.

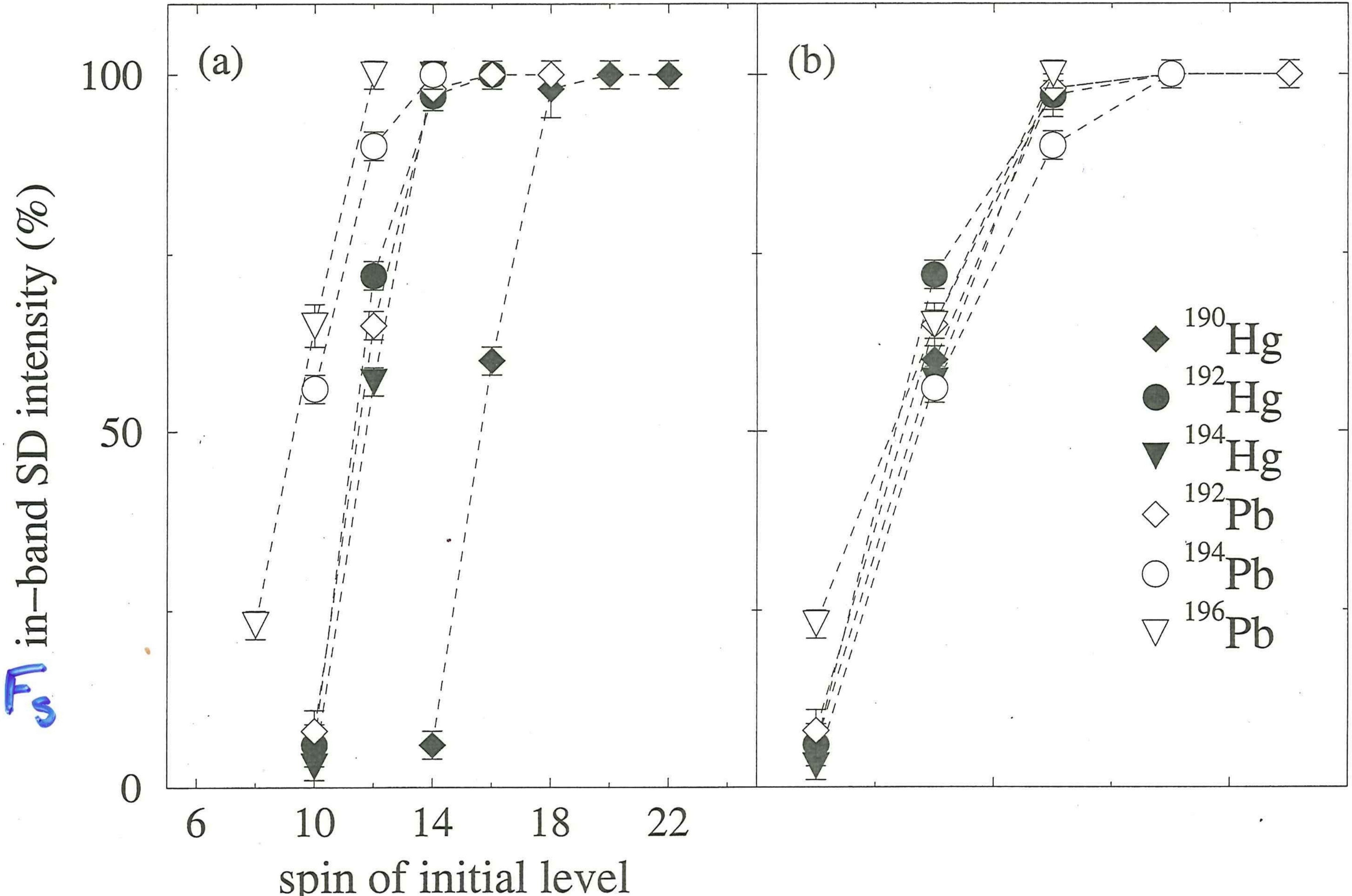
A γ cascade which flows through a SD minimum has three stages (Fig. 1): (a) feeding and trapping into the SD well, (b) intraband transitions within SD bands, and (c) decay from SD to ND states. In stage (a), hot compound nuclear states cool via γ emission. This stage involves the coupling of hot SD and ND states, which includes tunneling between hot states on either side of the barrier. A small fraction (typically around 1 %) of the cascades becomes trapped in the

Interesting Questions
A shopping list

- How many states do we need to keep in the ND well?
- How important is electromagnetic broadening?
- Can we extract information about the potential barrier from a decay experiment?
- Why are the decay profiles for $A \approx 190$ so similar?

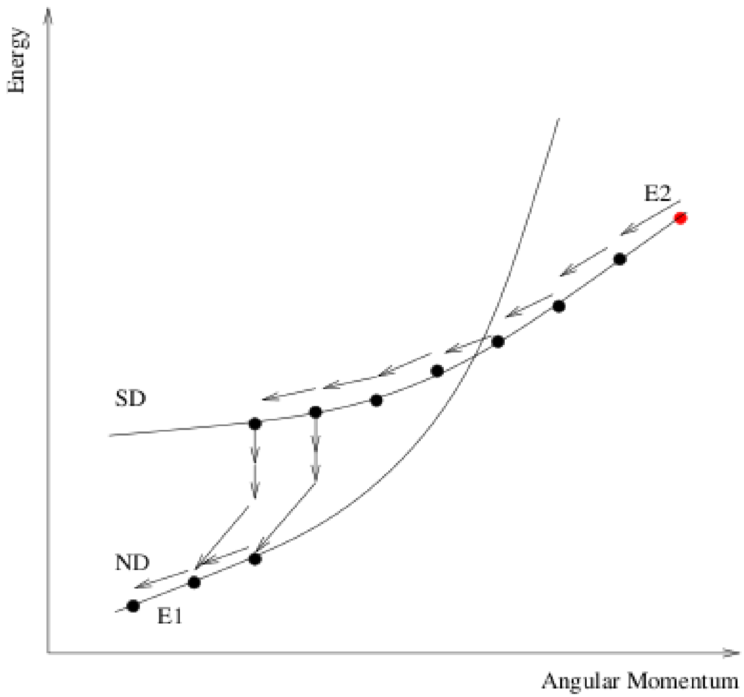


from Wilson *et al.* *PRC*, **71**, 34319 (2005).



A.N. Wilson et al. Phys Rev C 71, 034319 (2005)

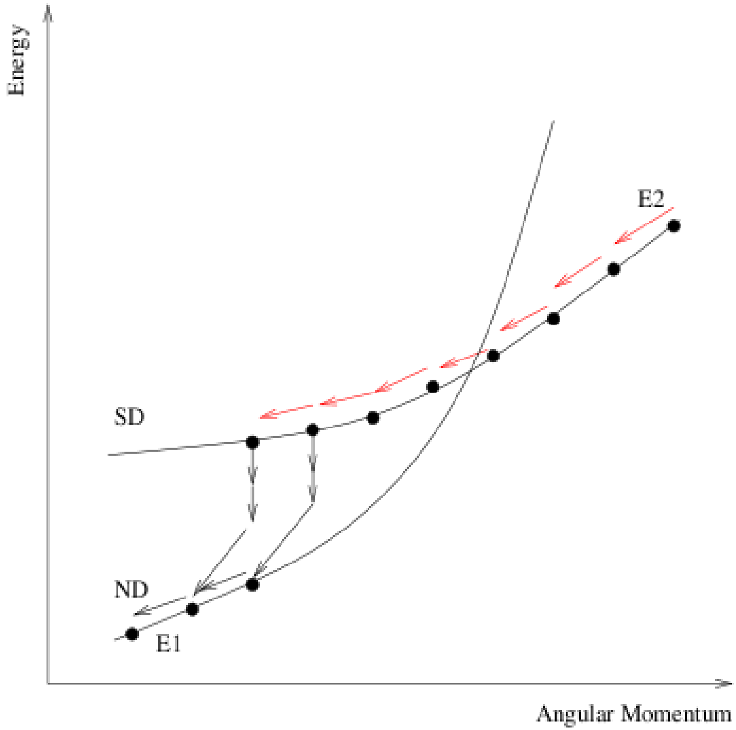
Life and Death of an SD Nucleus



Typical Decay Experiment

1. Nucleus is created in a high angular momentum SD yrast state.
2. Decay via E2 transitions along SD rotational band.
3. Transistion to a lower-lying ND band.
4. Decay down ND band via E1-dominated transitions.

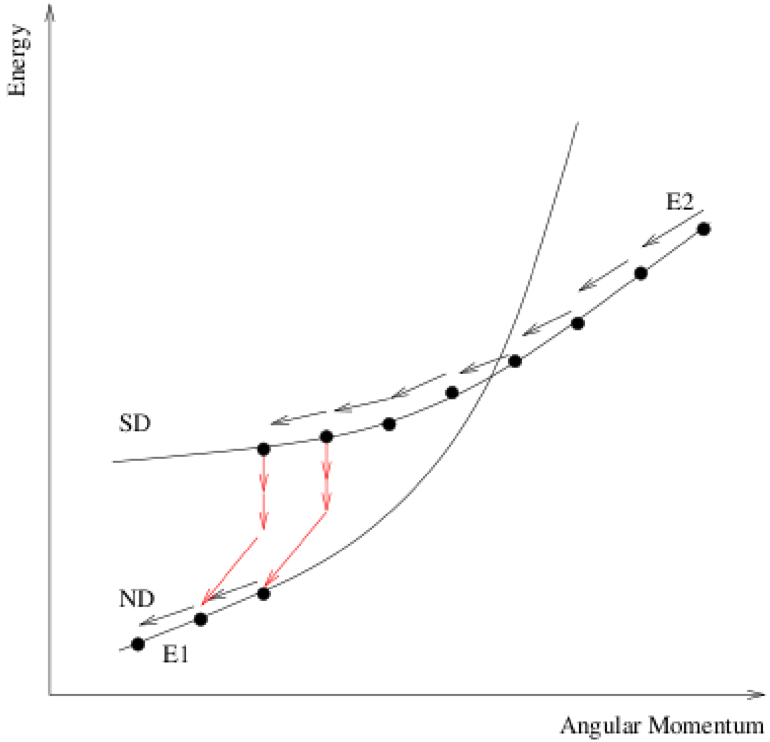
Life and Death of an SD Nucleus



Typical Decay Experiment

1. Nucleus is created in a high angular momentum SD yrast state.
2. Decay via E2 transitions along SD rotational band.
3. Transistion to a lower-lying ND band.
4. Decay down ND band via E1-dominated transitions.

