Effective field theory and the renormalization group for nuclear systems

Achim Schwenk

TRIUMF

CANADA'S NATIONAL LABORATORY FOR PARTICLE AND NUCLEAR PHYSICS

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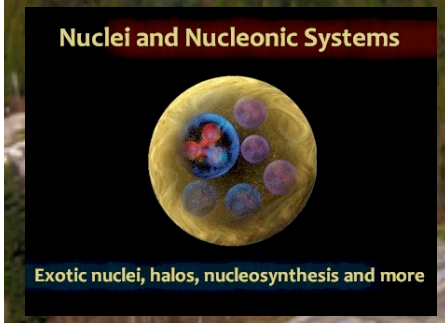

20th Chris Engelbrecht Summer School
Stellenbosch, ZA, Jan. 19-28, 2009

supported by

NSERC CRSNG

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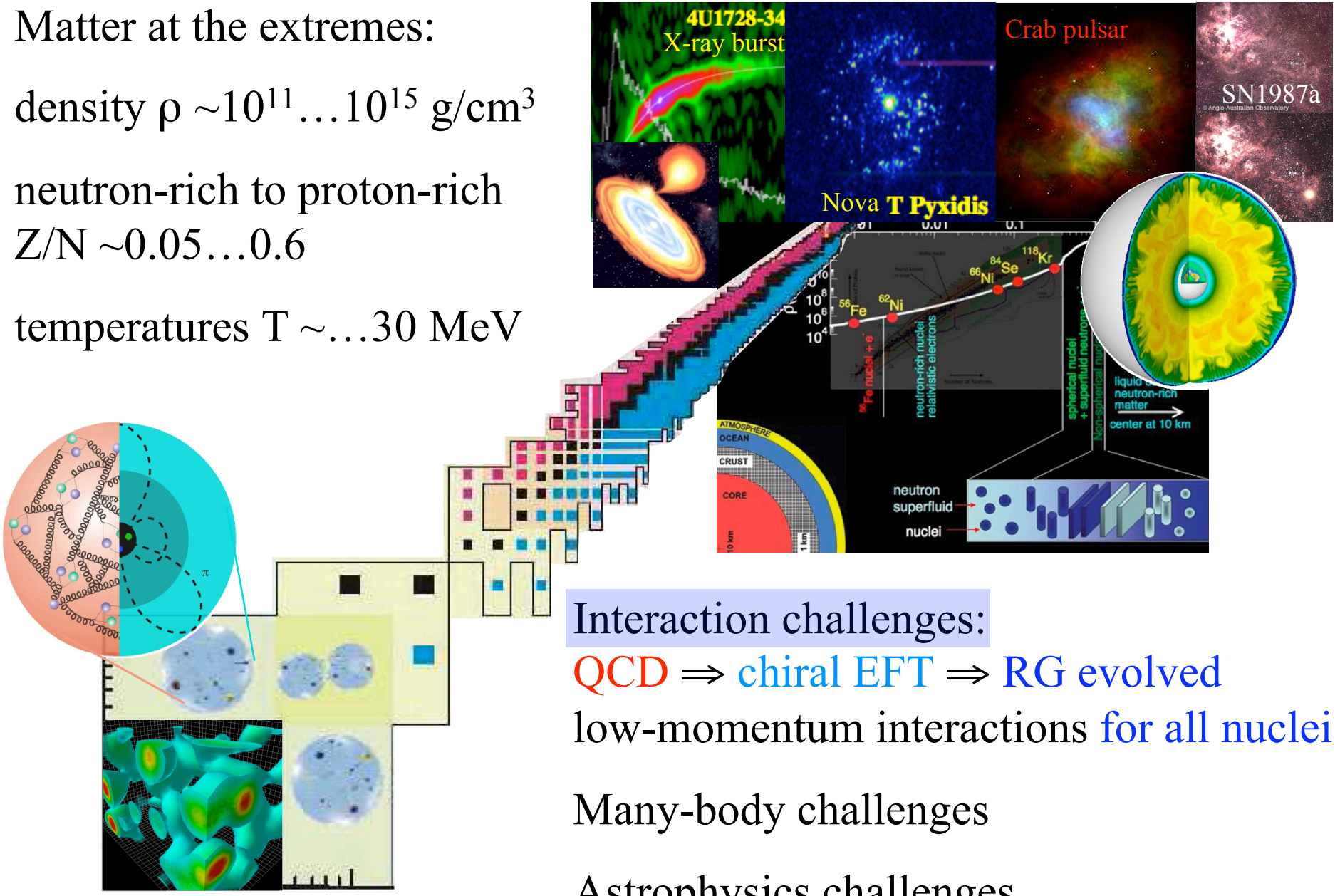
Strong interaction physics in the lab and cosmos

Matter at the extremes:

density $\rho \sim 10^{11} \ldots 10^{15}$ g/cm^3

neutron-rich to proton-rich
Z/N ~0.05…0.6

temperatures T ~…30 MeV



Interaction challenges:

QCD ⇒ chiral EFT ⇒ RG evolved low-momentum interactions for all nuclei

Many-body challenges

Astrophysics challenges

Frontiers in nuclear theory

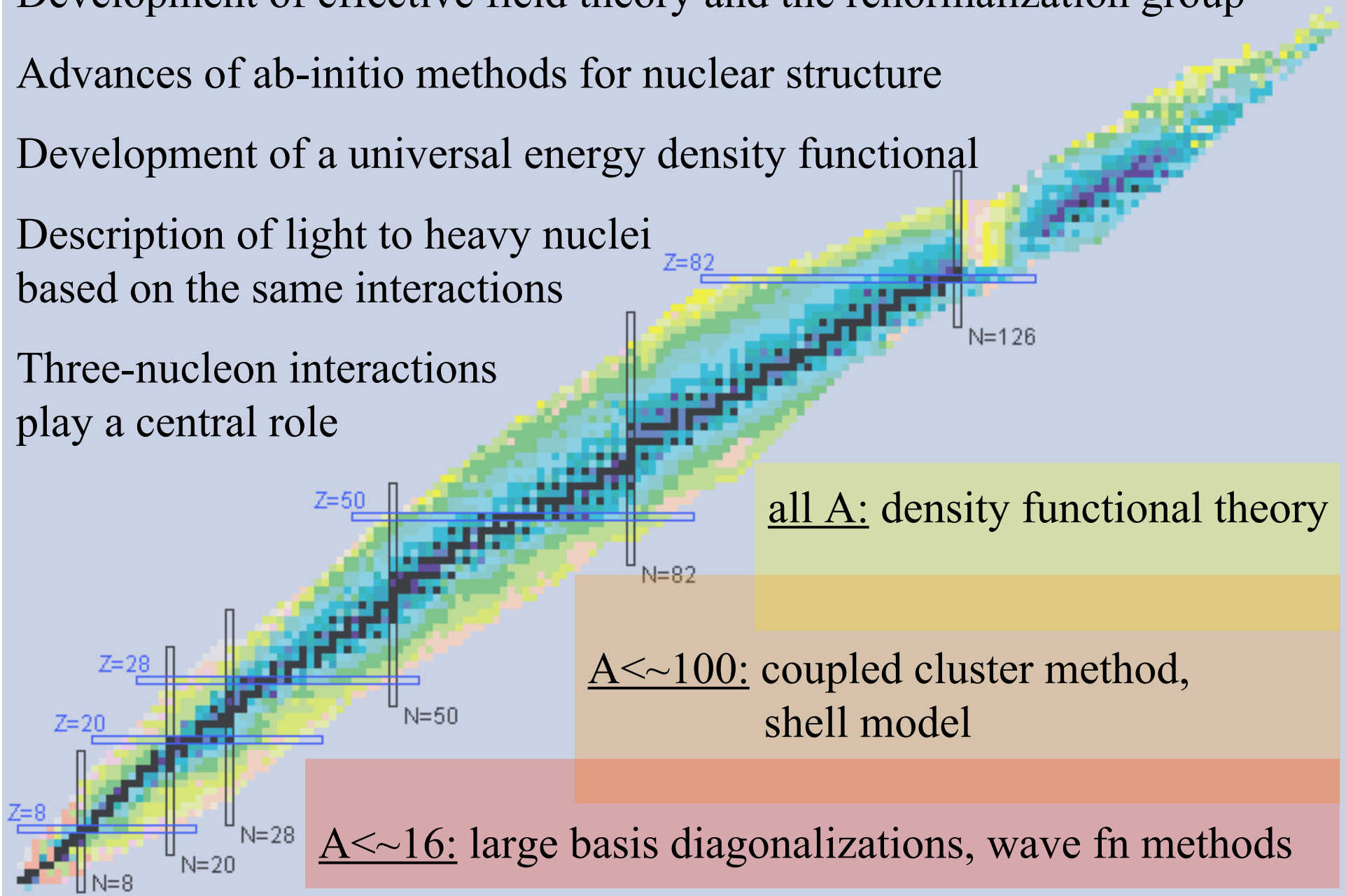
Development of effective field theory and the renormalization group

Advances of ab-initio methods for nuclear structure

Development of a universal energy density functional

Description of light to heavy nuclei based on the same interactions

Three-nucleon interactions play a central role



all A: density functional theory

A<~100: coupled cluster method, shell model

A<~16: large basis diagonalizations, wave fn methods

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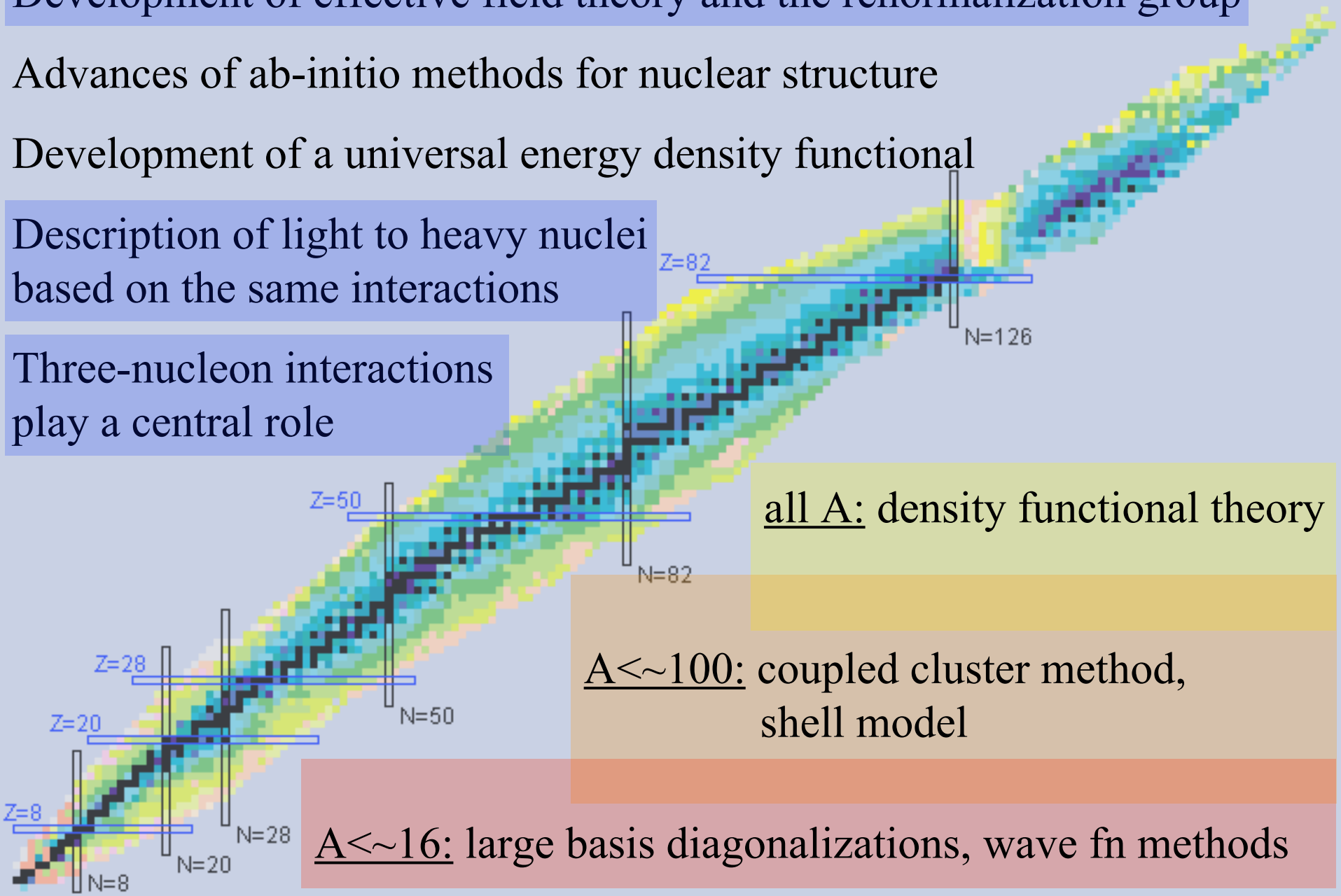

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<u>A<~100:</u> coupled cluster method, shell model

<u>A<~16:</u> large basis diagonalizations, wave fn methods

Outline

1. Introduction to effective field theory and the renormalization group

2. Chiral effective field theory

3. Renormalization group for nuclear forces

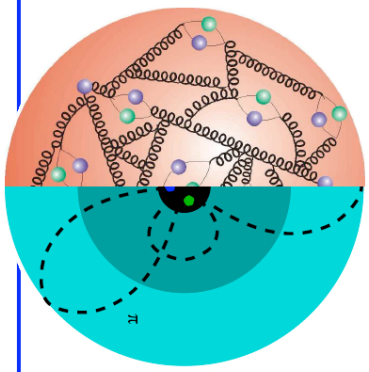
4. EFT and RG for nuclear matter

5. Nuclear matter in astrophysics

6. Neutrino processes in supernovae from chiral EFT

Λ / Resolution dependence of nuclear interactions

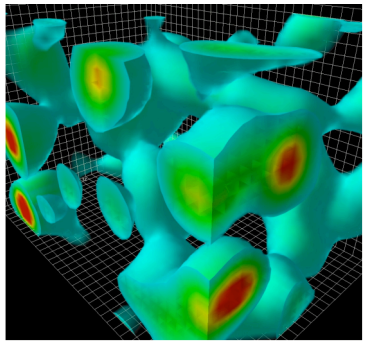
with high-energy probes:
quarks+gluons



Λ_{chiral}
momenta $Q \sim \lambda^{-1} \sim m_\pi$

$\Lambda_{pionless}$
$Q << m_\pi = 140$ MeV

at low energies:
complex QCD vacuum



lowest energy excitations:
pions, nearly massless, $m_\pi = 140$ MeV
"phonons" of QCD vacuum

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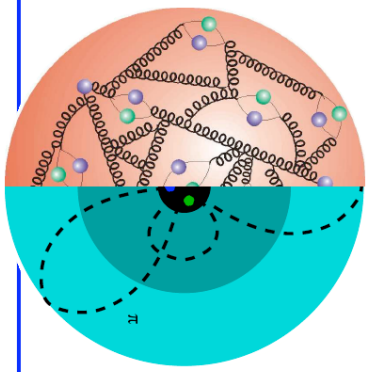
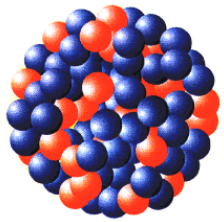


Effective theory for NN, many-N interactions,
operators depend on resolution scale Λ

$$H(\Lambda) = T + V_{\mathrm{NN}}(\Lambda) + V_{\mathrm{3N}}(\Lambda) + V_{\mathrm{4N}}(\Lambda) + \ldots$$

$\Lambda_{\mathrm{chiral}}$
momenta $Q \sim \lambda^{-1} \sim m_\pi$: chiral effective field theory
nucleons interacting via pion exchanges + contact interactions

typical Fermi momenta in nuclei ~ m_π



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$Q << m_\pi$=140 MeV: pionless effective field theory
nucleons interacting via contact interactions only

large scattering lengths + corrections
applicable to loosely-bound, dilute systems, reactions at astro energies

halo nuclei
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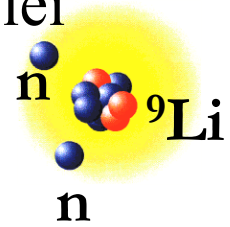
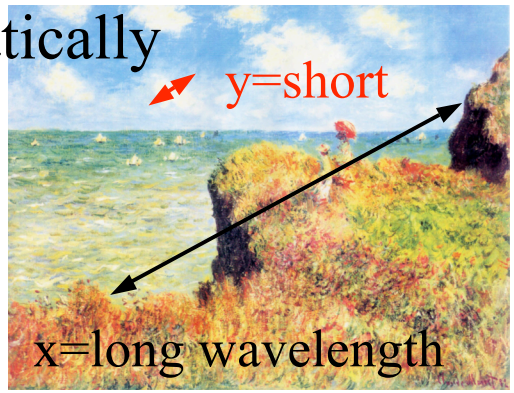