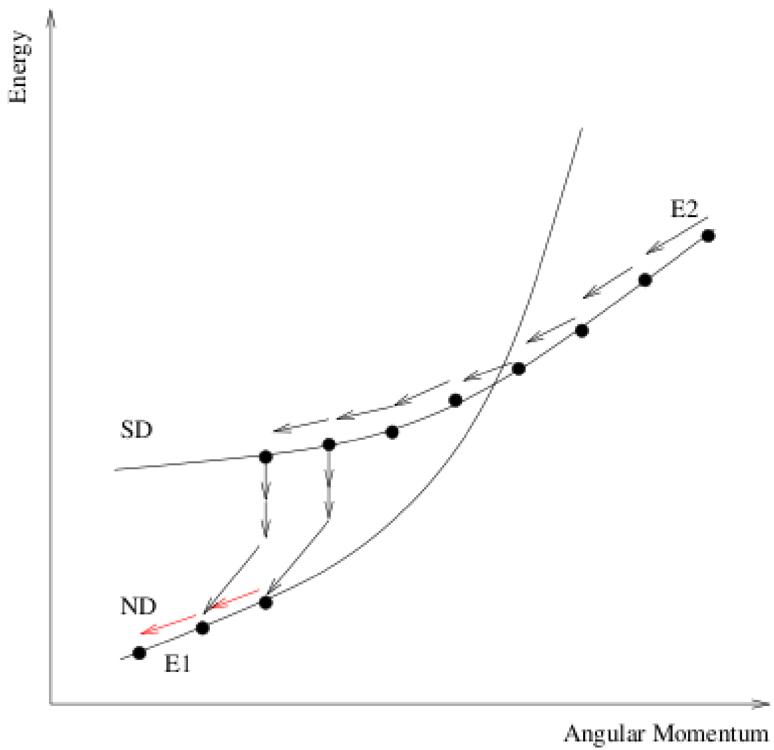
Life and Death of an SD Nucleus



Typical Decay Experiment

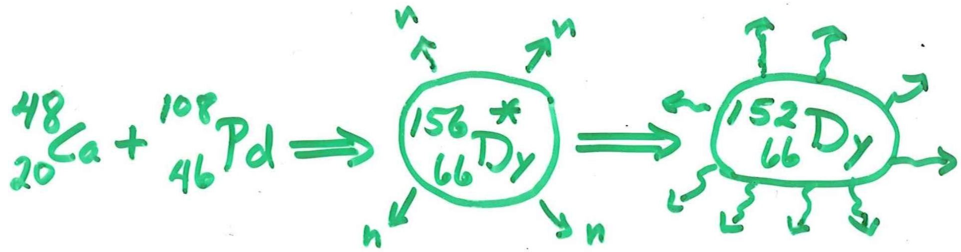
1. Nucleus is created in a high angular momentum SD yrast state.
2. Decay via E2 transitions along SD rotational band.
3. Transistion to a lower-lying ND band.
4. Decay down ND band via E1-dominated transitions.

Life and Death of an SD Nucleus



Typical Decay Experiment

1. Nucleus is created in a high angular momentum SD yrast state.
2. Decay via E2 transitions along SD rotational band.
3. Transistion to a lower-lying ND band.
4. Decay down ND band via E1-dominated transitions.



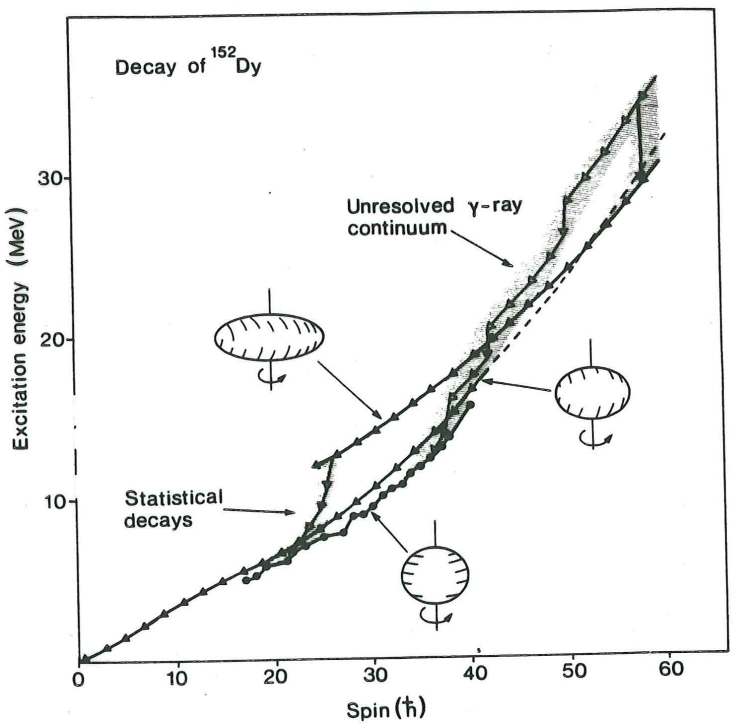
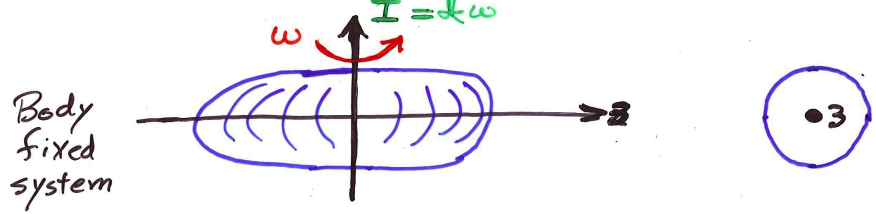


Figure 5 A schematic of the proposed γ-ray decay paths from a high-spin entry point in ^{152}Dy. The major initial decay flow occurs mainly via E2 transitions in the unresolved γ-ray continuum and reaches the oblate yrast structures between 30ħ and 40ħ. A small 1% branch feeds the superdeformed band, which is assumed to become yrast at a spin of 50–55ħ. The deexcitation of the superdeformed band around 26ħ occurs when the band is 3–5 MeV above yrast, and a statistical type of decay flow takes it into the oblate states between 19ħ and 2[illegible]ħ. The diagram also shows the low deformation prolate band.

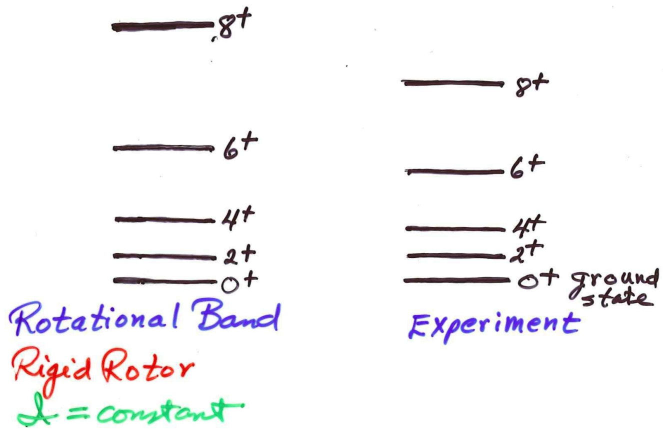
P.J. Twin et al. Phys. Rev. Lett. 57, 811 (1985)

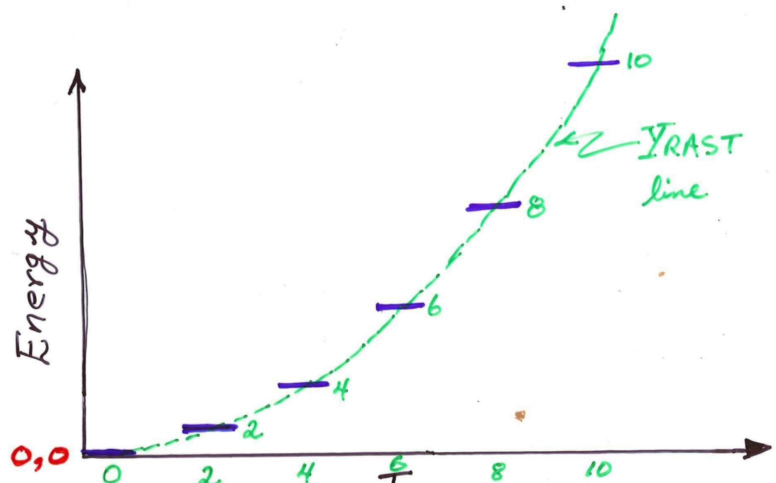
Collective Model

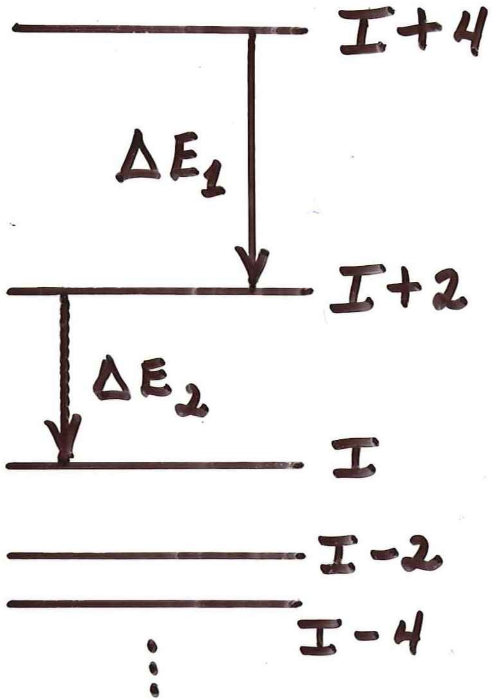
A. Bohr & B. Mottelson



$$E = \frac{1}{2}\mathcal{I}\omega^2 = \frac{\vec{I}^2}{2\mathcal{I}} = \frac{I(I+1)\hbar^2}{2\mathcal{I}}$$







$$\Delta E_1 = E(I+4) - E(I+2)$$

$$= \frac{[(I+4)(I+5) - (I+2)(I+3)]\hbar^2}{2\mathcal{I}}$$

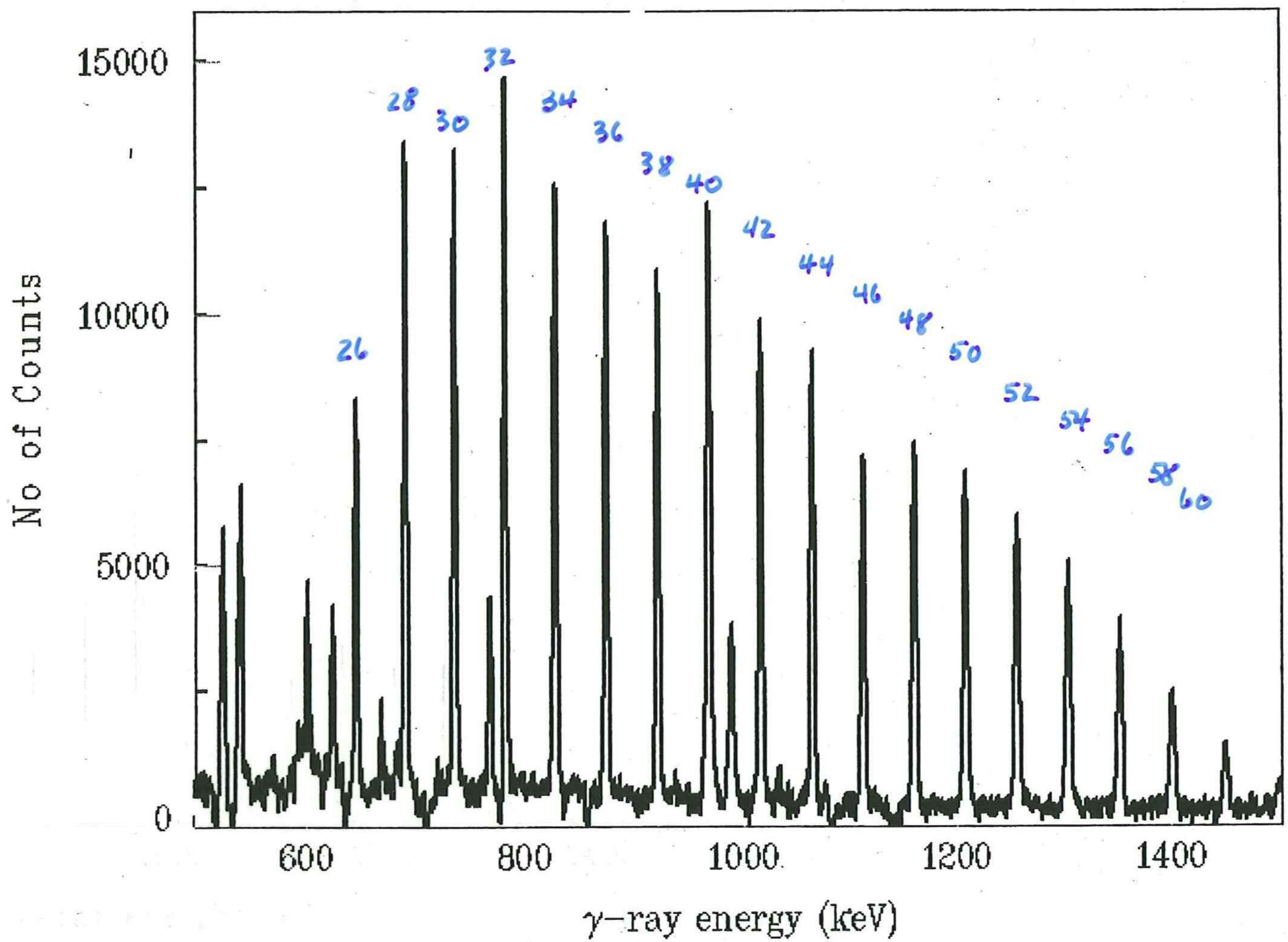
$$\Delta E_2 = E(I+2) - E(I)$$

$$= \frac{[(I+2)(I+3) - I(I+1)]\hbar^2}{2\mathcal{I}}$$

Double Difference

$$\delta E = \Delta E_1 - \Delta E_2 = \frac{4\hbar^2}{\mathcal{I}}$$

$\Rightarrow$ Equally spaced spectral lines, if $\mathcal{I}$ is constant.



Spectrum of the superdeformed band in ^{152}Dy

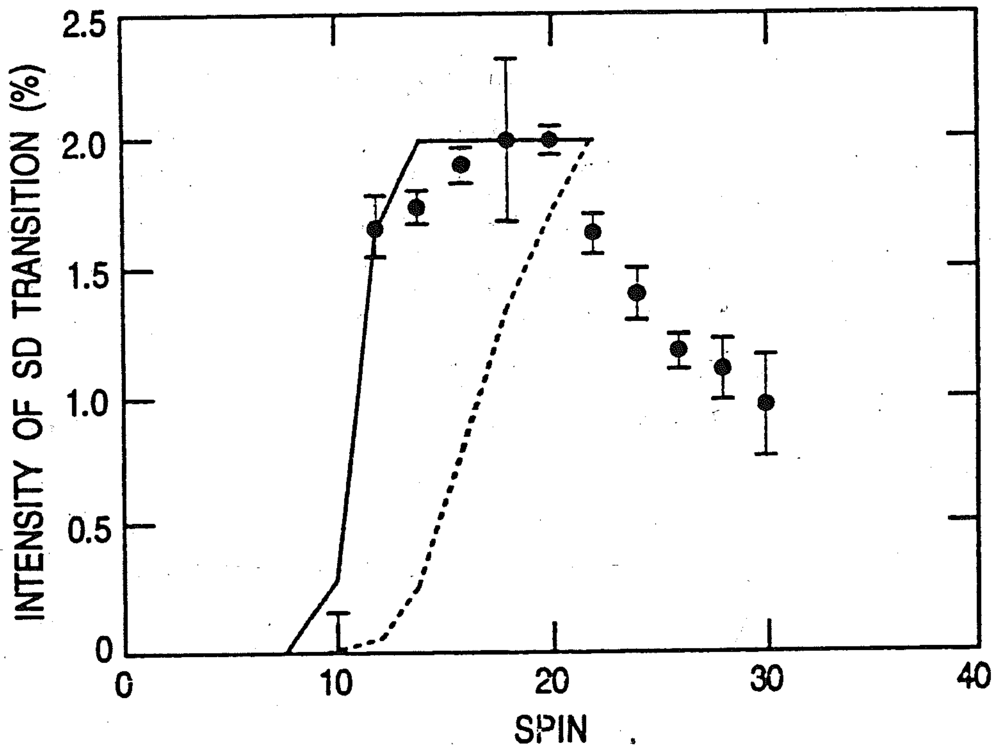


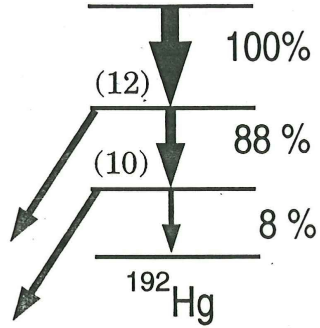
Figure 4: Intensities of SD-band transitions in ^{192}Hg as a function of initial spin, showing a sudden drop around spin 12. The solid and dashed lines are results of calculations using the Vigezzi model [20], with the SD well depth W(I) assumed to either increase with spin (solid line) or remain constant (dashed line).

Reiner Krücken, et al

$$\beta^2 = \frac{\lambda_{out}}{\lambda_n^{E1}}$$

Mischungsamplituden sind sehr klein !!!

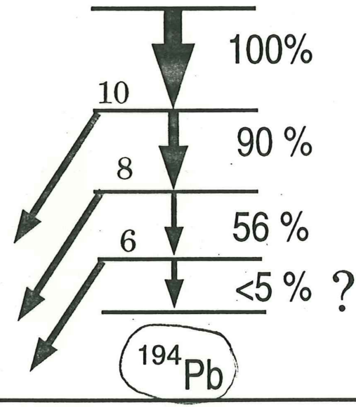
β^2		Q_t	
0.001		18.3	exp.
0.047		18.3	extrapol.



$F_s \equiv$ Nintra

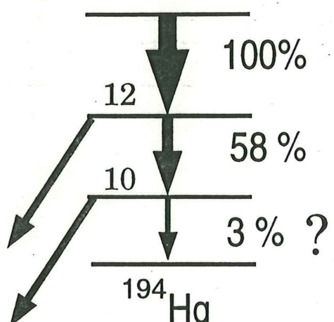
P. Wilsau et al.
A. Dewald et al.
I.Y. Lee et al.

β^2	Q_t	
0.003	18.8	exp.
0.005	18.8	exp.
0.036	18.8	extrapol.



This work

β^2	Q_t	
0.004	16.9	exp.
0.066	16.9	extrapol.



R. Kühn et al.

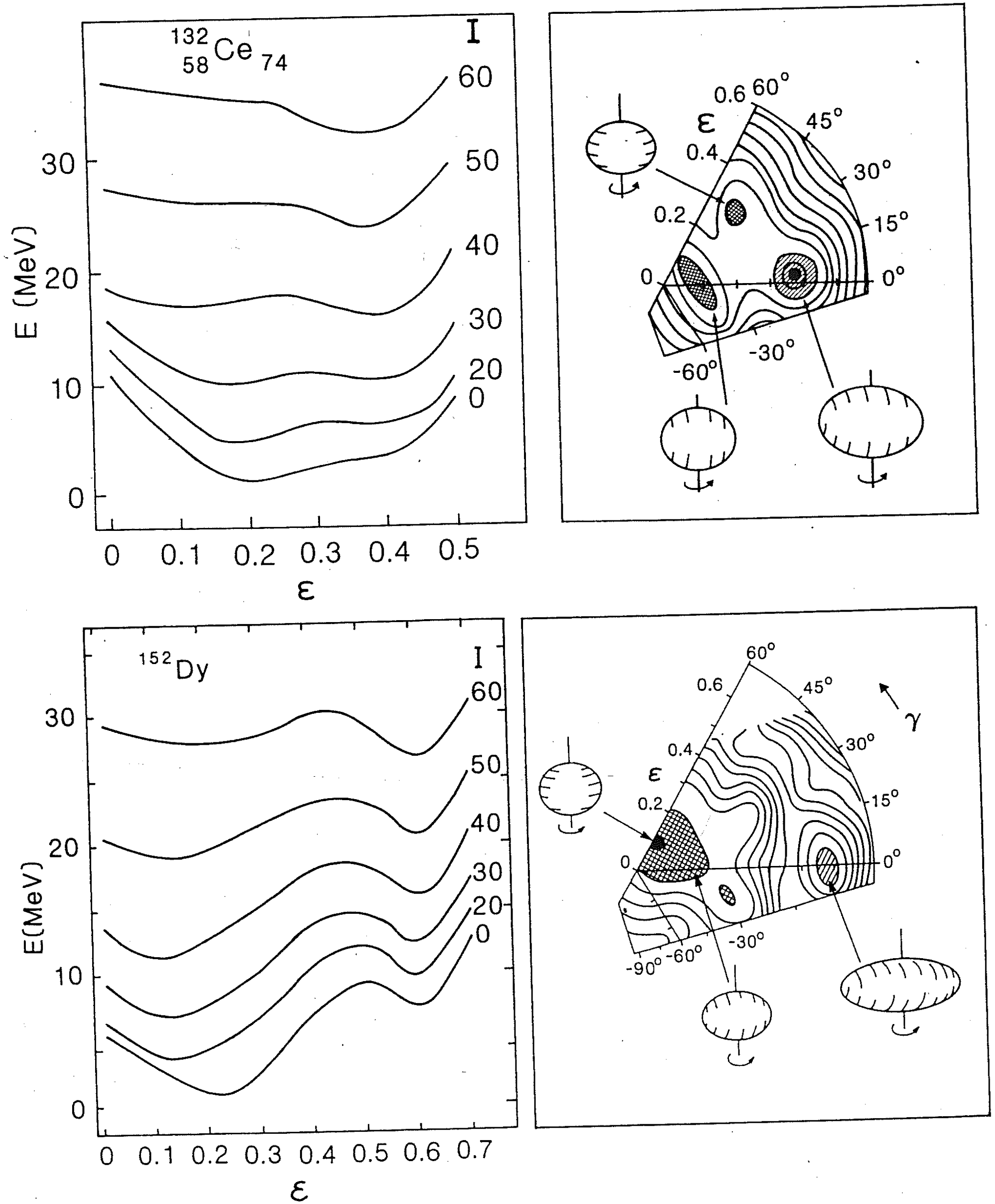
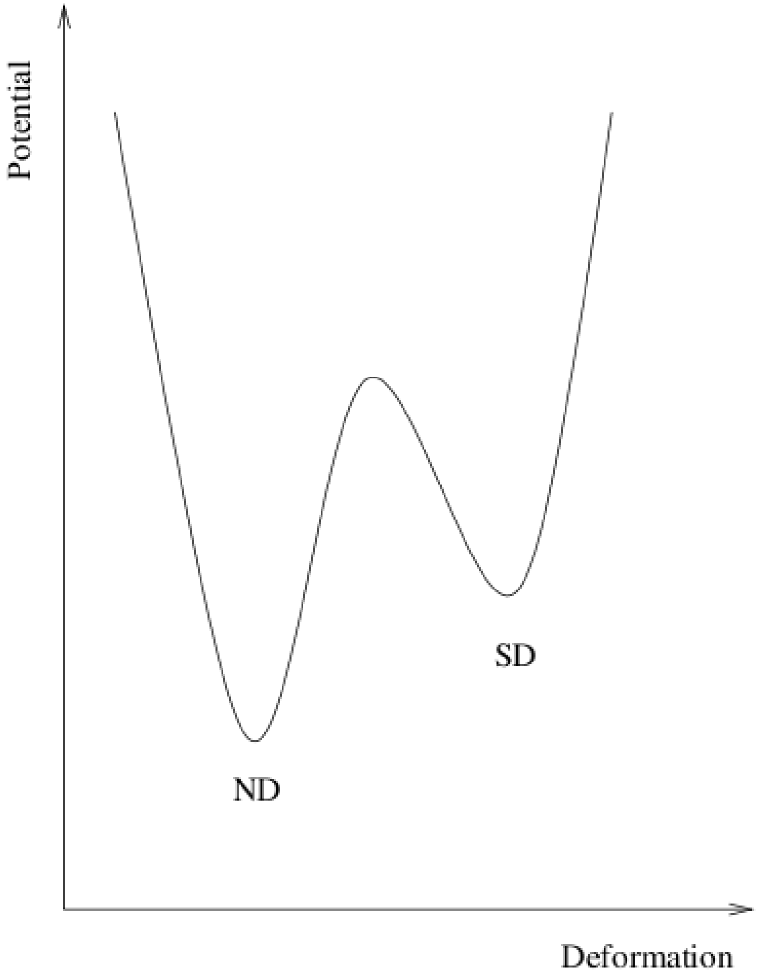