Idea of the renormalization group (RG)

integrate out high-momentum modes that are not resolved
and incorporate their effects in couplings of effective theory

schematically



$$Z = \int dx \int dy\, e^{-S(x,y)} = \int dx \int dy\, e^{-a(x^2+y^2)-b(x^2+y^2)^2}$$

$$= \int dx\, e^{-S_{\rm eff}(x)} = \int dx\, e^{-a'x^2-b'x^4-c'x^6+\ldots}$$

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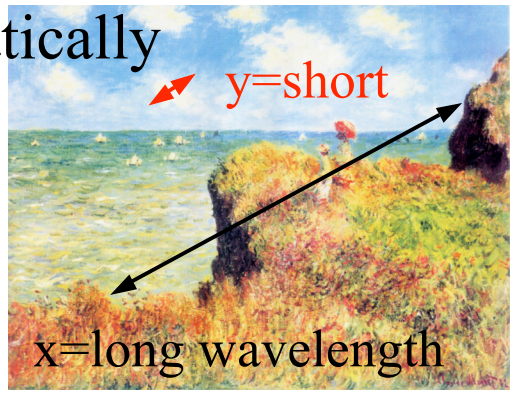

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separate into slow and fast modes $\phi(\omega,k) = \begin{cases} \phi_<(\omega,k) & \text{for } \omega,k < \Lambda \\ \phi_>(\omega,k) & \text{else} \end{cases}$

and integrate out fast modes

$$Z = \int \prod d\phi_<(\omega,k)\, e^{-S_{\text{free}}[\phi_<]} \int \prod d\phi_>(\omega,k)\, e^{-S_{\text{free}}[\phi_>]-S_{\text{int}}[\phi_<,\phi_>]}$$

$$= \int \prod d\phi_<(\omega,k)\, e^{-S_{\text{eff}}[\phi_<]}$$

when we integrate out momentum modes by $\delta\Lambda$ the couplings evolve

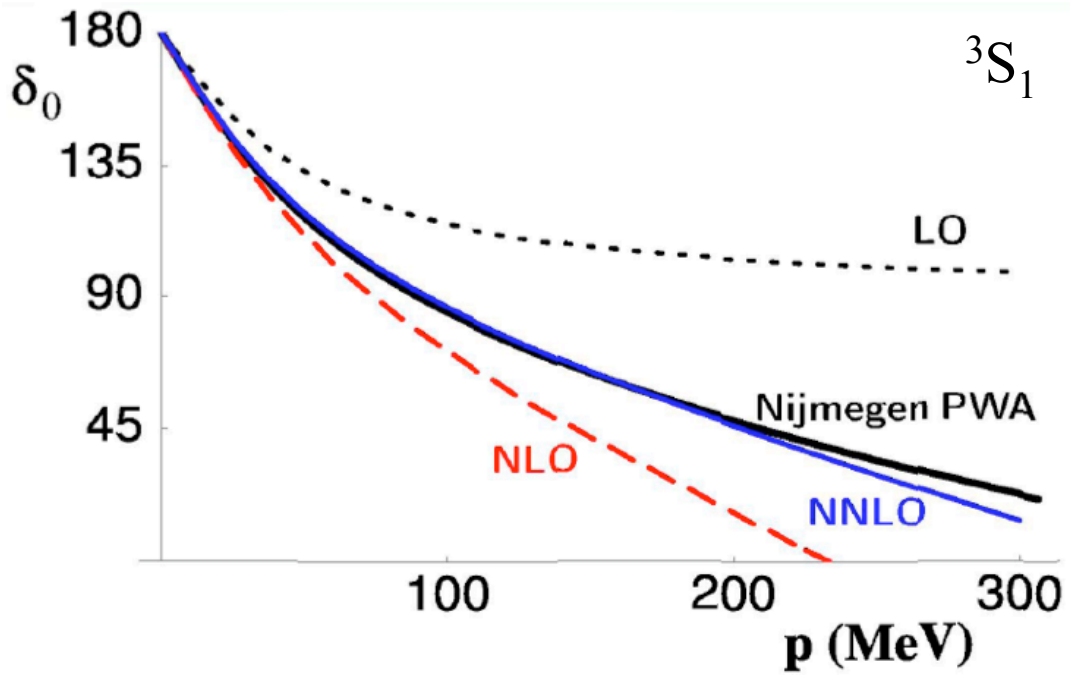
$$g_i(\Lambda-\delta\Lambda) = g_i(\Lambda) - \frac{\delta\Lambda}{\Lambda}\beta_i\big(\{g_j(\Lambda)\},\Lambda\big)$$

resolution/Λ-dependent couplings and RG eqns $\Lambda\frac{d}{d\Lambda}g_i(\Lambda) = \beta_i\big(\{g_j(\Lambda)\},\Lambda\big)$

Phase shifts in pionless EFT

converges to PWA
(partial wave analysis
of NN scattering data)

order-by-order improvement



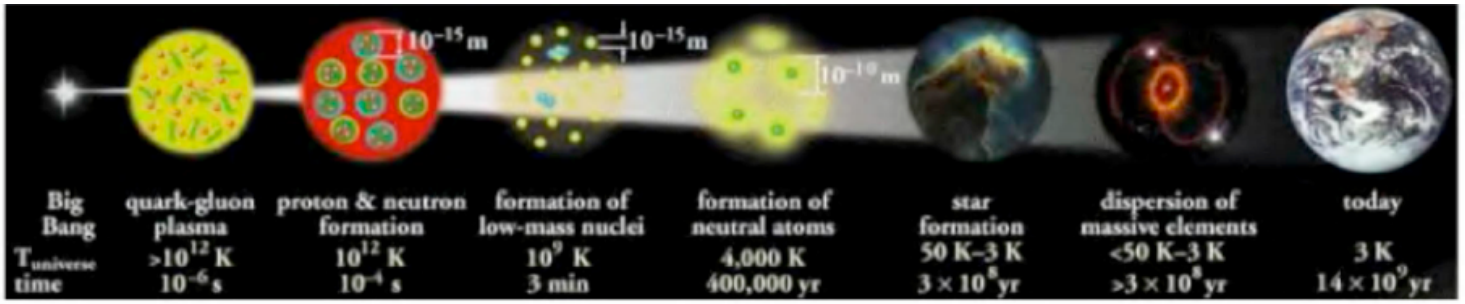
pionless EFT reproduces effective range expansion,
but EFT advantages:

systematic electroweak currents

systematic few-body calculations

systematic finite temperature, finite density,…

Pionless EFT and big-bang nucleosynthesis

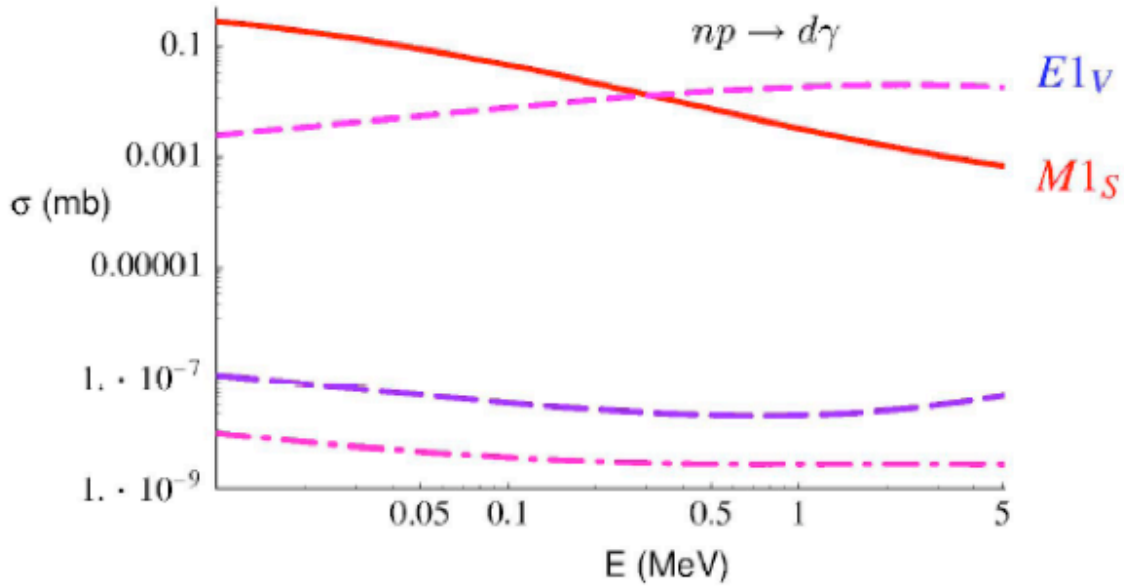


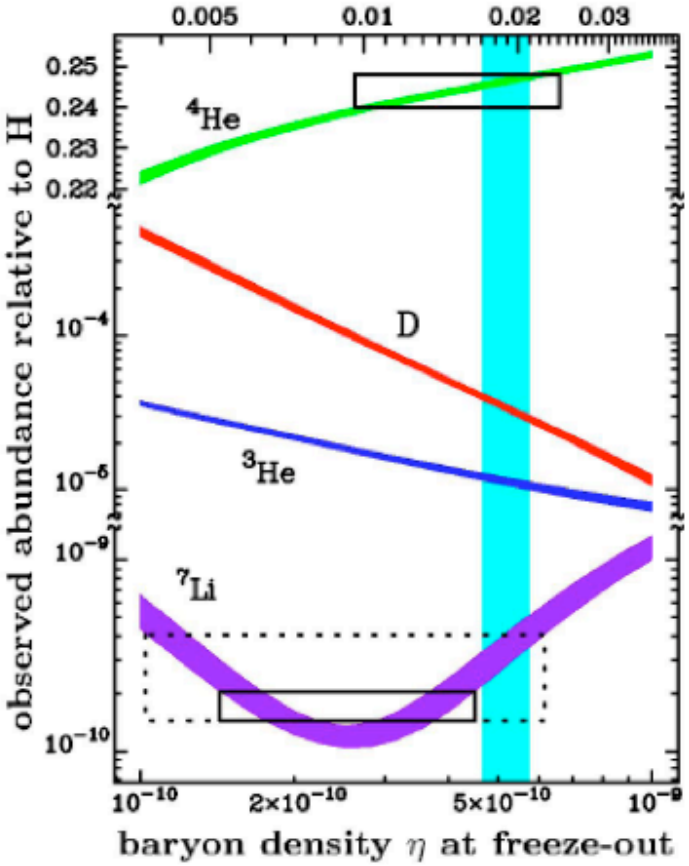
$E_{\rm typ} \approx 0.02 - 0.2$ MeV, light-element abundances sensitive to baryon density.

Accurate theoretical determination necessary: error-estimate!

$np \to d\gamma$ biggest uncertainty, but "impossible" to measure.

EFT($\not\pi$) to N^{4}LO in closed form: accuracy $\lesssim 1\%$. Rupak 1999





slide from Griesshammer; Chen, Savage (1999), Rupak (2000)

Pionless EFT applied to few-body systems

Leading-order NN contact interactions $^1S_0+^3S_1$ $C_0(\Lambda)$ lead to divergence in triton channel, cutoff dependence generates Phillips line Bedaque et al. (1999)

and band around Tjon line in A=3,4 system Platter et al. (2005)

promote 3N contact interaction $D_0(\Lambda)$ to leading order, coupling fixed by triton energy, exhibits RG limit cycle

$H(\Lambda) \sim D_0(\Lambda)$

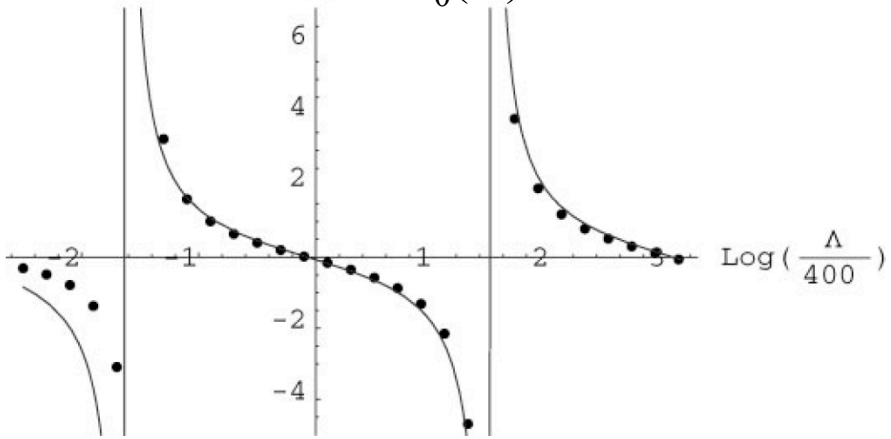


Figure 7 Three-body force coefficient $H(\Lambda)$ computed analytically (line) and numerically (points) as a function of $\log(\Lambda/400\ \text{MeV})$.

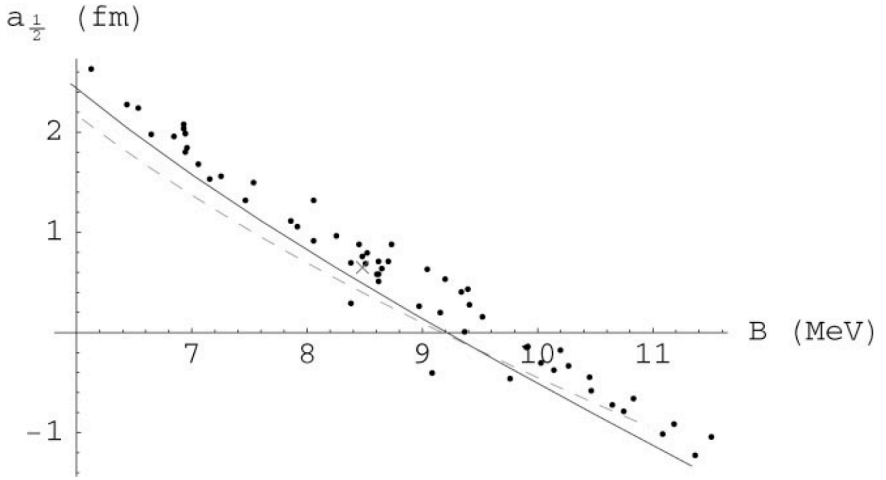
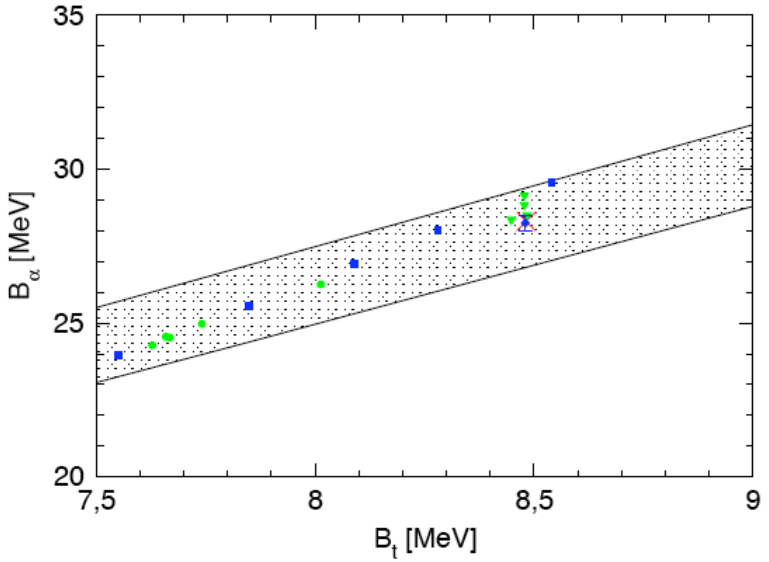
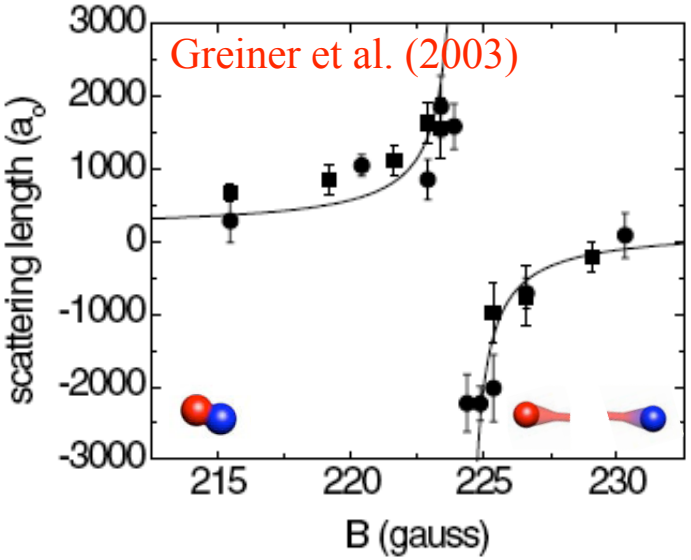


Figure 9 Correlation between the doublet *S*-wave nucleon-deuteron scattering length and the triton binding energy (Phillips line): predictions of different models (points), EFT at LO (light dashed line) and NLO (dark solid line), and experimental value (cross).