Figure 1 Calculated potential energies as a function of the quadrupole deformation for different spin values of ^{132}Ce (16) and ^{152}Dy (3). The various minima in the potential energy

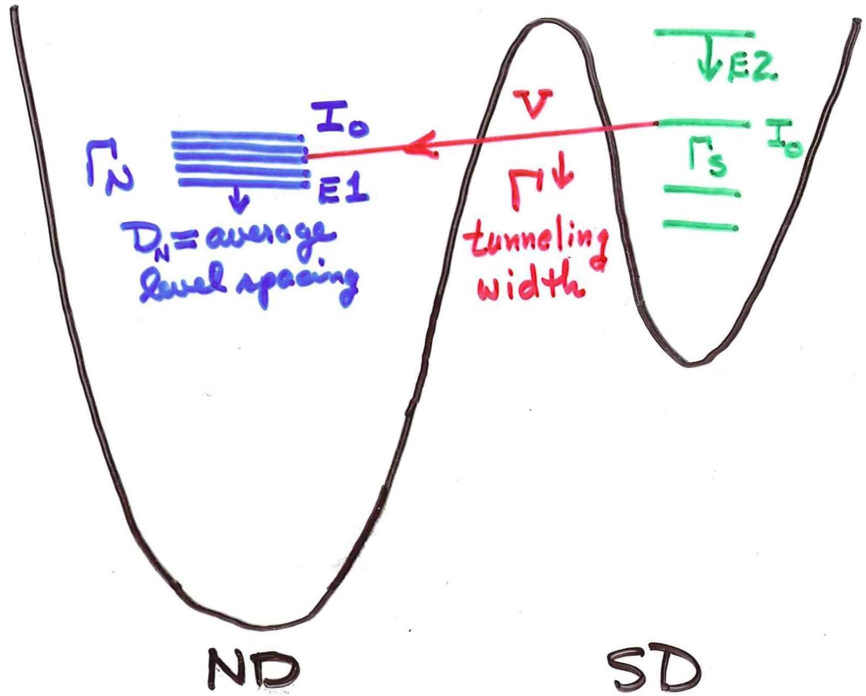
Modeling the Decay



Schematic Potential

- Double well.
- Function of angular momentum.

In principle, each SD state can decay to all ND states.



But $D_N > V >> \Gamma_N$

⟹ Nonoverlapping levels

⟹ Coupling mainly to nearest neighbor level in the ND potential.

From R. Kühn, et al., Phys. Rev. C 55 (1997) R1002 [Ref. (9)]

TABLE II. Properties of the decay out of the SD bands in ^{192}Hg, ^{194}Hg, and ^{194}Pb. Squared ND mixing amplitudes a_n^2 for the lowest states in the yrast SD bands in ^{192}Hg, ^{194}Hg, and ^{194}Pb calculated as the ratios of the partial decay probabilities out of the bands (λ_{out}) and the statistical $E1$ transition probabilities within the ND well (λ_n^{E1}). The latter were calculated applying a statistical model which was also used to derive the average level spacings in the ND well (D_n) at the excitation energies of the SD states and the upper limit for the mean interaction strength between ND and SD levels (v_{max}). Lifetimes and partial decay probabilities marked with a dagger were calculated assuming constant Q_t values at the bottom of each band. For further information see text.

Nucl.	I ($\hbar$)	N_{intra}	τ (ps)	λ_{out} (ps^{-1})	λ_n^{E1} (ps^{-1})	D_n (eV)	a_n^2 (%)	v_{max} (eV)
^{192}Hg	12	0.87(8)	4.9(7)	0.027(17)	15.7 0.0103eV	34	0.17	0.50
^{192}Hg	10	0.09(2)	1.1(4)†	0.83(5)†	15.7	30	5.3	2.44
^{194}Hg	12	0.60(4)	2.73(96)	0.147(55)	27.5	92	0.53	2.38
^{194}Hg	10	$\leqslant$0.05	$\leqslant$0.7†	$\geqslant$1.2†	27.9 0.018eV	79	$\geqslant$4.3	$\geqslant$5.8
^{194}Pb	10	0.85(9)	8.6(32)	0.018(10)	2.4	1699	0.74	51.67
^{194}Pb	8	0.75(16)	17.5(75)†	0.014(10)†	2.5	1549	0.56	40.98

Our Results for $\Gamma^\downarrow$

Nucleus	(10^{-4} eV) Γ_N	(in eV) D	(in 10^{-4} eV) Γ_S	(in 10^{-4} eV) $\Gamma^\downarrow$
^{192}Hg(12)	103	34	1.16	0.18
^{192}Hg(10)	103	30	0.54	5.44†
^{194}Hg(12)	181	92	1.44	0.97
^{194}Hg(10)	184	79	$\geq$0.47	$\geq$ 8.9†
^{194}Pb(10)	16	1699	0.66	0.11
^{194}Pb(8)	17	1549	0.28	0.09†

TABLE I. The spreading widths $\Gamma^\downarrow$ deduced from the data reviewed in Ref. [9] for a number of nuclei. The spin values of the decaying states are given in brackets. The units are eV for D, and 10^{-4} eV for Γ_N, Γ_S and $\Gamma^\downarrow$. The results indicated with † were calculated with estimated lifetimes in Ref. [9]. The total width $\Gamma = \Gamma_S + \Gamma^\downarrow$.

Nucl. Phys. A 557, 83c (1993)

inverse parabolic approximation, which characterizes the tunnelling. The action around spin 10 (from decay) and around spin 40 (from feeding) are shown on Fig. 7. For comparison, the theoretical action calculated by Shimizu et al. [11] is also shown. There is fair agreement at low spin, but the theoretical values at high spin are significantly lower than our inferred values. The origin of this discrepancy is not yet understood.

It is interesting to examine several of the quantities which affect the decay process. At the point of decay for ^{192}Hg Γ/D_n ~3 x 10^{-2}, Γ_{SD}/Γ_n ~5 x 10^{-3}, and the tunnelling probability is ~1.5 x 10^{-4}. (Our analysis for ^{152}Dy gives corresponding values of approximately 12 x 10^{-2}, 0.6, and 7 x 10^{-4}.) Thus, the decaying SD state is still very sharp and is not spread among several normal states; in fact, it acquires a small component of only the nearest neighboring one or two normal states. The small values of Γ/D_n and of the tunnelling probability imply that the coupling between SD and normal states is extremely small, indicating that the barrier is still sizeable. Despite the small mixing which occurs between the two classes of states, the decay nonetheless happens because Γ_{SD} is much smaller than Γ_n.

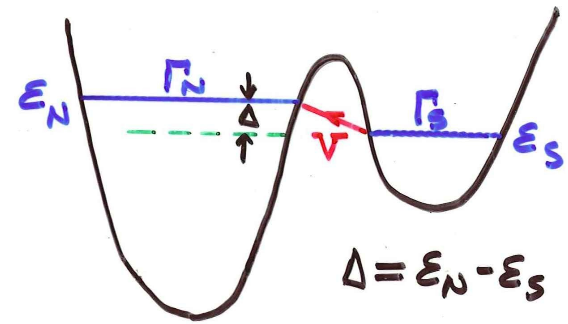
As the SD band cascades down, an unusual phenomenon occurs: we have a sharp state, isolated in its own pocket, which is embedded in a sea of states with increasing level density. By proximity to a normal state, it inevitably acquires a small normal component at low spin, through which it decays to the lower-lying normal states. Since the relevant parameters should be similar for SD states in each of the A = 150 or 190 region, the spin at which the decay occurs should be similar in each mass region. Thus the model naturally explains why SD states decay around spin 10 in the whole A = 190 region and around spin 25 in the A = 150 region.

IV. Summary

The feeding and decay mechanisms of SD bands is now believed to be well understood. We have developed a model which can account for almost all of the observables connected with the feeding process and shows that trapping in the SD well occurs when the γ cascade reaches 1-2 MeV below the barrier. Model calculations of the decay process reproduce the decay of SD bands and partially attribute the suddenness of the decay to a decrease of W as spin decreases. By comparing data and the results of the

Simple Two-Level Model for Decay of SD Nuclei

Charles Stafford and B.R.B., Phys. Rev. C $\underline{60}$, 051305 (1999)



$\Delta = \varepsilon_N - \varepsilon_S$

Parameters: V, Δ, Γ_N and Γ_S

This approach treats tunneling and EM decay on the same footing

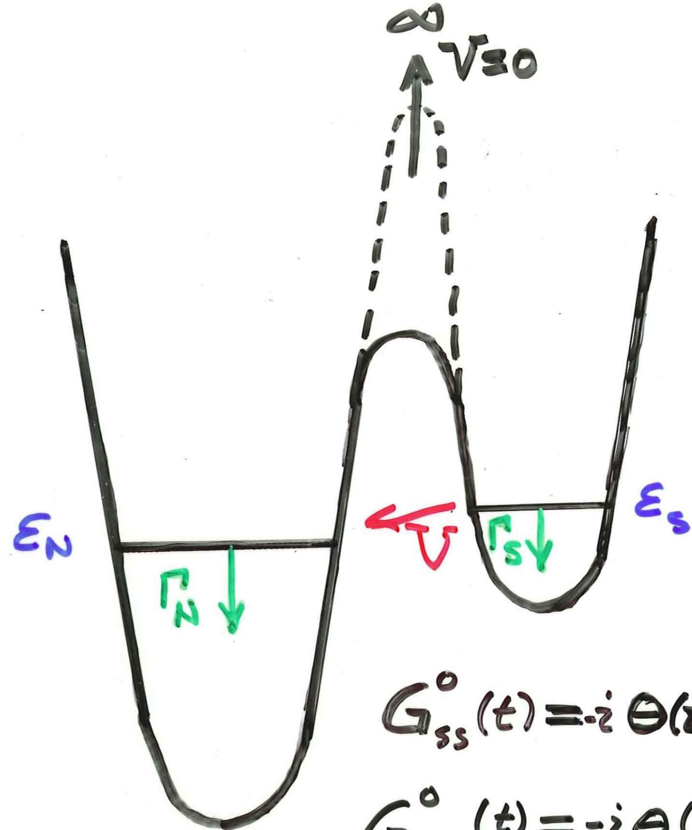
Consider the retarded Green's Function to include both the coherent "Rabi" oscillations due to V and the irreversible decays Γ_N and Γ_S.

$$G_{ij}(t) = -i\Theta(t) \langle \{ c_i(t), c_j^\dagger(0) \} \rangle$$

where $c_i = c_S$ or c_N ; $c_j^\dagger = c_S^\dagger$ or $c_N^\dagger$

and its Fourier transform

$$G_{ij}(E) = \int_{-\infty}^{+\infty} dt\, G_{ij}(t) e^{iEt}$$



$$G^0_{SS}(t) = -i\,\Theta(t)\,e^{-i\frac{\varepsilon_S t}{\hbar} - \frac{\Gamma_S t}{2}}$$

$$G^0_{NN}(t) = -i\,\Theta(t)\,e^{-i\frac{\varepsilon_N t}{\hbar} - \frac{\Gamma_N t}{2}}$$

$$G^0(E)^{-1} = \begin{pmatrix} E-\varepsilon_S + i\Gamma_S/2 & 0 \\ 0 & E-\varepsilon_N + i\Gamma_N/2 \end{pmatrix}$$

$$G(E)^{-1} = G^0(E)^{-1} - \begin{pmatrix} 0 & V \\ V & 0 \end{pmatrix}$$

or $G = G^0 + G^0 V G$: Dyson Equation

The full retarded Green's function is then

$$G = \begin{pmatrix} G_{SS} & G_{SN} \\ G_{NS} & G_{NN} \end{pmatrix}$$

$$= \frac{1}{\left[(E-\varepsilon_S+i\Gamma_S/2)(E-\varepsilon_N+i\Gamma_N/2)-V^2\right]} \begin{pmatrix} E-\varepsilon_N+i\frac{\Gamma_N}{2} & V \\ V & E-\varepsilon_S+i\frac{\Gamma_S}{2} \end{pmatrix}$$

We can study the dynamics of the coupled SD-ND system by looking at

$P_S(t) = |G_{SS}(t)|^2$, the probability that the nucleus is in state $|S\rangle$ at a later time t, if it started in $|S\rangle$ at time $t=0$.

and

$P_N(t) = |G_{NS}(t)|^2$, the probability that the nucleus is in state $|N\rangle$ at a later time t.

$$P_N(t) = \frac{2V^2}{|\omega|^2} e^{-\bar{\Gamma}t/\hbar} (\cosh \omega_i t - \cos \omega_r t)$$

and

$$P_S(t) = \frac{V^2}{|\omega|^2} e^{-\bar{\Gamma}t/\hbar} \left(\frac{\omega_i + \Gamma'}{\Gamma' - \omega_i} e^{\omega_i t/\hbar} + \frac{\Gamma' - \omega_i}{\omega_i + \Gamma'} e^{-\omega_i t/\hbar} + \frac{i\omega_r + \Gamma'}{i\omega_r - \Gamma'} e^{i\omega_r t/\hbar} + \frac{i\omega_r - \Gamma'}{i\omega_r + \Gamma'} e^{-i\omega_r t/\hbar} \right)$$

$$\bar{\Gamma} = \frac{\Gamma_N + \Gamma_S}{2}, \quad \Gamma' = \frac{\Gamma_N - \Gamma_S}{2}$$

$$\omega \equiv \omega_r + i\omega_i = \sqrt{4V^2 + (\Delta - i\Gamma')^2}$$

so $\omega_r = \sqrt{4V^2 + \Delta^2}$

phys. stat. sol. (b) **230**, No. 2, 419–423 (2002)

Coherence and Decoherence in Tunneling between Quantum Dots

D. M. CARDAMONE[1]), C. A. STAFFORD, and B. R. BARRETT

Physics Department, University of Arizona, 1118 East 4th Street, Tucson, Arizona 85721, USA

(Received October 12, 2001; accepted December 5, 2001)

Subject classification: 73.20.Jc; 73.40.Gk; 73.63.Kv

Coupled quantum dots are an example of the ubiquitous quantum double potential well. In a typical transport experiment, each quantum dot is also coupled to a continuum of states. Our approach takes this into account by using a Green's function formalism to solve the full system. The time-dependent solution is then explored in different limiting cases. In general, a combination of coherent and incoherent behavior is observed. In the case that the coupling of each dot to the macroscopic world is equal, however, the time evolution is purely coherent.

The double-well potential is one of the simplest and best understood problems in modern quantum mechanics. Its utility is likewise unparalleled. Potential applications of double-well devices have been noted in Refs. [1–5]. For such devices to be useful, an understanding of the processes which couple the microscopic device to the macroscopic environment is paramount. That is to say, the decoherence processes of such systems must be well understood. To this end, we consider a simple exactly solvable model of two coupled quantum dots. An excellent review of related systems and some approximate solutions are given in Ref. [6].

Each dot is coupled to an environment (a triple-barrier system) as in Fig. 1. The environment consists of a continuum of states, as would be appropriate for a macroscopic lead. Only one state in each quantum dot is considered, which amounts to the assumption that the tunneling parameters connecting our two states to any neglected state are much less than the energy differences with that state.

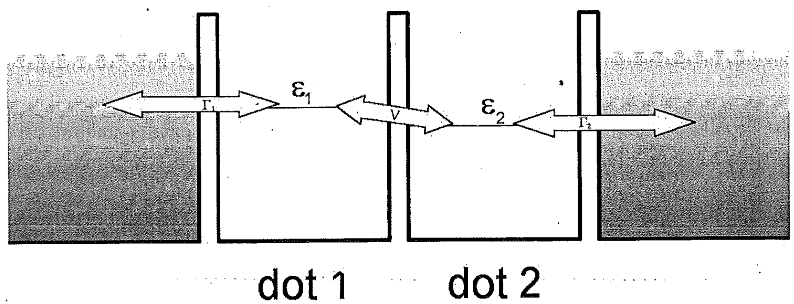


Fig. 1. Schematic diagram of the problem

[1]) Corresponding author; e-mail: dmcard@physics.arizona.edu

 0370-1972/02/23004-0419 $ 17.50+.50/0

Fermi's Golden Rule

$$\Gamma^{\downarrow} = 2\pi V^2 \int_{-\infty}^{+\infty} dE\, \rho_S(E)\, \rho_N(E) \qquad (1)$$

where the lifetime-broadened densities of states of the SD and ND levels are

$$\rho_S(E) = \frac{\Gamma_S/2\pi}{(E-\varepsilon_S)^2 + \Gamma_S^2/4} \qquad (2)$$

$$\rho_N(E) = \frac{\Gamma_N/2\pi}{(E-\varepsilon_N)^2 + \Gamma_N^2/4} \qquad (3)$$

Doing the integral (1) yields

$$\Gamma^{\downarrow} = \frac{2\bar{\Gamma} V^2}{\Delta^2 + \bar{\Gamma}^2}, \quad \Delta = \varepsilon_N - \varepsilon_S, \quad \bar{\Gamma} = (\Gamma_S + \Gamma_N)/2$$

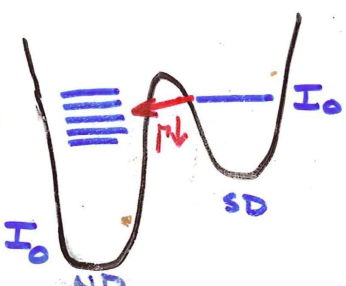
Calculate the average value of $\Gamma^{\downarrow}$ for an average level spacing D_N

$$\langle \Gamma^{\downarrow} \rangle = \frac{1}{D_N} \int_{-D_N/2}^{D_N/2} \frac{2\bar{\Gamma} V^2}{\bar{\Gamma}^2 + \Delta^2}\, d\Delta = \frac{2\bar{\Gamma}\langle V^2 \rangle}{D_N} \frac{1}{\bar{\Gamma}}\, 2 \underbrace{\tan^{-1}\frac{D_N}{\bar{\Gamma}}}_{\sim \frac{\pi}{2} \text{ since } D_N \gg \bar{\Gamma}}$$

or

$$\langle \Gamma^{\downarrow} \rangle = 2\pi \langle V^2 \rangle / D_N$$

D_N = average level spacing of ND states of ang. mom. I_0



Decay Branching Ratios

Time dependent rates to decay in the S and N channels

$$\tilde{\Gamma}_S(t) = \Gamma_S |G_{SS}(t)|^2 = \Gamma_S P_S(t)$$
$$\tilde{\Gamma}_N(t) = \Gamma_N |G_{NS}(t)|^2 = \Gamma_N P_N(t)$$

F_N = fraction of nuclei that decay via E1 processes in the ND band

$$F_N = \frac{\int_0^\infty dt\, \tilde{\Gamma}_N(t)}{\int_0^\infty dt\, [\tilde{\Gamma}_N(t) + \tilde{\Gamma}_S(t)]} = \Gamma_N \int_0^\infty dt\, P_N(t)$$

$$F_N = \frac{(1 + \Gamma_N/\Gamma_S)\, V^2}{\Delta^2 + \bar{\Gamma}^2 (1 + 4V^2/\Gamma_N \Gamma_S)} \qquad \bar{\Gamma} = \frac{\Gamma_N + \Gamma_S}{2}$$

$$= \frac{\dfrac{\Gamma_N \Gamma^\downarrow}{\Gamma_N + \Gamma^\downarrow}}{\Gamma_S + \dfrac{\Gamma_N \Gamma^\downarrow}{\Gamma_N + \Gamma^\downarrow}}, \text{ where } \Gamma^\downarrow = \frac{2\bar{\Gamma} V^2}{\Delta^2 + \bar{\Gamma}^2}$$

transition or tunneling width

$$F_S = 1 - F_N = \frac{\Gamma_S}{\Gamma_S + \Gamma_N \Gamma^\downarrow/(\Gamma_N + \Gamma^\downarrow)} = \frac{\Gamma_S}{\Gamma_S + \Gamma_{out}}$$

$\Gamma^\downarrow$ is a real, measurement decay rate

$$\Gamma^\downarrow = \frac{F_N \Gamma_S \Gamma_N}{\Gamma_N - F_N(\Gamma_S + \Gamma_N)}$$

Direct Decay from the Superdeformed Band to the Yrast Line in $^{152}_{66}Dy_{86}$