Strong interactions with cold atoms

Controlled strong interactions:
large scattering lengths via
Feshbach resonances



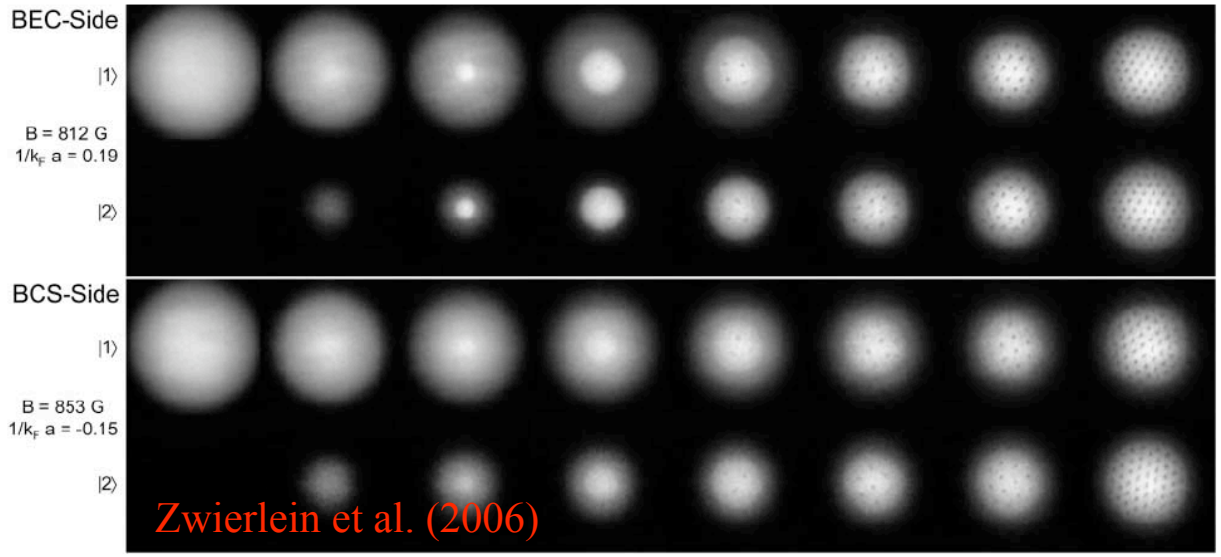
Many control knobs:
wide/narrow, S-wave, P-wave,…

Variable composition:
spin-polarization, m/M,
Bose-Fermi mixtures

Rotation

Optical lattices

……



Large scattering lengths lead to Efimov effect

leading-order two- and three-body contact interactions:

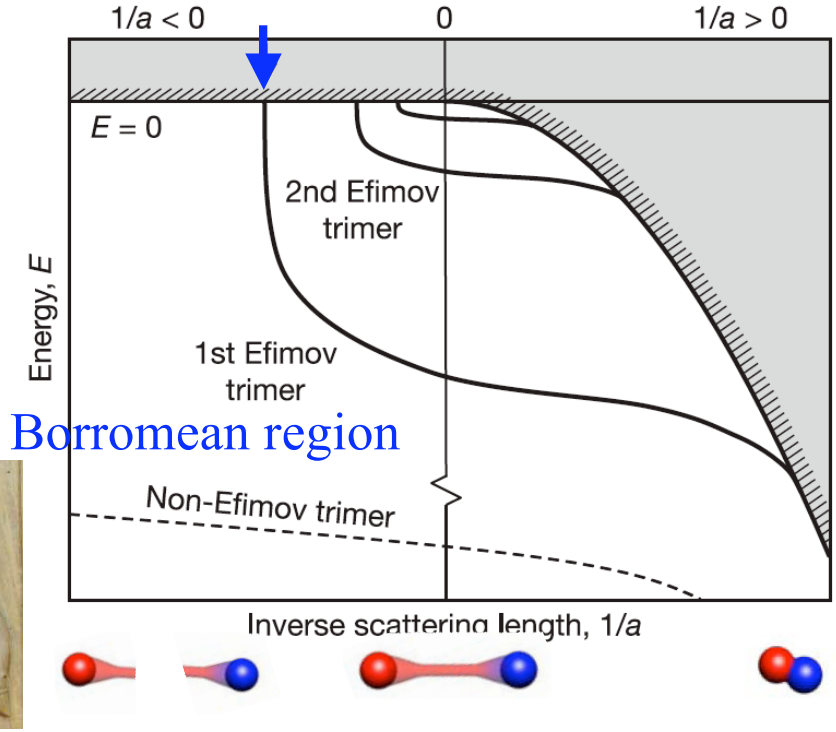
Efimov spectrum for 3 distinguishable particles or 3 bosons with identical pair-wise scattering lengths

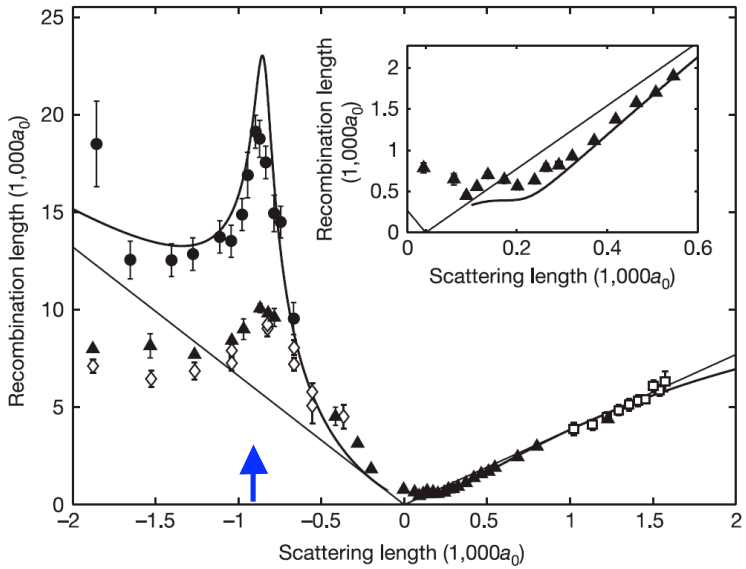
predicts Borromean states ~three rings quantum three-body bound states, all two-body subsystems unbound

Universal physics on resonance: infinite tower of bound excited states with universal scaling $E_n/E_{n+1} = 515$

observed first Efimov resonance in 3-body losses for Cs bosons

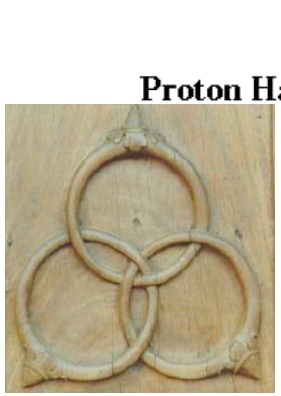
Kraemer et al. (2006)



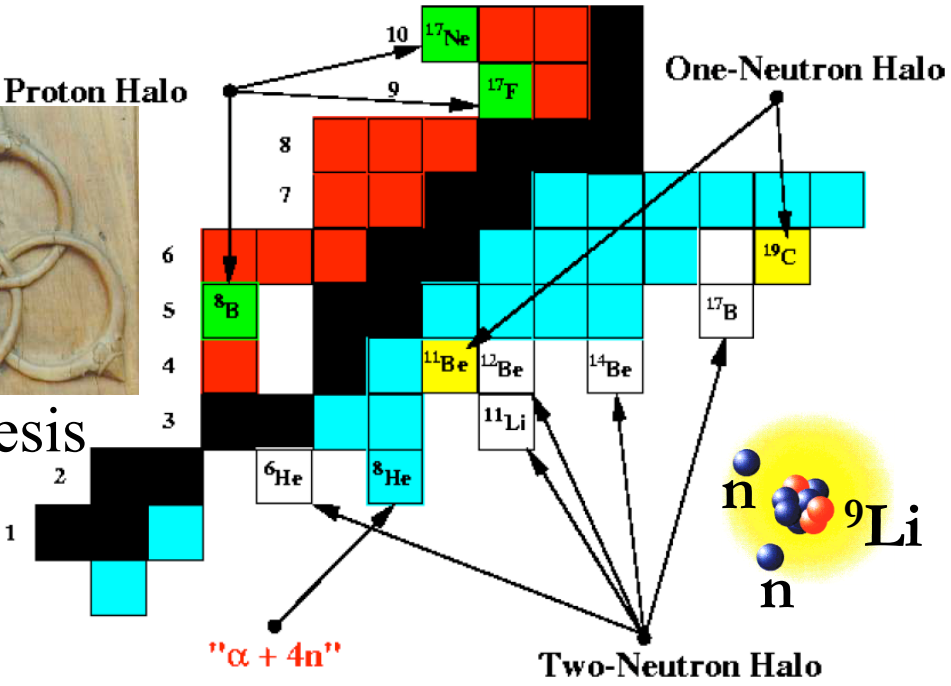


Borromean states in nuclei

nn halos near neutron drip-line are the only known examples of Borromean states in nature

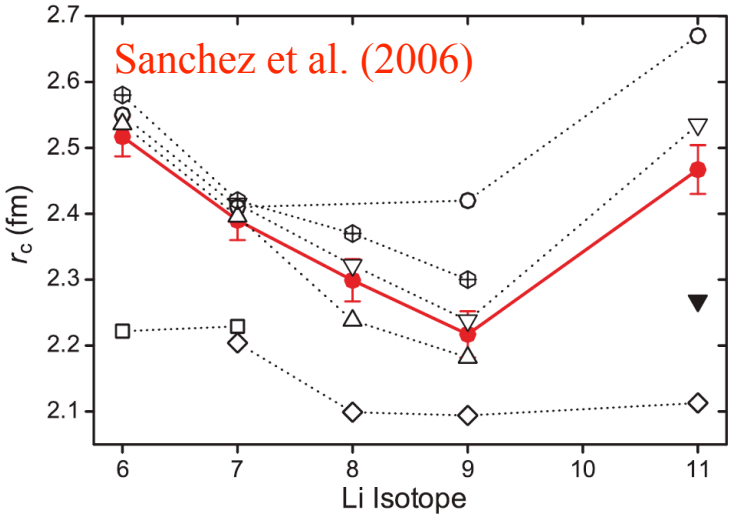


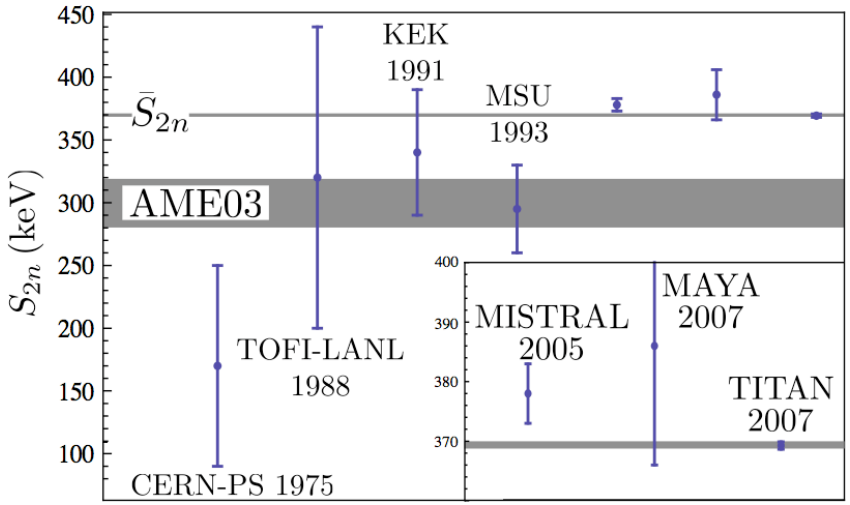
Partly responsible for A=5 mass gap, important for nucleosynthesis



Recent results for short-lived ^{11}Li: charge radius and mass

Smith et al. (2008)





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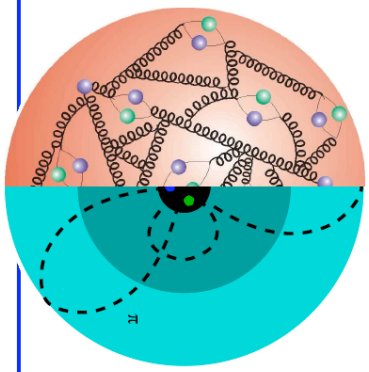
Λ / Resolution dependence of nuclear interactions

with high-energy probes:
quarks+gluons

cf. scale/scheme dependence
of parton distribution functions



Lattice QCD

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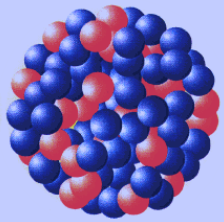

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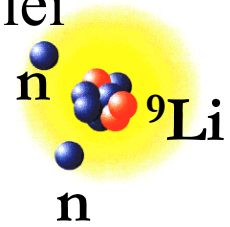
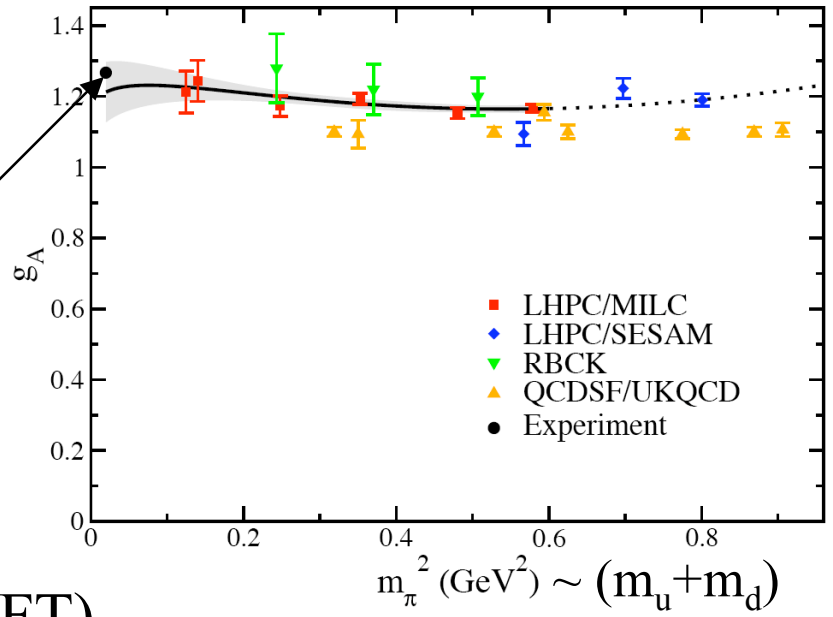


Lattice QCD and nuclear forces

Long-range couplings:

pion-NN coupling g_a from full QCD
Edwards et al. (2006)

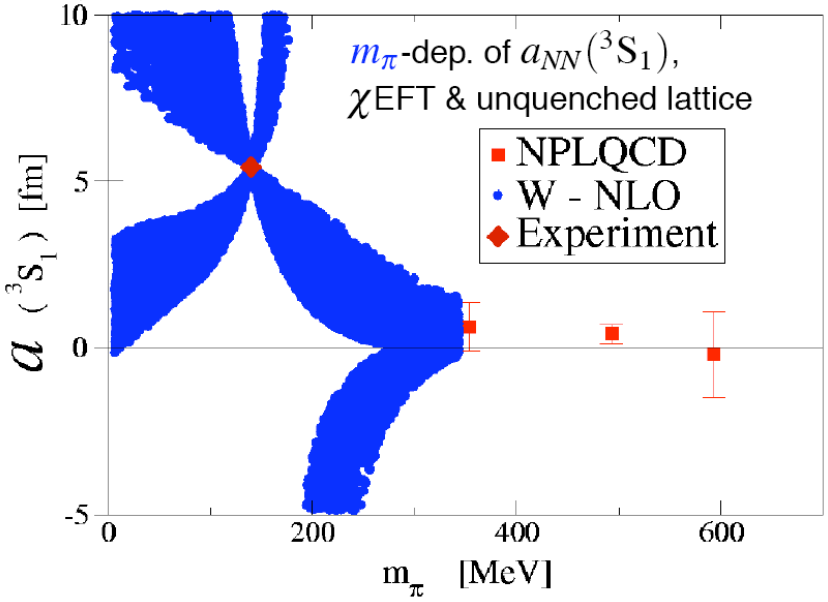


chiral EFT extrapolation to physical pion mass agrees with experiment

constrain higher-order long-range pion-, N-,… couplings (scheme: chiral EFT)

Few-nucleon observables:

NN scattering lengths from full QCD,
dependence on quark masses Beane et al. (2006)



Constrain experimentally difficult observables: 3-neutron properties
first steps: 3 pions on a lattice Detmold et al. (2008)

Lattice QCD and nuclear forces

NN potential:

not unique, especially short-range

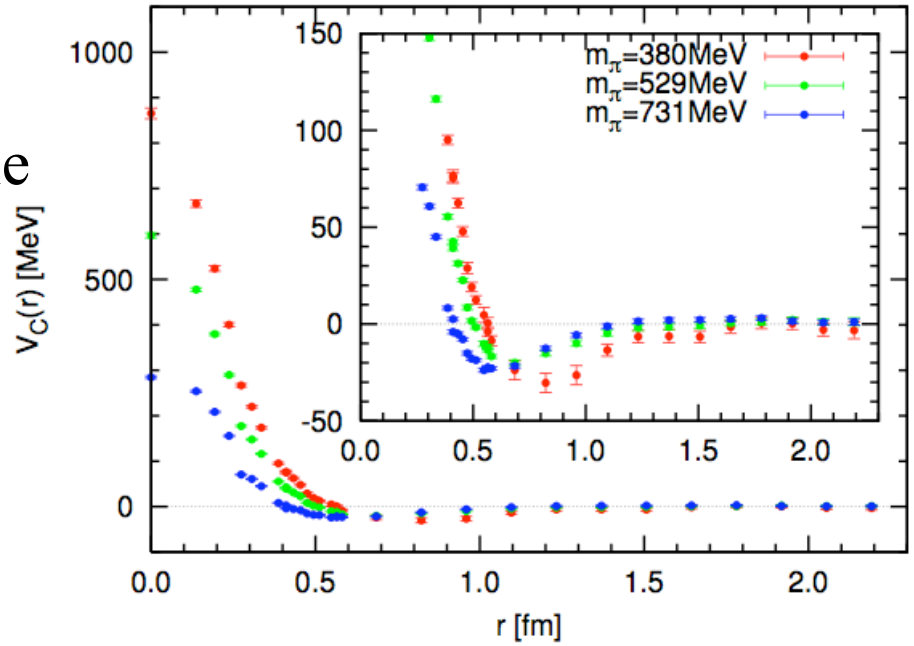
properties depend on scheme and scale

Ishii, Aoki, Hatsuda (2007)

scheme: quasi-local potential,

nucleon interpolating field,…

(not chiral EFT)



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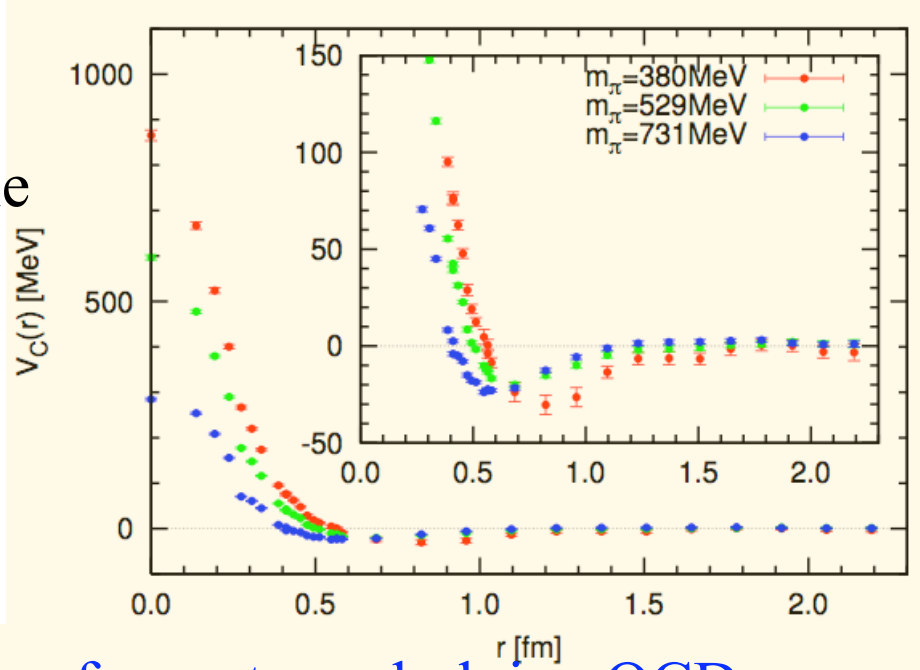
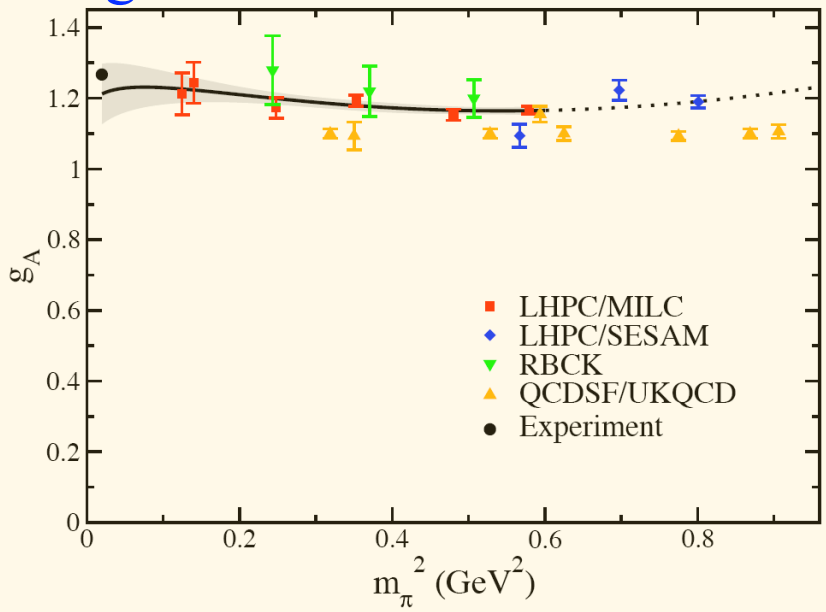
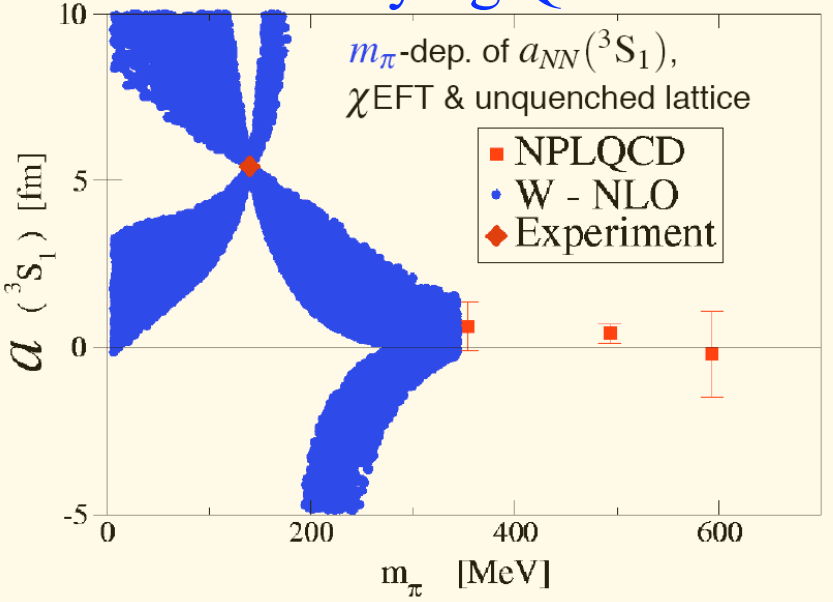


First, exciting efforts to connect nuclear forces to underlying QCD





References

P. Lepage, How to renormalize the Schroedinger equation, nucl-th/9706029.

P. Bedaque and B. van Kolck, EFT for few-nucleon systems, Ann. Rev. Nucl. Part. Sci. **52**, 339 (2002), nucl-th/0203055.

D. Phillips, Building light nuclei from neutrons, protons and pions, Czech J. Phys. **52**, B49 (2002), nucl-th/0203040.

D. Kaplan, Five lectures on EFT, nucl-th/0510023.

E. Braaten and H.-W. Hammer,
Universality in few-body systems with large scattering length,
Phys. Rept. **428**, 259 (2006), cond-mat/0410417.

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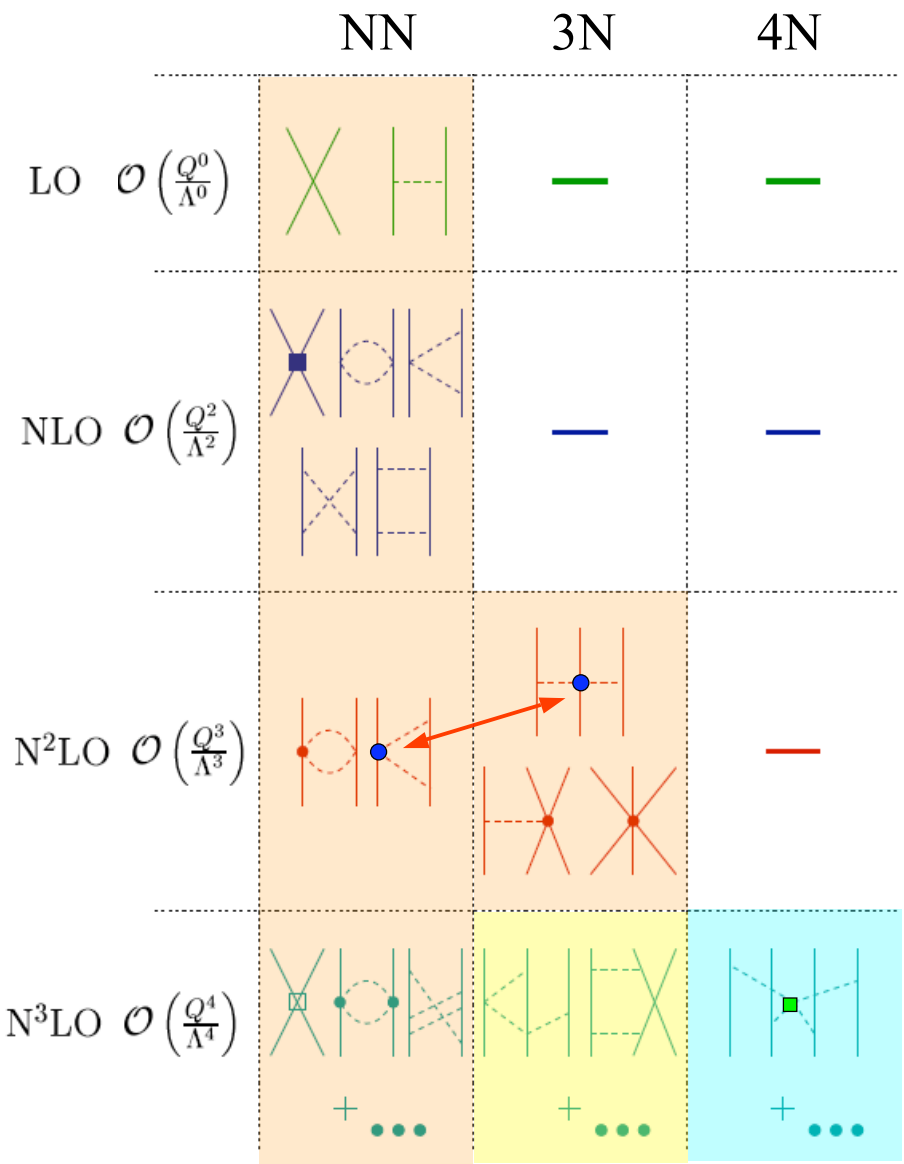
Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



limited resolution at low energies, can expand in powers Q/Λ_b

include long-range pion physics

details of short-distance physics not resolved

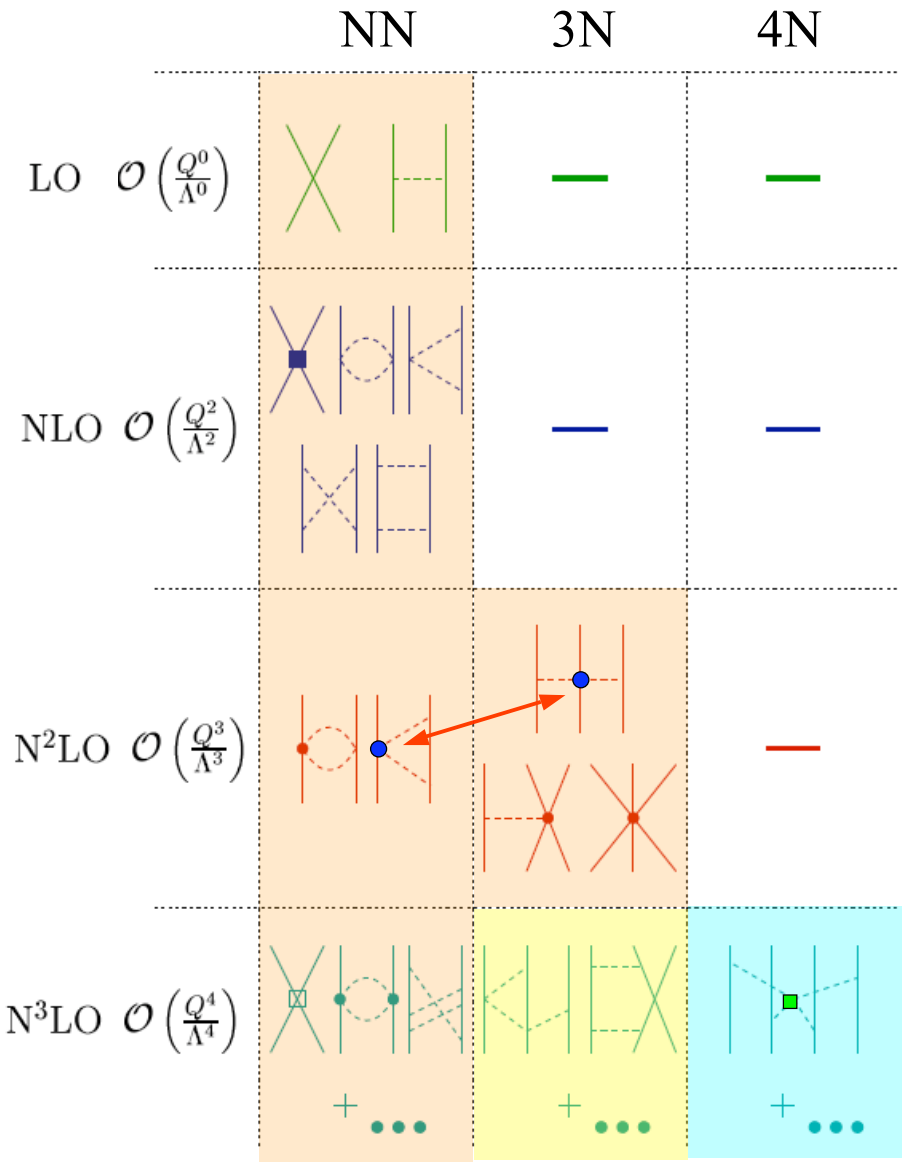
capture in few short-range couplings, fit to experiment once (experiment includes all short-range effects)

systematic: can work to desired accuracy and obtain error estimates

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



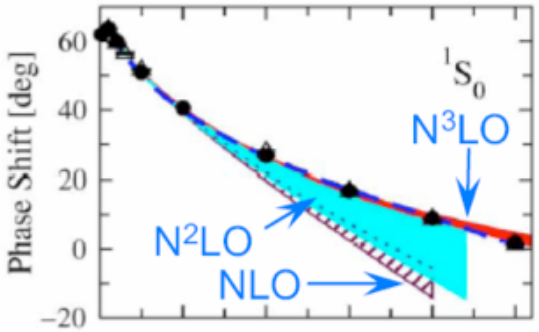
explains pheno hierarchy:
NN > 3N > 4N > …