T. Lauritsen,[1] M. P. Carpenter,[1] T. Døssing,[2] P. Fallon,[3] B. Herskind,[2] R. V. F. Janssens,[1] D. G. Jenkins,[1] T. L. Khoo,[1] F. G. Kondev,[1] A. Lopez-Martens,[4] A. O. Macchiavelli,[3] D. Ward,[3] K. S. Abu Saleem,[1] I. Ahmad,[1] R. Clark,[3] M. Cromaz,[3] J. P. Greene,[1] F. Hannachi,[4] A. M. Heinz,[1] A. Korichi,[4] G. Lane,[3] C. J. Lister,[1] P. Reiter,[1,5] D. Seweryniak,[1] S. Siem,[1] R. C. Vondrasek,[1] and I. Wiedenhöver[1,6]

VOLUME 88, NUMBER 4 PHYSICAL REVIEW LETTERS 28 JANUARY 2002

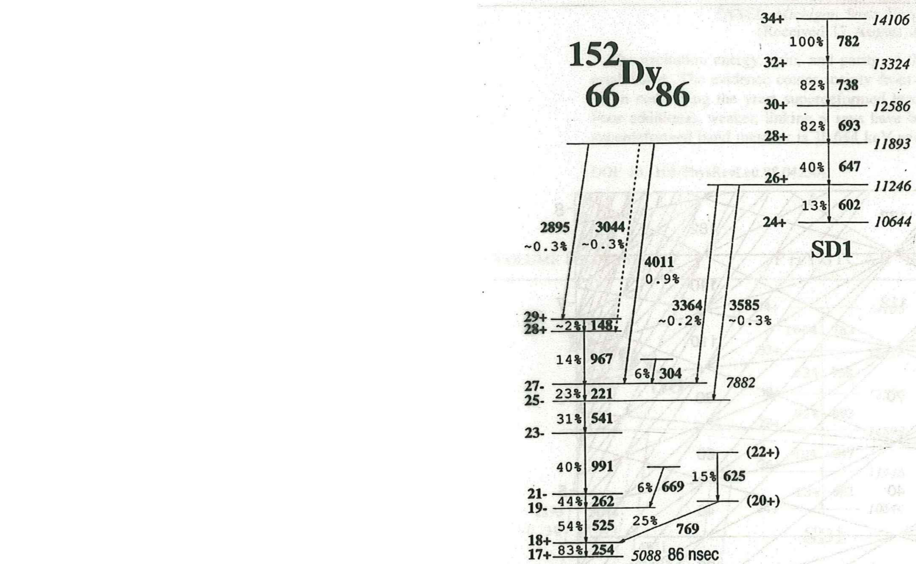


FIG. 3. Partial level scheme of ^{152}Dy showing the lowest part of the yrast SD band and normal states to which the SD band mainly decays. The transition intensities, given in %, reflect the requirement of the isomer tag.

originates from the 11 893 keV SD level. However, in this case the decay could not be traced all the way into the yrast or near yrast states, as the deexcitation fragments after the first transition into several paths involving γ rays with intensities below the detection threshold.

It is worth pointing out that the SD band spin values firmly assigned here are two units higher than those proposed by Twin *et al.* [1] following the discovery of the band. Several systematic theoretical investigations of all SD bands in the $A = 140$–150 region are also available. For ^{152}Dy, cranked Nilsson-Strutinsky calculations by Ragnarsson [22], and relativistic mean field calculations by Afanasjev *et al.* [23] suggested spins of either $28\hbar$ or $30\hbar$ for the 11 893-keV level, while the cranking calculations with a Woods-Saxon potential of Dudek *et al.* [24] propose a spin of $26\hbar$.

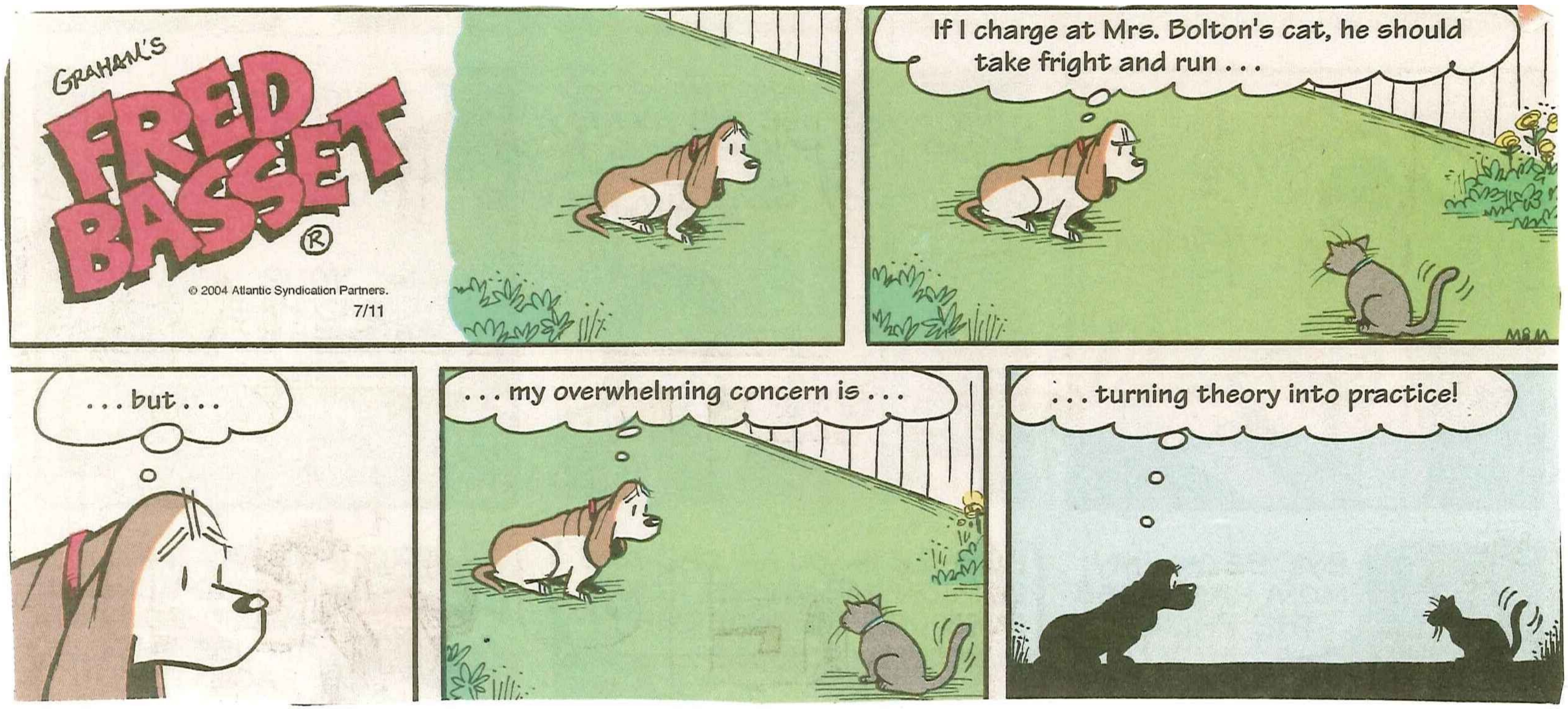
A solid understanding of superdeformed rotational mo-

$38\hbar$ before decreasing smoothly back to $84.8\hbar^2$ MeV^{-1} at the highest spin of $68\hbar$. Results of calculations without pairing [25] are higher by 7%–5%, suggesting some persistence of pairing at the lower spins. The inclusion of pairing improves the agreement (see, e.g., [24,26,27]), but a simultaneous reproduction of $\Im^{(1)}$ and $\Im^{(2)}$ still has not been achieved.

Extrapolated to zero spin, the excitation energy of the SD band is $E^{SD}(0^+) = 7.5$ MeV. The extrapolation was performed with a functional form of the excitation energy written as $E^{SD}(I) = E^{SD}(0^+) + a[I(I+1)] + b[I(I+1)]^2$, where $E^{SD}(I)$ is the energy of the SD level at spin I. This procedure is less accurate here than in the $A = 190$ mass region because the spin of the lowest observed SD level is $24\hbar$ rather than $10\hbar$. The extrapolated zero spin energy is slightly higher than the values found in ^{192,194}Hg: 5.3(5) and 6.0 MeV [2,16]. For ^{152}Dy, a relativistic mean field prediction of the excitation energy is 8.32 MeV [28], while the cranked Strutinsky calculation of [21] obtains a value of ~8.8 MeV. A recent Hartree-Fock-Bogoliubov calculation estimates the excitation energy to be ~7 MeV [29].

The 4011 keV one-step decay line carries only 0.9(2)% of the intensity of the SD band. The quadrupole moment of the SD band in ^{152}Dy, 17.5(2) e b [30], gives a partial lifetime of the 647 keV in-band transition of 66 fs and a partial lifetime of the 4011 keV transition of 2.9 ps, equivalent to a strength in Weisskopf units (W.u.) of $\approx 2 \times 10^{-6}$. Just as in the $A = 190$ mass region [2], the decay-out transition is very retarded. This retardation can be understood [2] in terms of the decay mechanism out of the SD state first proposed by Vigezzi *et al.* [31]. In their interpretation, the SD level mixes with one (or a few) of the adjacent closely spaced levels in the normal well, on the other side of the SD barrier, and the decay occurs through the admixed component of the normal state in the wave function.

The model of Vigezzi *et al.* [31,32] was used to fit the SD transition intensities (i.e., decay-out profiles) in ^{152}Dy and ^{194}Hg, where the SD excitation energies are known. In this model, the probability for decay of a SD state to the normal well depends on the gamma decay widths Γ_s and Γ_n, the average separation D_n between excited normal-deformed states, and the width Γ for tunneling across the barrier. Γ_s, the decay width within a SD band, is obtained from the measured transition quadrupole moments; Γ_n, the $E1$ width for statistical decay from an excited ND state, and D_n are estimated by scaling [33] values obtained from neutron spectroscopy [34]. Values of the one free parameter Γ, which reproduce the


GRAHAM'S
FRED BASSET
© 2004 Atlantic Syndication Partners.
7/11
If I charge at Mrs. Bolton's cat, he should take fright and run . . .
. . . but . . .
. . . my overwhelming concern is . . .
. . . turning theory into practice!

TABLES

TABLE I. Tunneling widths $\Gamma^{\downarrow}$ extracted from Eq. (2) compared with the results given in Table I of Ref. [2]. The quantities used in the two computations, *i.e.*, $F_N \equiv P_{out}$ (a), Γ_S, Γ_N, and D_N are same as those in Table I of Ref. [2]. The spin values of the decaying states are given in parentheses. Footnotes: (a) Refers to quantities from Ref. [2], (b) Our results.

Nucleus	F_N $\equiv P_{out}$ (a)	Γ_S (meV)	Γ_N (meV)	D_N (eV)	$\Gamma^{\downarrow}$ (b) our result (meV)	$\Gamma^{\downarrow}$ (a) Vigezzi approach (meV) Ref (2)
^{152}Dy(28)	0.40	10.0	17	220	11	41,000
^{152}Dy(26)	0.81	7.0	17	194	-40	220,000
^{194}Hg(12)	0.40	0.108	21	344	0.072	560
^{194}Hg(10)	0.97	0.046	20	493	1.6	37,000

$$\Gamma^{\downarrow}_{\text{2-level}} = \frac{F_N \Gamma_S \Gamma_N}{\Gamma_N - F_N(\Gamma_S + \Gamma_N)} \quad \text{vs} \quad \langle \Gamma^{\downarrow} \rangle = 2\pi \frac{\langle V^2 \rangle}{D_N}$$

$$\Gamma^{\downarrow}_{\text{2-level}} \Rightarrow \Gamma_N - F_N(\Gamma_S + \Gamma_N) > 0$$

$$\text{or} \quad \frac{\Gamma_N}{\Gamma_S + \Gamma_N} > F_N$$

Γ_N is not well-determined, perhaps within a factor of 2.

Adding a Second ND State

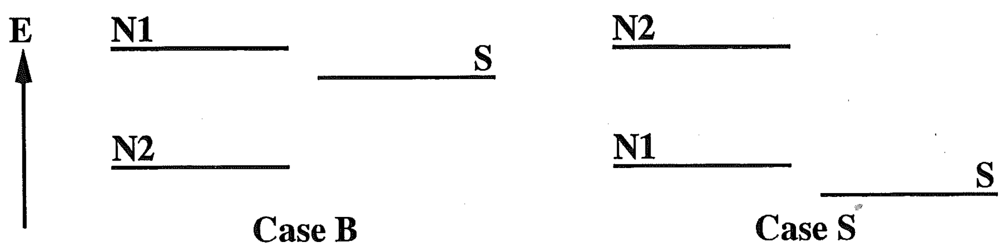
By the same method as before:

$$G^{-1} = \begin{pmatrix} E + i\Gamma_S/2 & -V & -V \\ -V & E - \Delta_1 + i\Gamma_N/2 & 0 \\ -V & 0 & E - \Delta_2 + i\Gamma_N/2 \end{pmatrix}$$

$$F_i = \Gamma_i \int_{-\infty}^{\infty} \frac{dE}{2\pi} |\langle S|G|i\rangle|^2$$

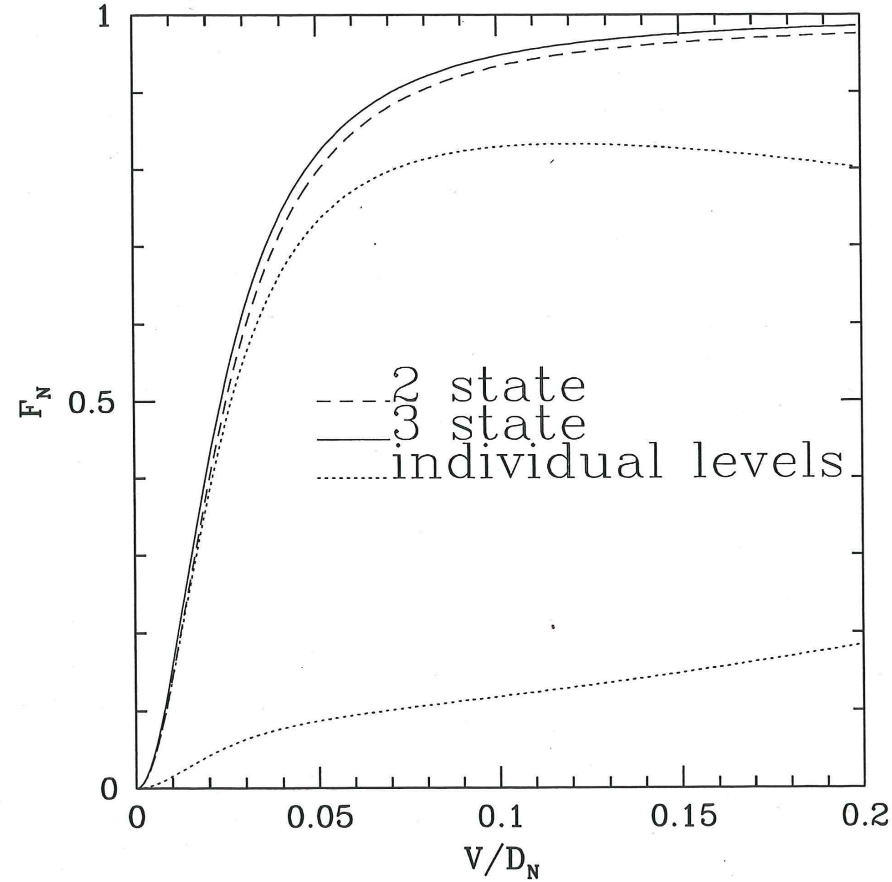
- Semiclassically, we expect the second level to draw some strength from the first, but that the total F_N will be increased.

- BUT, this does not take into account the possibility of quantum effects, *i.e.* interference between the two ND states.



D.M. Cardamone, C.A. Stafford, B.R.B.
Phys. Rev. Lett. 91, 102502 (2003)

$A \approx 190$ **Region**



$\Gamma_S/\Gamma_N = 10^{-2}$

Determination of V

$$\Gamma^{\downarrow} = \frac{2\bar{\Gamma} V^2}{\Delta^2 + \bar{\Gamma}^2} \quad \text{where } \bar{\Gamma} = \frac{\Gamma_N + \Gamma_S}{2}$$

But $\Delta >> \Gamma_N, \Gamma_S$

$$\Gamma^{\downarrow} \approx (\Gamma_N + \Gamma_S) \frac{V^2}{\Delta^2}$$

or

$$V \approx \sqrt{\frac{\Gamma^{\downarrow}}{\Gamma_N + \Gamma_S}} \sqrt{\Delta^2}$$

$$\langle V \rangle \approx \sqrt{\frac{\Gamma^{\downarrow}}{\Gamma_N + \Gamma_S}} \, \frac{1}{D} \int_{-D/2}^{D/2} d\Delta \, |\Delta|$$

$$= \sqrt{\frac{\Gamma^{\downarrow}}{\Gamma_N + \Gamma_S}} \, \frac{D}{4}$$

For SD nuclei in the mass 190 region

$$\Gamma_N \gg \Gamma_S$$

$$\therefore \Gamma^{\downarrow} = \frac{F_N \Gamma_S \Gamma_N}{\Gamma_N - F_N(\Gamma_S + \Gamma_N)} \approx \frac{F_N}{1-F_N} \frac{\Gamma_S \Gamma_N}{\Gamma_N} = \frac{F_N}{1-F_N} \Gamma_S$$

$$\Gamma_{out} = \frac{\Gamma^{\downarrow}}{\Gamma^{\downarrow} + \Gamma_N} \Gamma_N$$ compared with $\Gamma_{out} = |a_N|^2 \Gamma_N$

$$\Rightarrow |a_N|^2 = \frac{\Gamma^{\downarrow}}{\Gamma^{\downarrow} + \Gamma_N}$$

In the mass 150 region $\Gamma^{\downarrow} \approx \Gamma_N$

so that

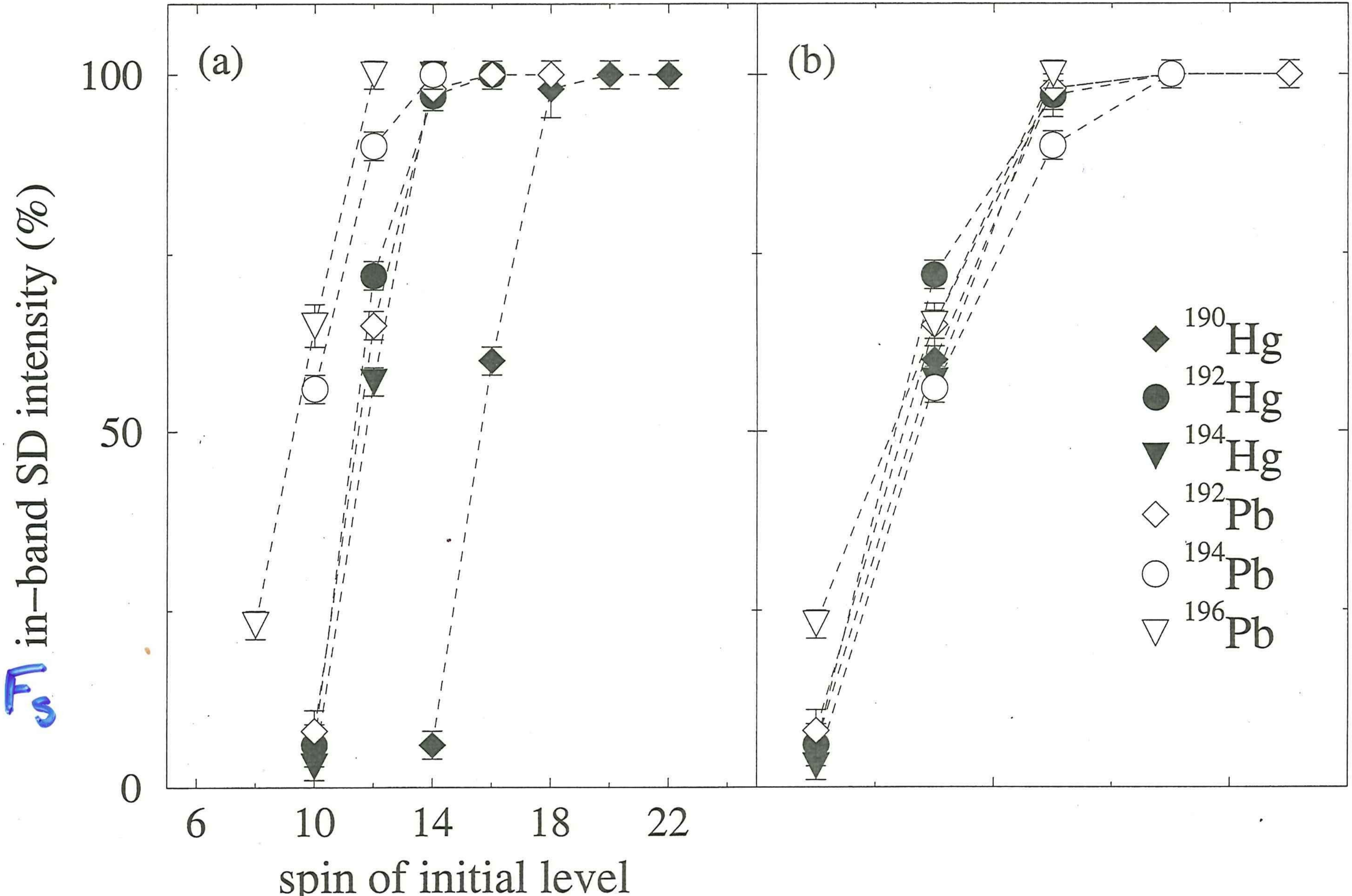
$$|a_N|^2_{150} \approx \frac{1}{2}$$

TABLE I: Results of the two-level model, for all SD decays for which sufficient data (branching ratios, Γ_S, Γ_N, and D_N) are known. The right-most column gives the source of the experimental inputs and the estimates of Γ_N and D_N.