consistent NN-3N, πN, ππ, electroweak operators

3N,4N: 2 new couplings to N³LO

resolution/Λ-dep. contact interactions

error estimates from truncation order and resolution/Λ-variation



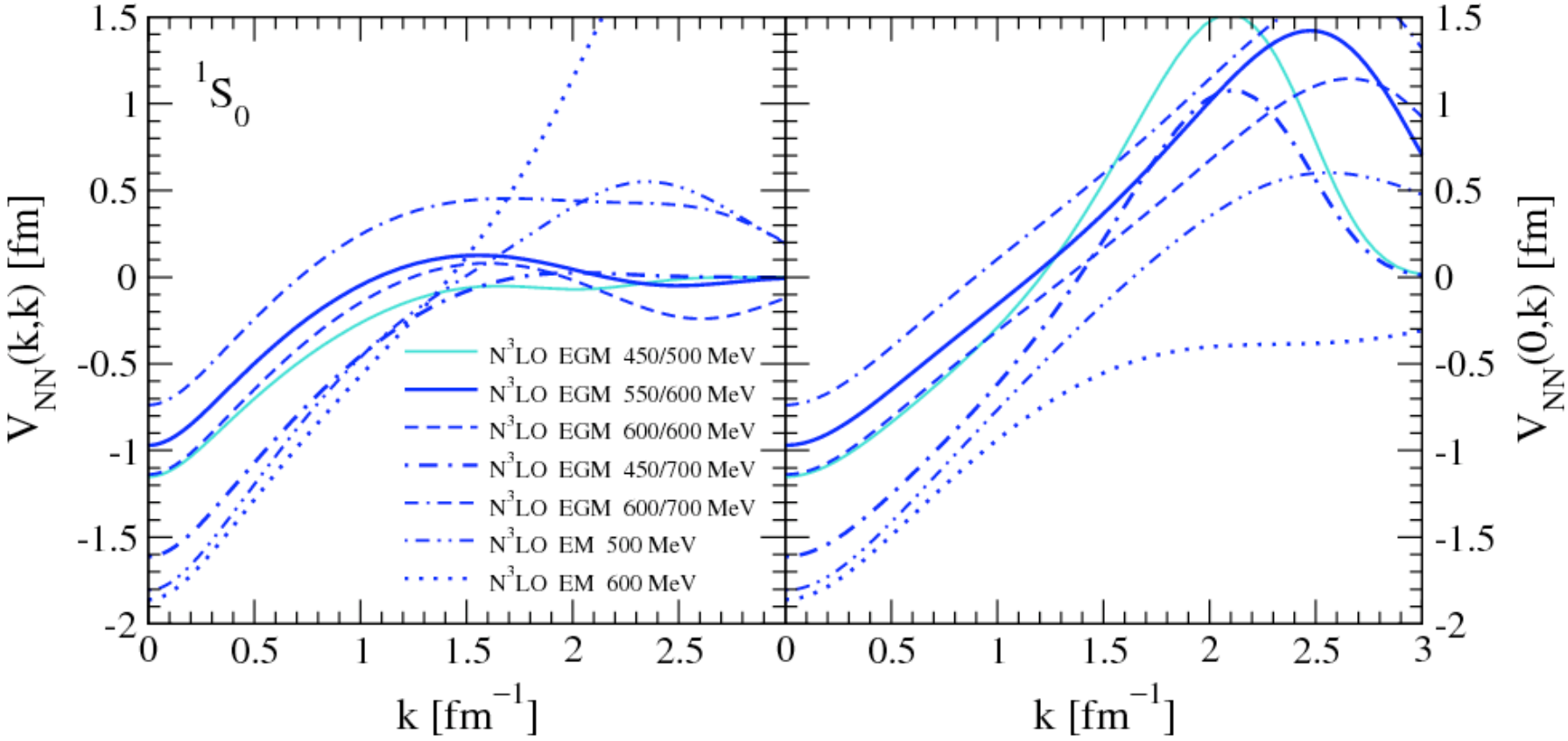
Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

References

E. Epelbaum, Few-nucleon forces and systems in chiral EFT, Prog. Part. Nucl. Phys. **57**, 654 (2006).

R. Machleidt, Nuclear forces from chiral EFT, arXiv:0704.0807.

Chiral EFT interactions at N^{3}LO

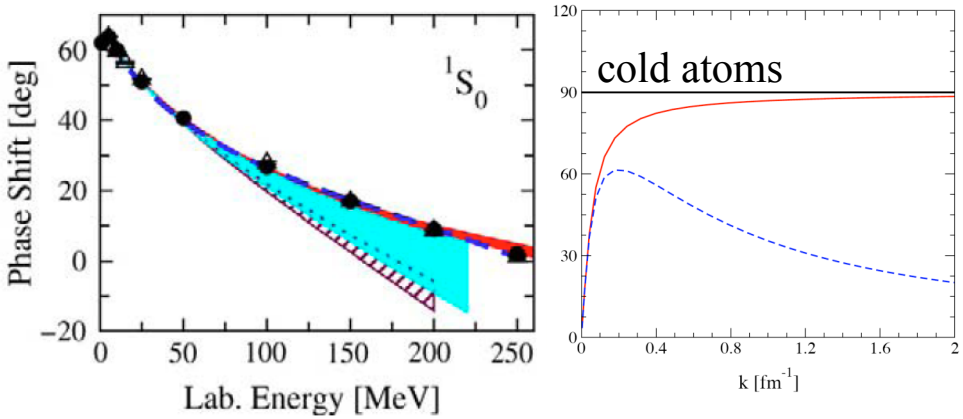


regulator and renormalization scheme and scale dependence

There is not just one N^{3}LO potential!

Chrial EFT phase shifts and NN PWA

Nijmegen PWA93 (filled circles) http://www.nn-online.org



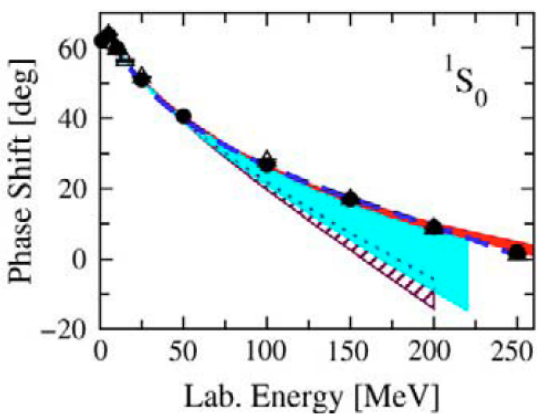
a_{nn} only

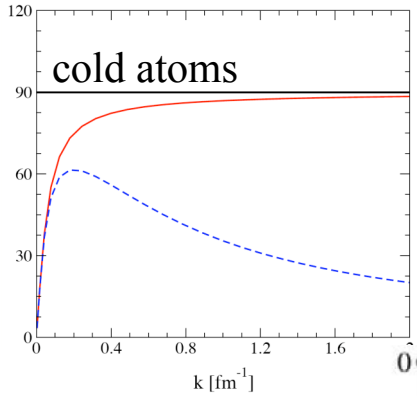
a_{nn}+effective range
r_{nn}=2.7fm

S-waves weaker at higher energies:
fewer particles interact strongly
at finite density

Chrial EFT phase shifts and NN PWA

Nijmegen PWA93 (filled circles) http://www.nn-online.org





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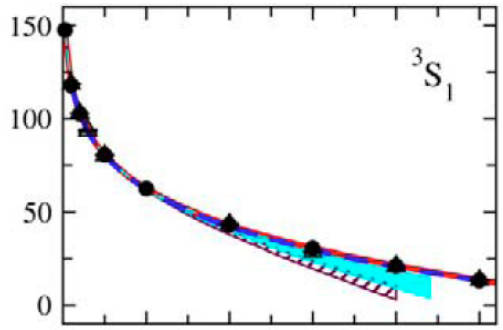
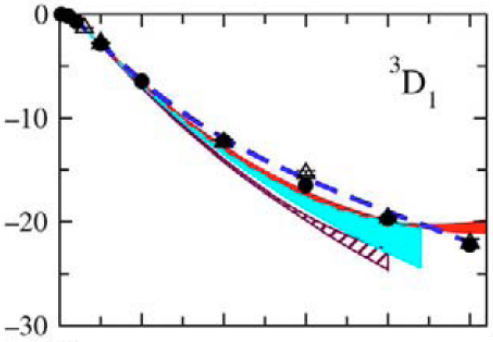
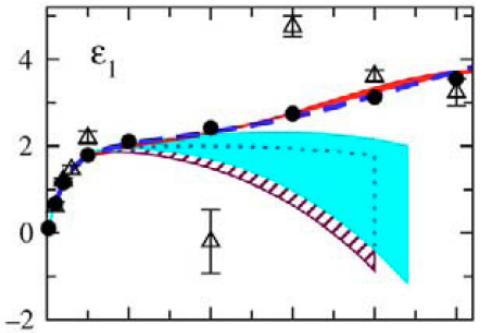






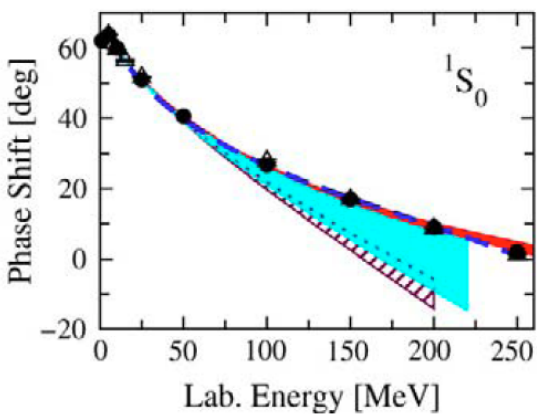
S-waves weaker at higher energies:
fewer particles interact strongly
at finite density

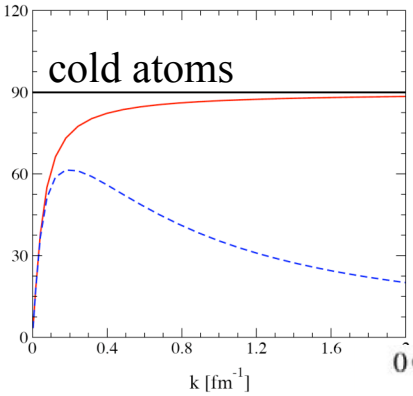
Tensor force from $^3S_1$-$^3D_1$ mixing ε_1 in spin=1 channel

Epelbaum (2006)
Machleidt (1998)

Chrial EFT phase shifts and NN PWA

Nijmegen PWA93 (filled circles) http://www.nn-online.org





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a_{nn}+effective range
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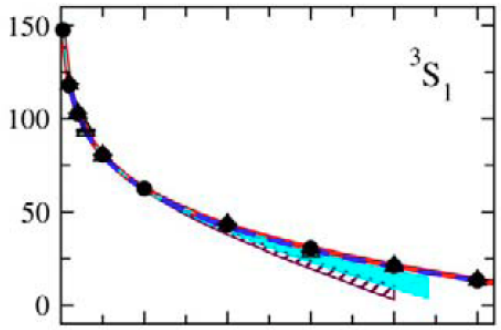

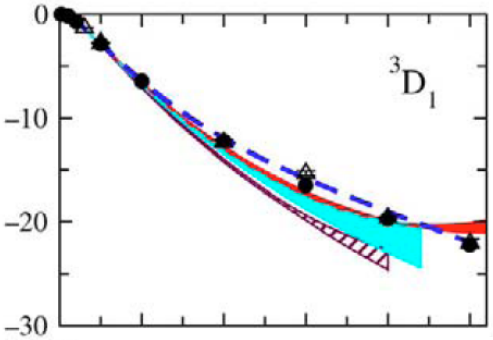

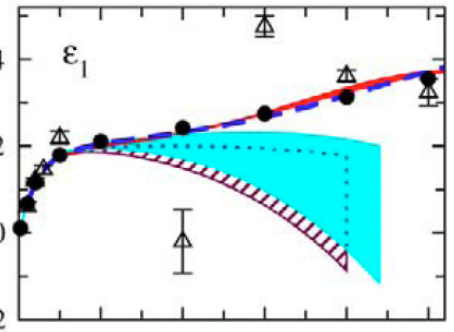

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Tensor force from $^3S_1$-$^3D_1$ mixing ε_1 in spin=1 channel

Epelbaum (2006)
Machleidt (1998)

Spin-orbit (LS) force
from triplet $^3P_{J=0,1,2}$
phase shifts
C=central, T=tensor only

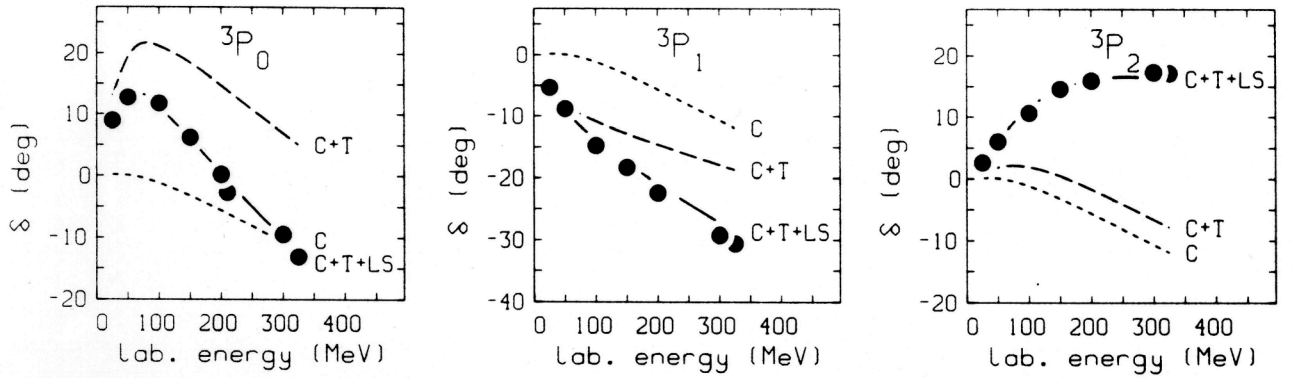
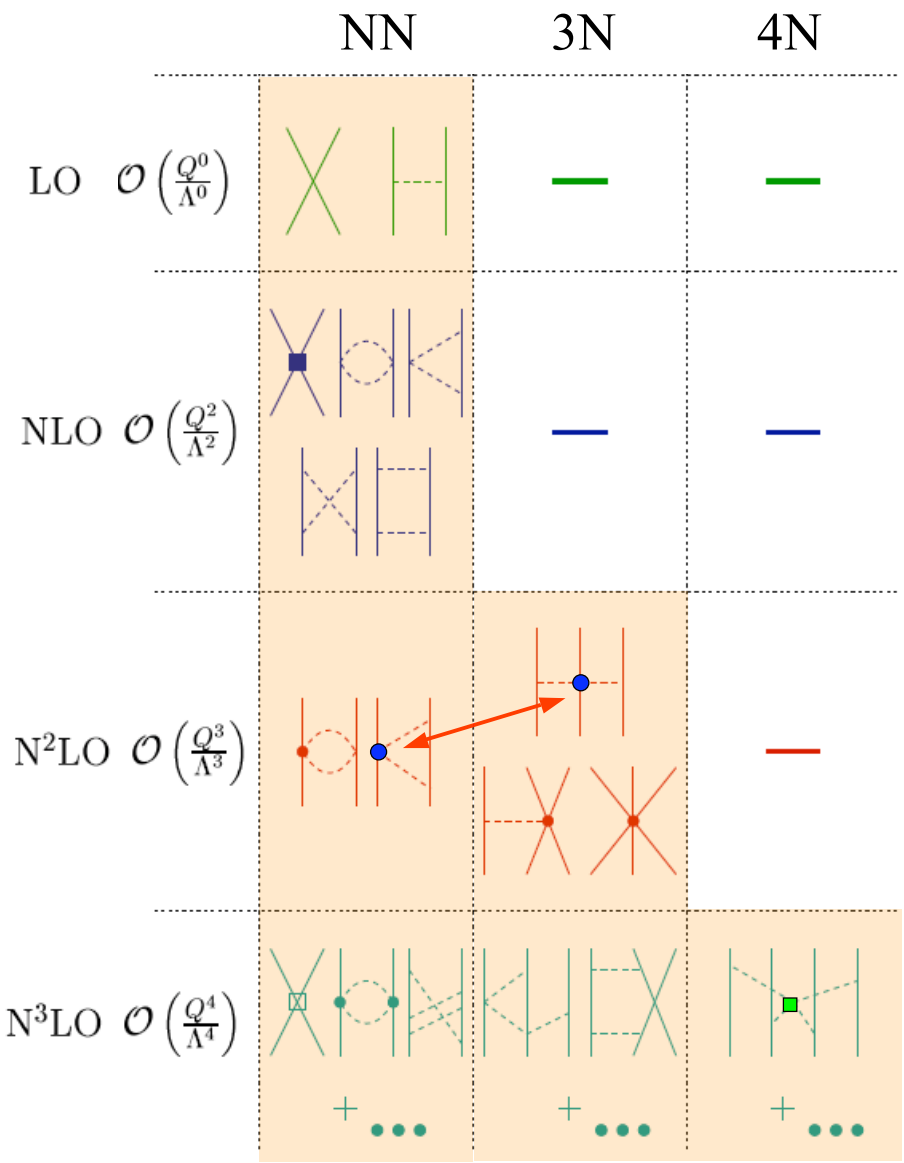