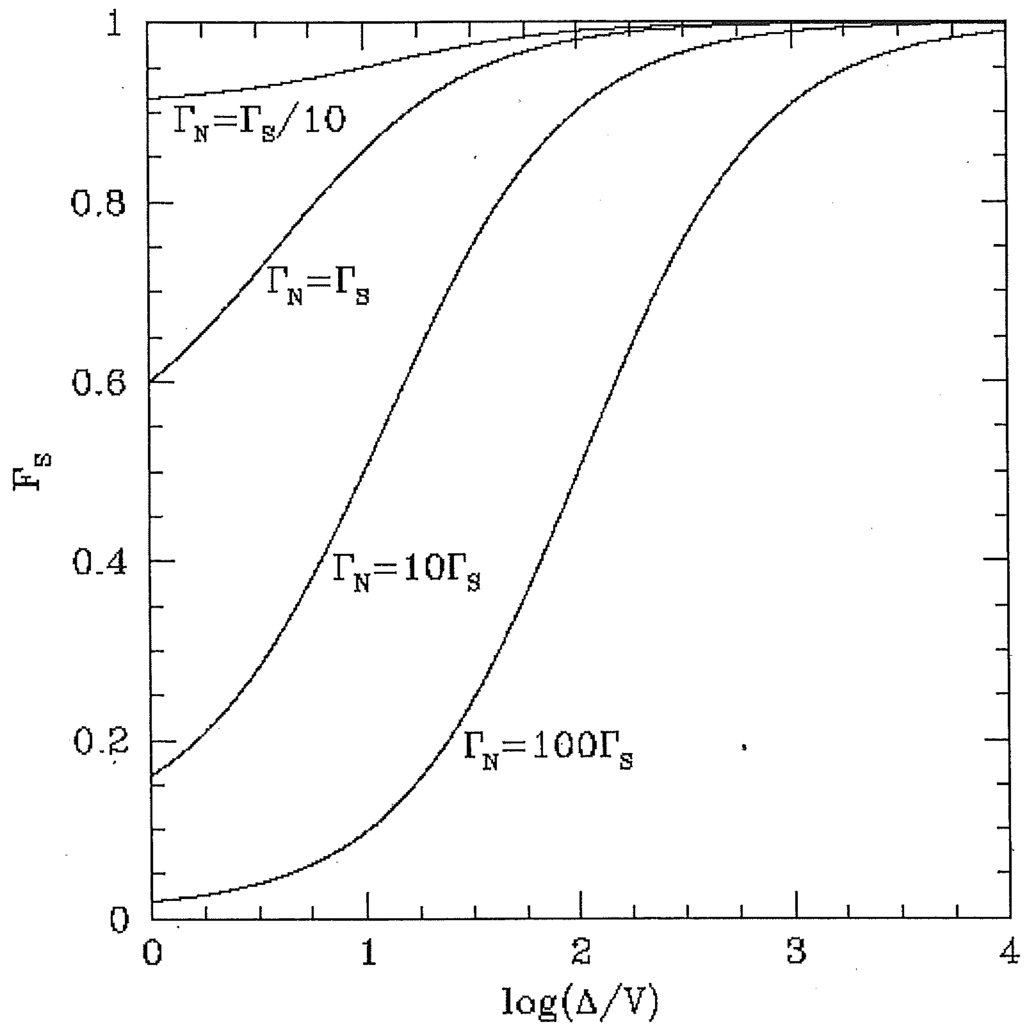
nucleus(I)	F_N	Γ_S (meV)	Γ_N (meV)	D_N (eV)	$\Gamma^{\downarrow}$ (meV)	Γ_{out} (mev)	$\langle V \rangle$ (eV)	$\frac{\langle V \rangle}{D_N}$	$\frac{\Gamma_N}{\Gamma_S+\Gamma_N}$	Refs.
^{192}Hg(12)	0.26	0.128	0.613	135.	0.049	0.045	8.7	0.064	0.827	[1, 10]
^{192}Hg(10)	0.92	0.050	0.733	89.	0.37	0.25	15.	0.17	0.936	[1, 10]
^{192}Pb(16)	<0.01	0.487	0.192	1,362.	<0.0050	<0.0049	<29.	< 0.021	0.283	[11, 12]
^{192}Pb(14)	0.02	0.266	0.201	1,258.	0.0056	0.0054	34.	0.027	0.430	[11, 12]
^{192}Pb(12)	0.34	0.132	0.200	1,272.	0.10	0.067	170.	0.13	0.602	[11, 12]
^{192}Pb(10)	0.88	0.048	0.188	1,410.	1.9†	0.17	1000.	0.71	0.797	[11, 12]
^{192}Pb(8)	>0.75	0.016	0.169	1,681.	>0.067	>0.048	>250.	>0.15	0.914	[11, 12]
^{194}Hg(12)	0.42	0.097	4.8	16.3	0.071	0.070	0.49	0.030	0.980	[13–16]
^{194}Hg(10)	>0.91	0.039	4.1	26.2	>0.44	>0.40	>2.1	>0.080	0.99	[13–16]
^{194}Hg(12)	0.40	0.108	21.	344.	0.072	0.072	5.0	0.015	0.99	[17]
^{194}Hg(10)	0.97	0.046	20.	493.	1.6	1.5	35.	0.071	1.0	[17]
^{194}Hg(12)	0.40	0.086	1.345	19.	0.060	0.057	0.97	0.051	0.94	[1, 15]
^{194}Hg(10)	≥0.95	0.033	1.487	14.	≥1.1	≥0.63	≥3.0	≥0.21	0.98	[1, 15]
^{194}Hg(15)	0.10	0.230	4.0	26.5	0.026	0.026	0.52	0.020	0.95	[15, 16]
^{194}Hg(13)	0.16	0.110	4.5	19.9	0.021	0.021	0.34	0.017	0.98	[15, 16]
^{194}Hg(11)	>0.93	0.048	6.4	7.2	>0.71	>0.64	>0.60	>0.083	0.99	[15, 16]
^{194}Pb(10)	0.10	0.045	0.08	21,700.	0.0053	0.0050	1100.	0.051	0.64	[16, 18–20]
^{194}Pb(8)	0.38	0.014	0.50	2,200.	0.0087	0.0086	72.	0.031	0.97	[16, 18–20]
^{194}Pb(6)	>0.91	0.003	0.65	1,400.	>0.032	>0.030	>77.	>0.055	1.0	[16, 18–20]
^{194}Pb(12)	<0.01	0.125	0.476	236.	<0.0013	<0.0013	<2.7	<0.011	0.792	[12, 16]
^{194}Pb(10)	0.10	0.045	0.470	244.	0.0051	0.0050	6.1	0.025	0.913	[12, 16]
^{194}Pb(8)	0.35	0.014	0.445	273.	0.0077	0.0076	8.8	0.032	0.969	[12, 16]
^{194}Pb(6)	>0.96	0.003	0.405	333.	>0.088	>0.072	>39.	>0.12	0.993	[12, 16]
^{152}Dy(28)	0.40	10.0	17.	220.	11.	6.7	35.	0.16	0.63	[17]
^{152}Dy(26)	0.81	7.0	17.	194.	140.†	15.	120.	0.62	0.71	[17]

†Calculated statistically, as explained in the text.

D.M. Cardmone Ph.D. Thesis



A.N. Wilson et al. Phys Rev C 71, 034319 (2005)



1
0.8
0.6
0.4
0.2
0
F_S
0
1
2
3
4
log(Δ/V)
$\Gamma_N=\Gamma_S/10$
$\Gamma_N=\Gamma_S$
$\Gamma_N=10\Gamma_S$
$\Gamma_N=100\Gamma_S$

Conclusions

1. A simple two-level model accurately describes the SD decay out process.

2. $$F_N = \frac{\Gamma_N \Gamma^{\downarrow}/(\Gamma_N + \Gamma^{\downarrow})}{\Gamma_S + \Gamma_N \Gamma^{\downarrow}/(\Gamma_N + \Gamma^{\downarrow})}$$

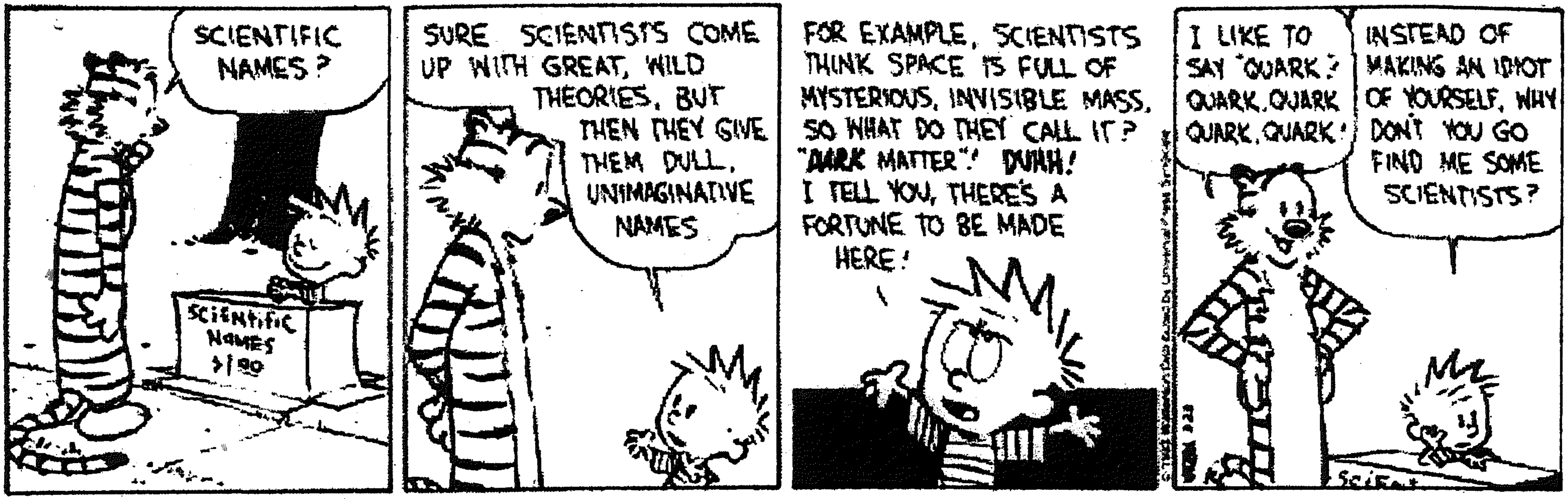
   where $\Gamma_{out} = \Gamma_N \Gamma^{\downarrow}/(\Gamma_N + \Gamma^{\downarrow})$

3. Thus $$\Gamma^{\downarrow} = \frac{F_N \Gamma_S \Gamma_N}{\Gamma_N - F_N(\Gamma_S + \Gamma_N)} = \frac{2\bar{\Gamma} V^2}{\Delta^2 + \bar{\Gamma}^2}$$

4. The universality of the SD decay out intensity is determined by

   Γ_N/Γ_S and $\frac{V}{D_N}$.

NON SEQUITUR By Wiley


SCIENTIFIC NAMES?
SCIENTIFIC NAMES
SURE. SCIENTISTS COME UP WITH GREAT, WILD THEORIES, BUT THEN THEY GIVE THEM DULL, UNIMAGINATIVE NAMES.
FOR EXAMPLE, SCIENTISTS THINK SPACE IS FULL OF MYSTERIOUS, INVISIBLE MASS. SO WHAT DO THEY CALL IT? "DARK MATTER"! DUHH! I TELL YOU, THERE'S A FORTUNE TO BE MADE HERE!
I LIKE TO SAY "QUARK." QUARK, QUARK, QUARK, QUARK!
INSTEAD OF MAKING AN IDIOT OF YOURSELF, WHY DON'T YOU GO FIND ME SOME SCIENTISTS?