Fig. 3.3. *NN* phase shifts in triplet *P* waves. Shown are predictions using a central

Chiral EFT for nuclear forces

Separation of scales: low momenta $\frac{1}{\lambda} = Q \ll \Lambda_b$ breakdown scale ~500 MeV



Open questions:

Power counting with singular pion exchanges, tensor parts ~$1/r^3$ for OPE
more singular for higher order pion exch.

promotion of contact interactions
Nogga, Timmermans, van Kolck (2005)

Delta-full m_Δ-$m_N \sim m_\pi$
vs. Delta-less chiral EFT

one-Δ 3N interaction starts at NLO

Counting of 1/m corrections,
$m \sim \Lambda^2$ or Λ, could be important for A_y

Lepage plots and breakdown scale

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Lepage plots

Lepage (1997)

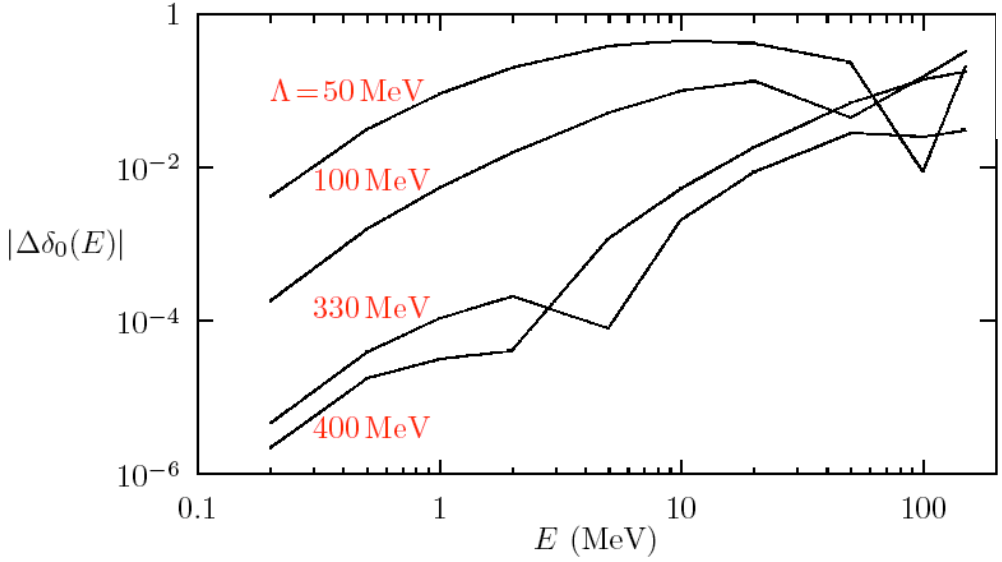
Log-log plots of relative errors vs. E

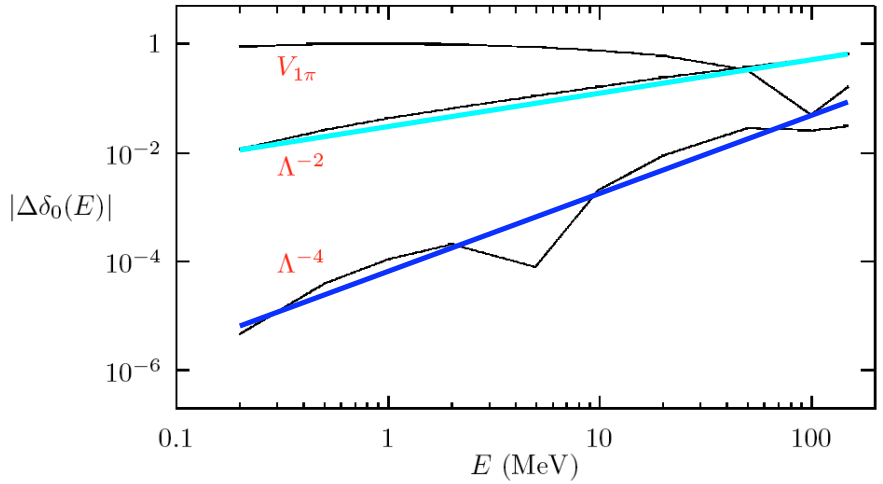
steeper slope with subsequent orders, same in all channels: S,P,D,… waves

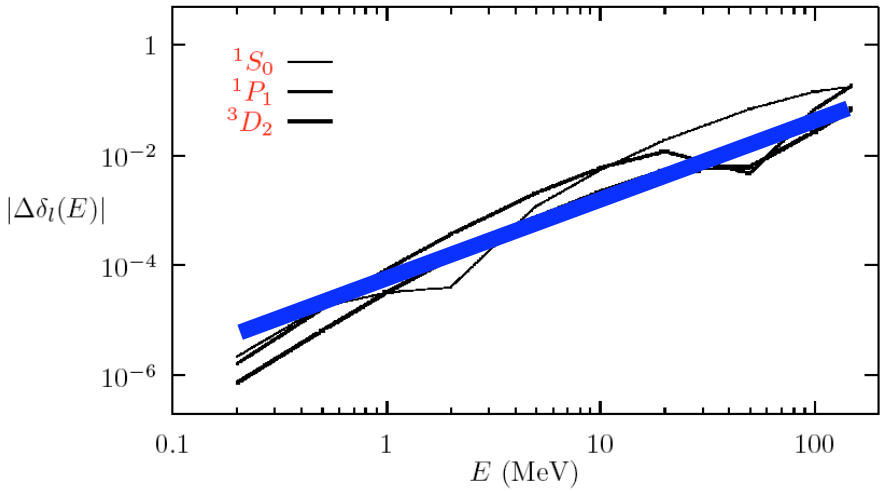
$$V(r) = -\alpha_\pi v_\Lambda(r) + c\frac{\delta^3_{1/\Lambda}(\mathbf{r})}{\Lambda^2} - d\frac{\nabla^2\delta^3_{1/\Lambda}(\mathbf{r})}{\Lambda^4}$$

$$P: \quad V_{\pi,P} + \frac{\nabla\delta^3_{1/\Lambda}\nabla}{\Lambda^4} + \mathcal{O}(1/\Lambda^6)$$

$$D: \quad V_{\pi,D} + \mathcal{O}(1/\Lambda^6)$$







Cutoff dependence:

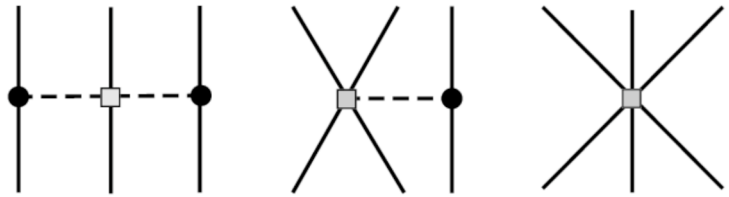
truncation errors decrease with increasing cutoff

no advantages for cutoffs >> breakdown scale (nonlinearities)

Chiral EFT 3N interactions

leading $N^2LO \sim (Q/\Lambda)^3$ van Kolck (1994), Epelbaum et al. (2002)

long (2π) intermed. (π) short-range



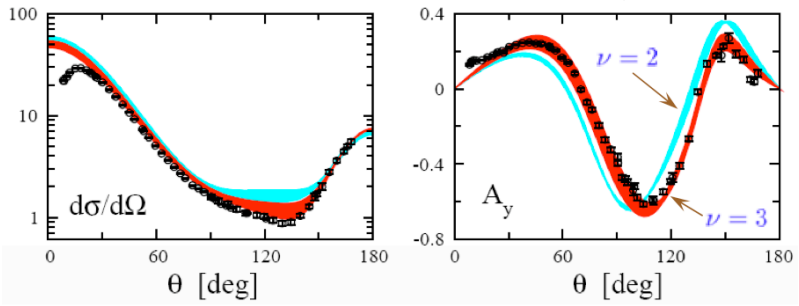
c_1,c_3,c_4 terms $D(\Lambda)$ term $E(\Lambda)$ term

$$= \frac{g_A^2}{8F_\pi^4}\frac{(\vec{\sigma}_1\cdot\vec{q}_1)(\vec{\sigma}_3\cdot\vec{q}_3)}{[\vec{q}_1{}^2+M_\pi^2][\vec{q}_3{}^2+M_\pi^2]}\left\{\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3(-4c_1M_\pi^2+2c_3\vec{q}_1\cdot\vec{q}_3)+c_4[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_3]\cdot\boldsymbol{\tau}_2[\vec{q}_1\times\vec{q}_3]\cdot\vec{\sigma}_2\right\} + \text{perm.}$$

$$= -\frac{g_A D}{8F_\pi}\frac{(\vec{\sigma}_3\cdot\vec{q}_3)}{\vec{q}_3{}^2+M_\pi^2}(\boldsymbol{\tau}_2\cdot\boldsymbol{\tau}_3)(\vec{\sigma}_2\cdot\vec{q}_3) + \text{perm.}$$

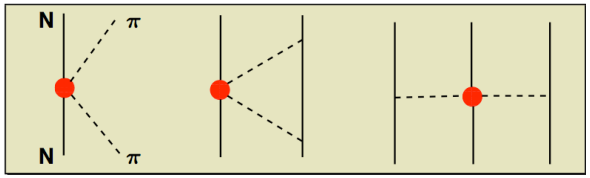
$$= \frac{1}{2}E(\boldsymbol{\tau}_1\cdot\boldsymbol{\tau}_3) + \text{perm.}$$

3N scattering pd @ 65MeV



c_i relate πN, NN, 3N: determination from πN

Meissner (2007) $c_1 = -0.9^{+0.2}_{-0.5}$, $c_3 = -4.7^{+1.2}_{-1.0}$, $c_4 = 3.5^{+0.5}_{-0.2}$



consistent with NN: c_1=-0.76(7), c_3=-4.78(10), c_4=3.96(22)

Rentmeester et al. (2003)

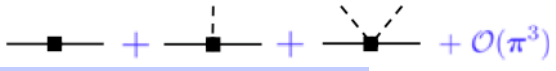
c_3,c_4 important for structure, but large uncertainties at present

D,E couplings fixed by A>2 data see Barrett's lectures and next lecture

D term can be fixed by tritium beta decay Gardestig, Phillips (2006) Gazit et al. (2008)

Subleading chiral EFT 3N interactions

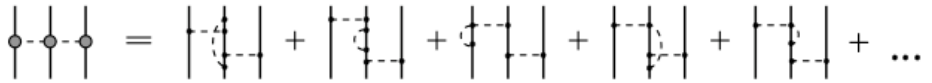
parameter-free N^{3}LO Bernard et al. (2007), Ishikawa, Robilotta (2007)

- 1/m-corrections to 1 insertion from $\mathcal{L}^{(2)}_{1/m}$ =  $+ \mathcal{O}(\pi^3)$

  — rich operator structure (includes spin-orbit interactions)

- 1-loop diagrams with all vertices from $\mathcal{L}^{(0)}_{\rm eff}$

2π – exchange

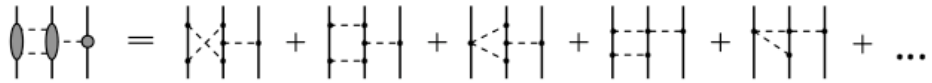


decrease c_i constants

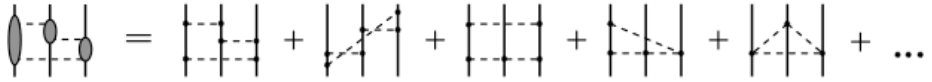
The calculated corrections simply shift the LECs c_i as follows:

$$\delta c_1 = \frac{g_A^2 M_\pi}{64\pi F_\pi^2} \sim 0.13\ \text{GeV}^{-1} \qquad \delta c_3 = \frac{3 g_A^4 M_\pi}{16\pi F_\pi^2} \sim 2.5\ \text{GeV}^{-1} \qquad \delta c_4 = -\frac{g_A^4 M_\pi}{16\pi F_\pi^2} \sim -0.85\ \text{GeV}^{-1}$$

2π-1π – exchange

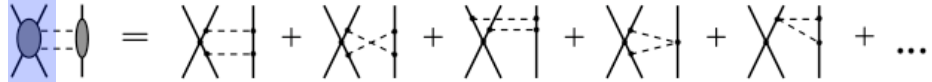


ring diagrams



contact-1π – exchange

contact-2π – exchange



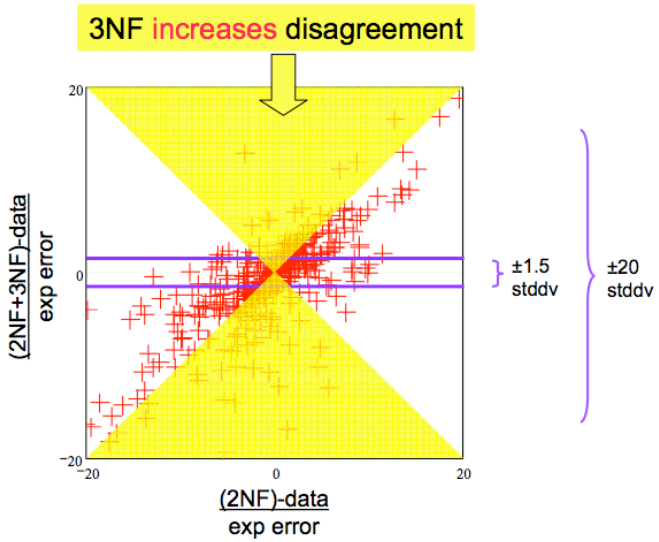
3N interactions involving NN contacts in progress Epelbaum et al.

3N interactions: a frontier

from H.-O. Meyer @ TRIUMF 3N workshop (2007)

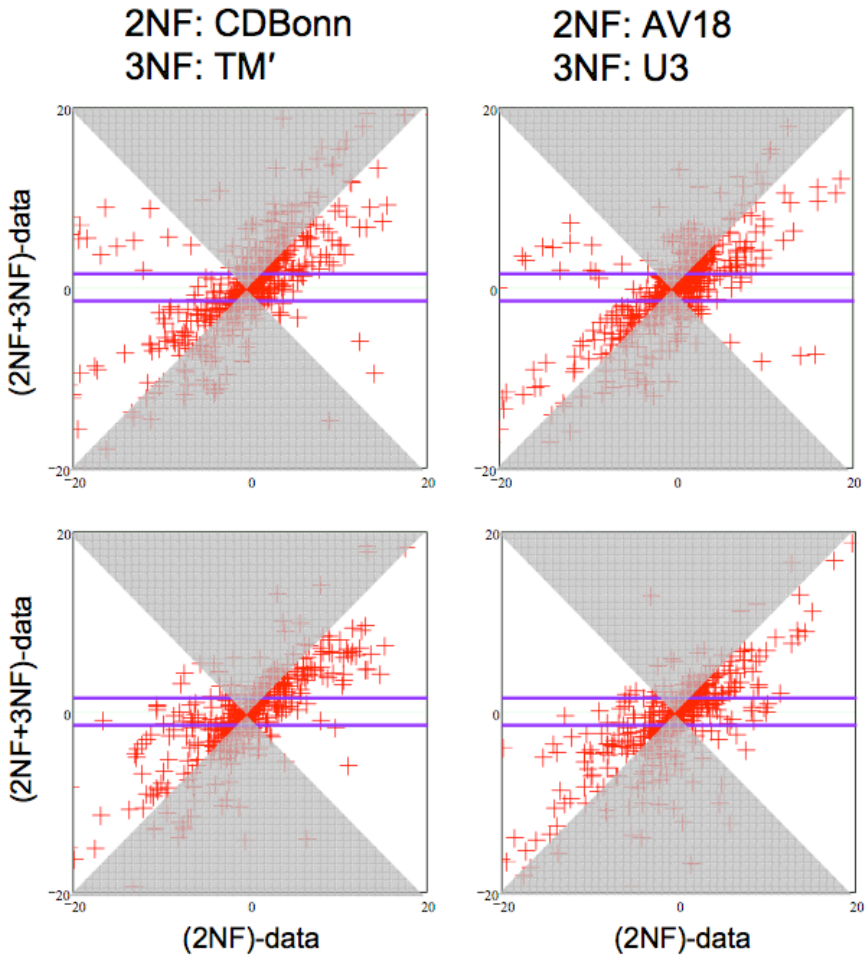
pd scattering

a way to look at
880 data points…

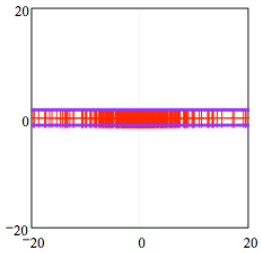


135 MeV

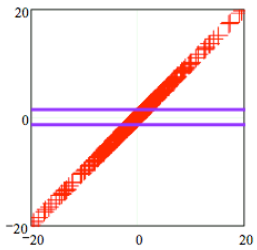
200 MeV



b) 3NF explains data



a) 3NF has no effect



coherent 3N effort needed with theoretical uncertainties

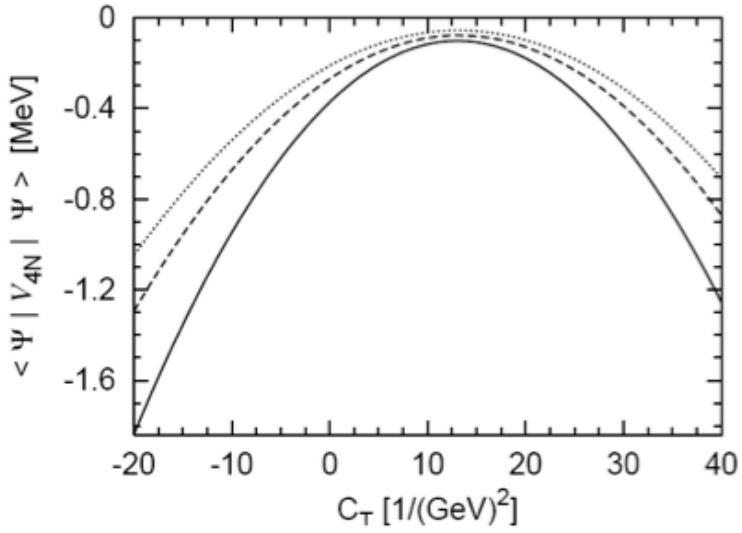
Chiral EFT 4N interactions - first estimate

from Epelbaum @ TRIUMF 3N workshop (2007)

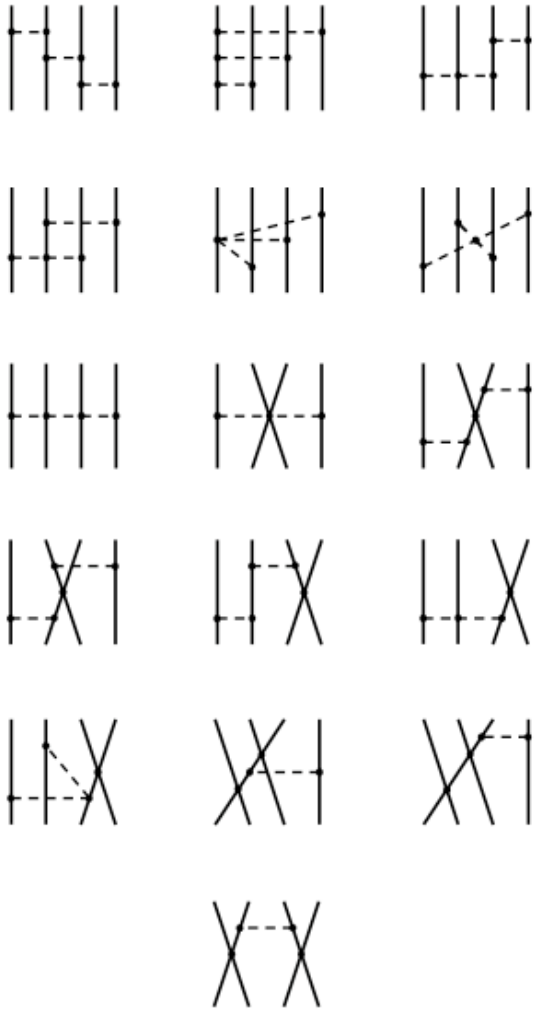
Four-nucleon force *(E.E. '05)*

- first shows up at order $\nu = 4$
- chiral symmetry plays a crucial role
- parameter-free

Contribution of the 4NF to the ^{4}He BE is attractive and of the order of few 100 keV
(Rozpedzik et al. ´06)

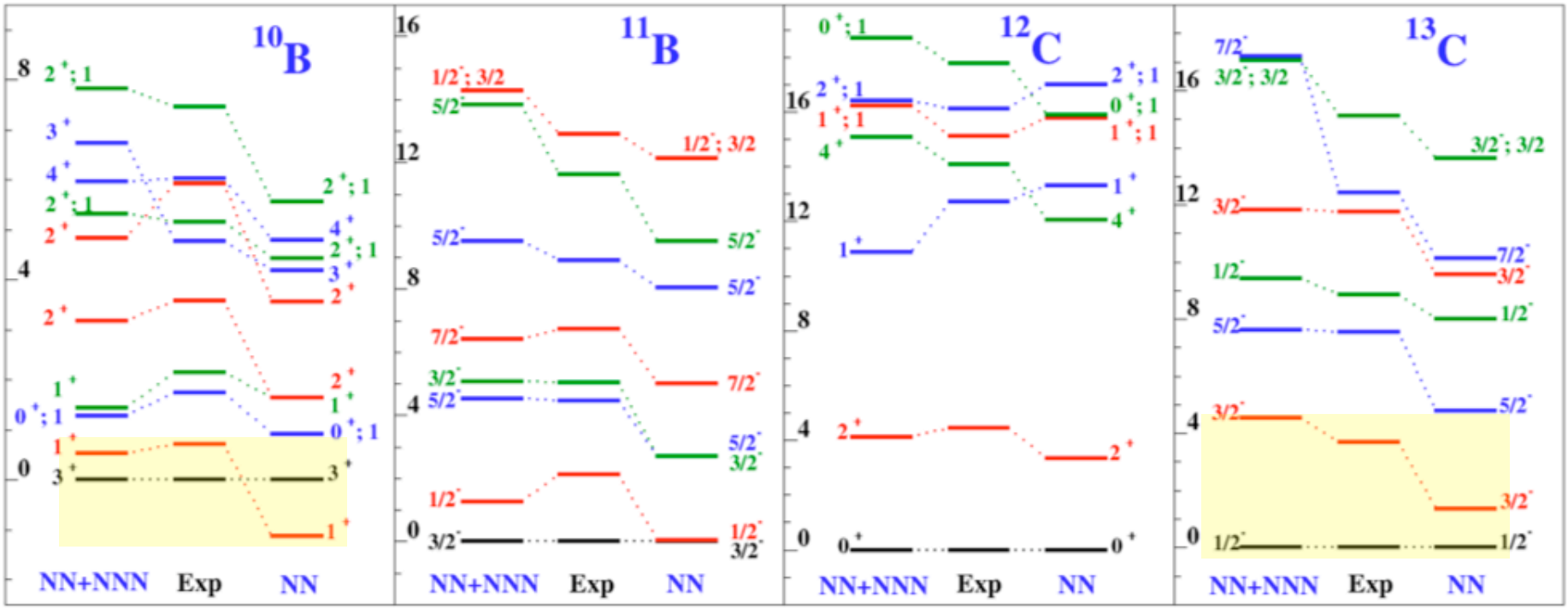


Results from: Rozpedzik et al., nucl-th/0606017



4N contributions ~ 1 MeV at saturation density not unreasonable

3N interactions and nuclear structure



No-Core Shell Model: Navratil et al. (2007)

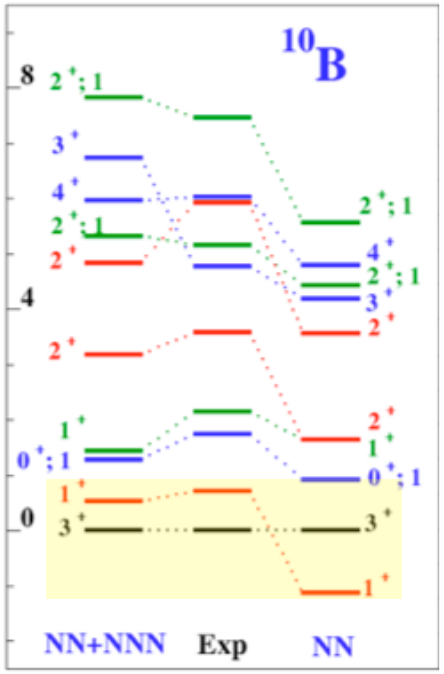
agreement supports chiral EFT interactions

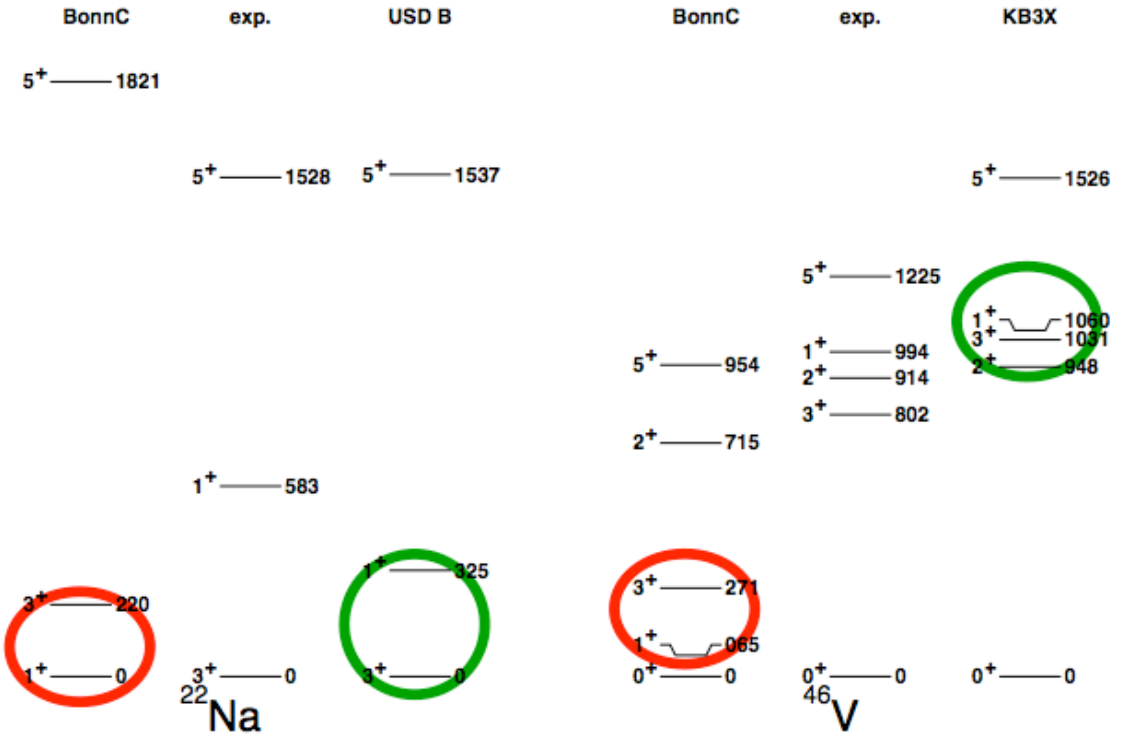
highlights the importance of 3N interactions

3N interactions and nuclear structure

microscopic calculations highlight the importance of 3N interactions

Navratil et al. (2007)





same 1^+ vs. 3^+ inversion in closed shell +3p+3n without 3N interactions

Nowacki @ Oslo 2008

3N crucial for T=1 spin-orbit shell closures in ^{22}O, ^{48}Ca,…

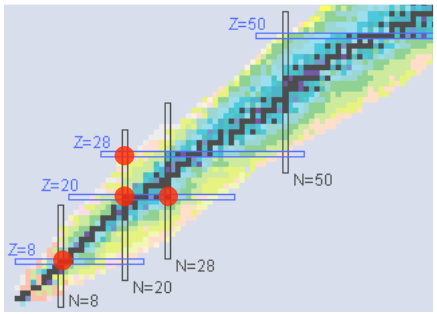
see e.g., AS, Zuker (2006)

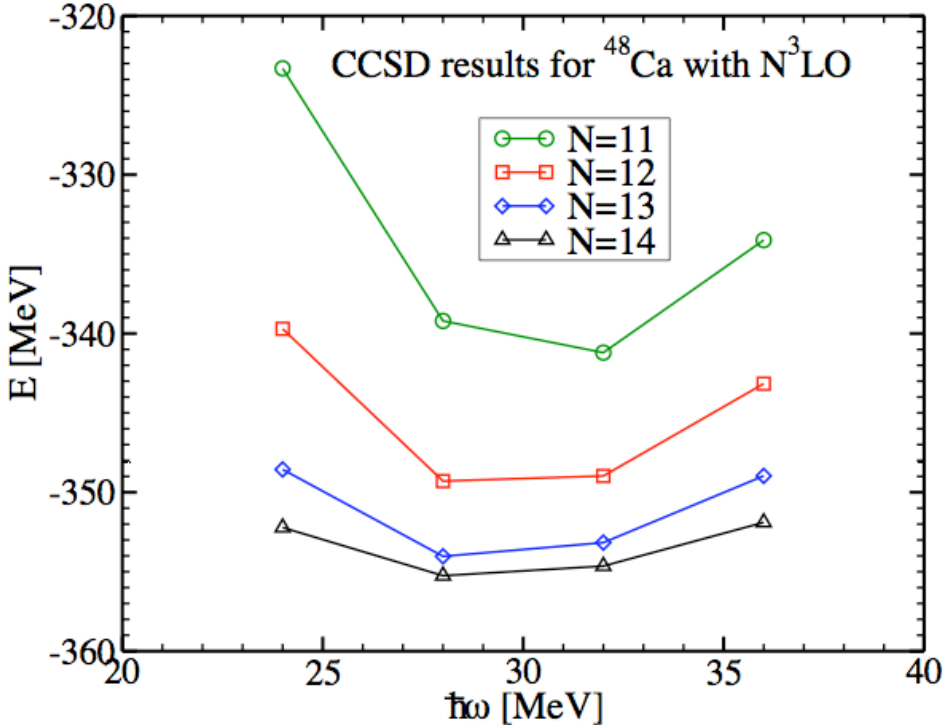
Medium-mass nuclei from chiral NN interactions Hagen et al. (2008)

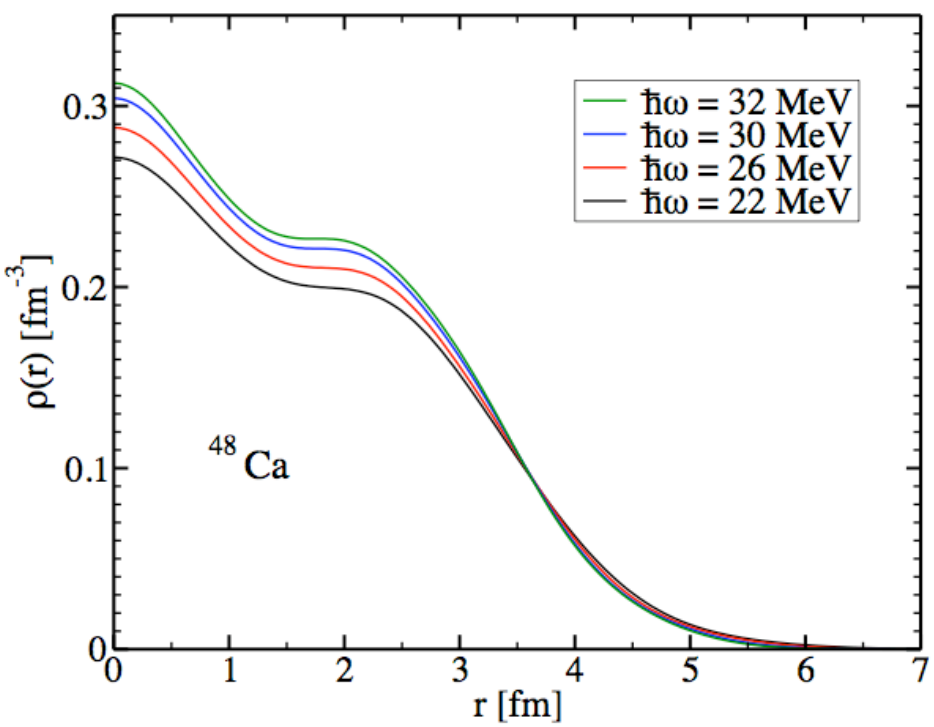
spherical coupled-cluster theory: ~10^5 speed-up

^{40,48}Ca and ^{48}Ni in 15 major shells on single processor

near converged ground states for N^{3}LO NN potentials







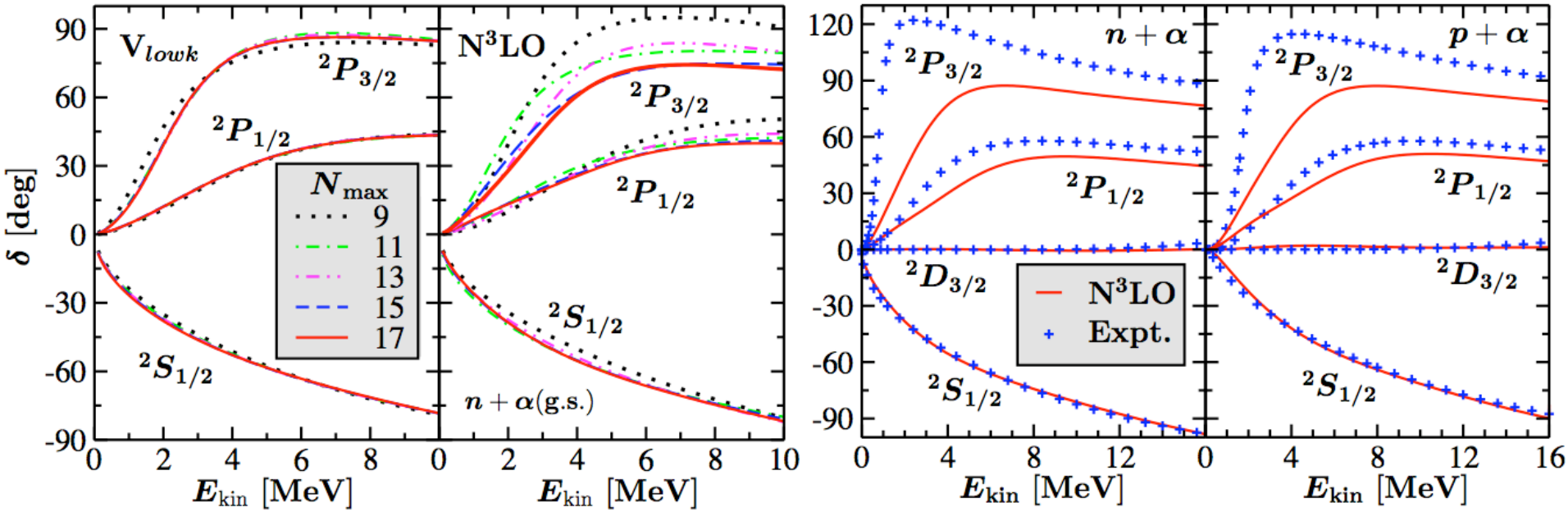
future: first calculations with 3N interactions for medium-mass nuclei

^{100}Sn and ^{208}Pb, ab-initio calc of Pb neutron radius from chiral EFT

First nuclear reactions from chiral interactions Quaglioni, Navratil (2008)

Resonating group method combined with No-Core Shell Model

n/p-^{4}He scattering,… n-^{10}Be scattering



future: include 3N interactions, inelastic scattering,…

Outline

1. Introduction to effective field theory and the renormalization group

2. Chiral effective field theory

3. Renormalization group for nuclear forces

4. EFT and RG for nuclear matter

5. Nuclear matter in astrophysics

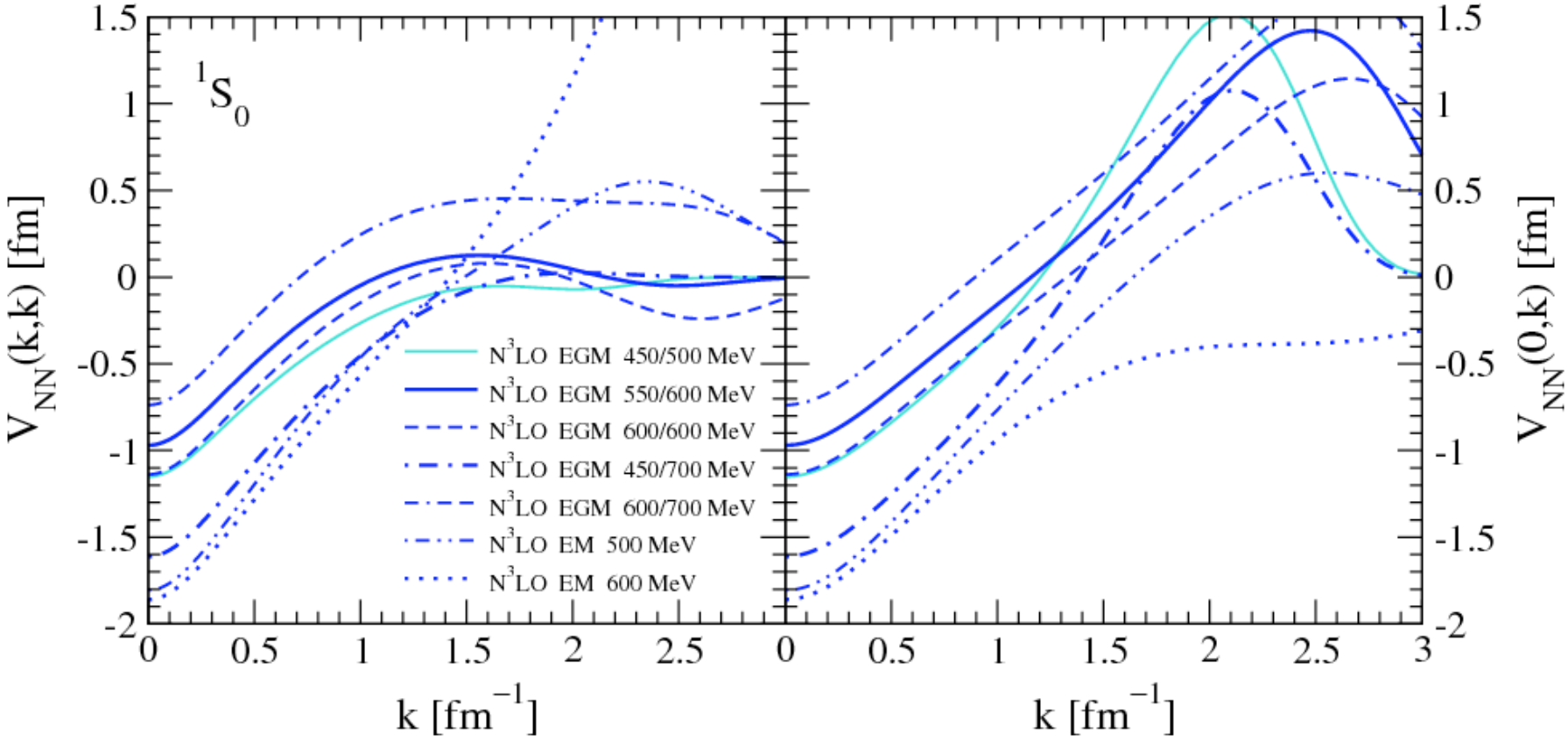
6. Neutrino processes in supernovae from chiral EFT

References

S.K. Bogner, T.T.S. Kuo and AS, Phys. Rept. **386**, 1 (2003).

S.K. Bogner, R.J. Furnstahl, S. Ramanan and AS,
Low-momentum interactions with smooth cutoffs,
Nucl. Phys. **A784**, 79 (2007).

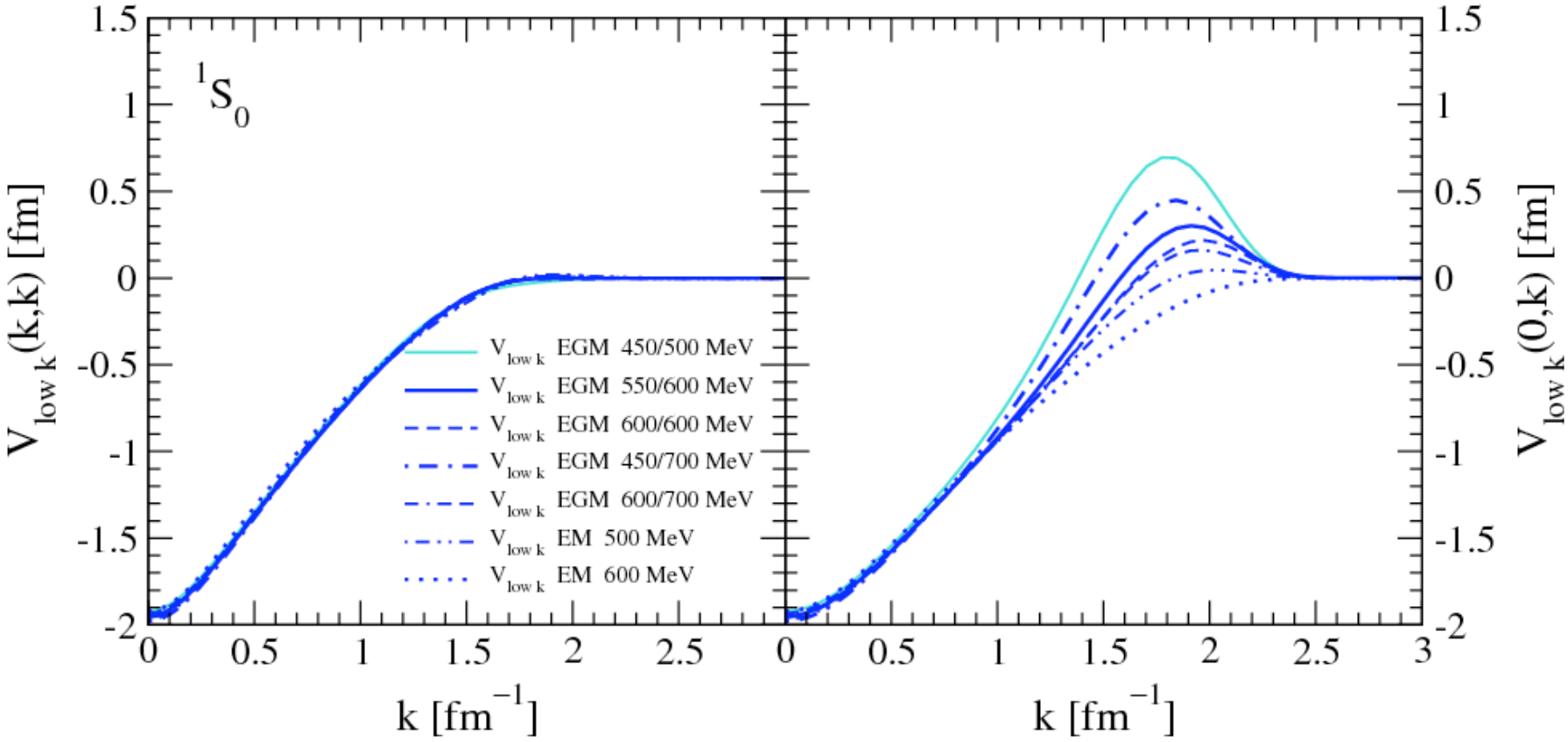
Chiral EFT interactions at N^{3}LO



regulator and renormalization scheme and scale dependence

There is not just one N^{3}LO potential!

RG evolution of chiral EFT interactions at N^{3}LO



find ≈ universality from different N^{3}LO potentials

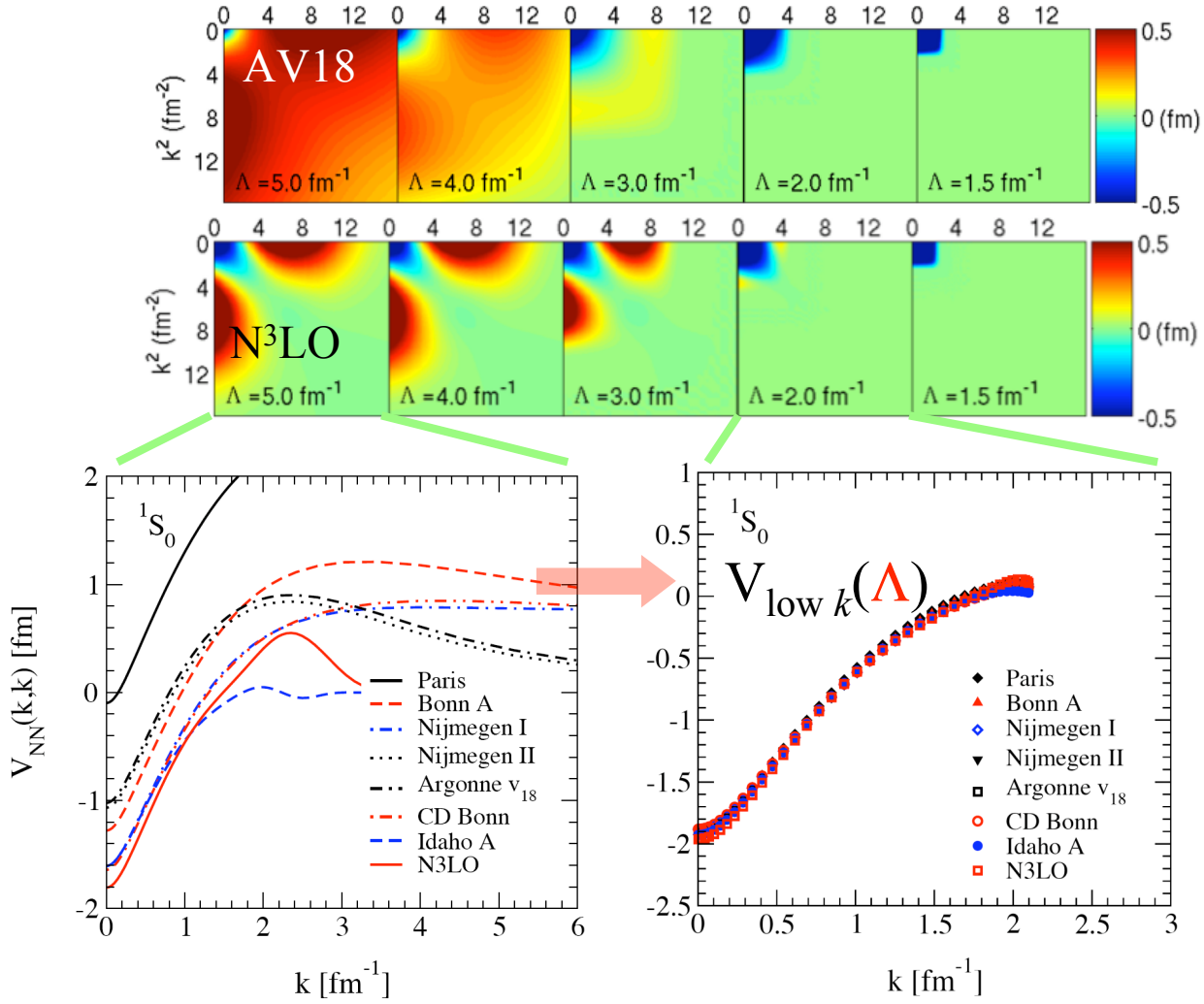
RG evolution weakens off-diagonal coupling,
decouples high momenta

Low-momentum interactions from the Renormalization Group

RG evolution to lower resolution/cutoffs

$$H(\Lambda) = T + V_{\rm NN}(\Lambda) + V_{\rm 3N}(\Lambda) + V_{\rm 4N}(\Lambda) + \ldots$$

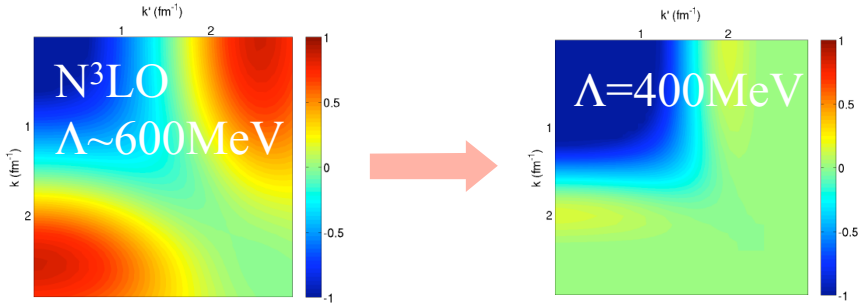
≈ universal interaction for low momenta $V_{\text{low } k}(\Lambda)$ Bogner, Kuo, AS (2003)



Advantages of low-momentum interactions for nuclei

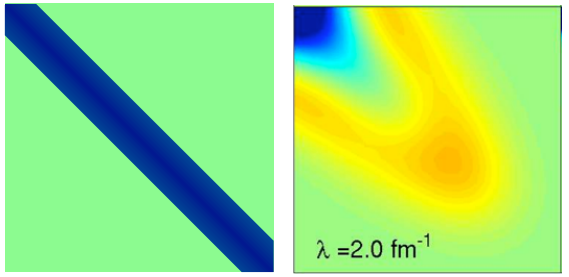
lower cutoffs need smaller basis

Bogner et al. (2007…)



Similarity Renormalization Group (SRG)

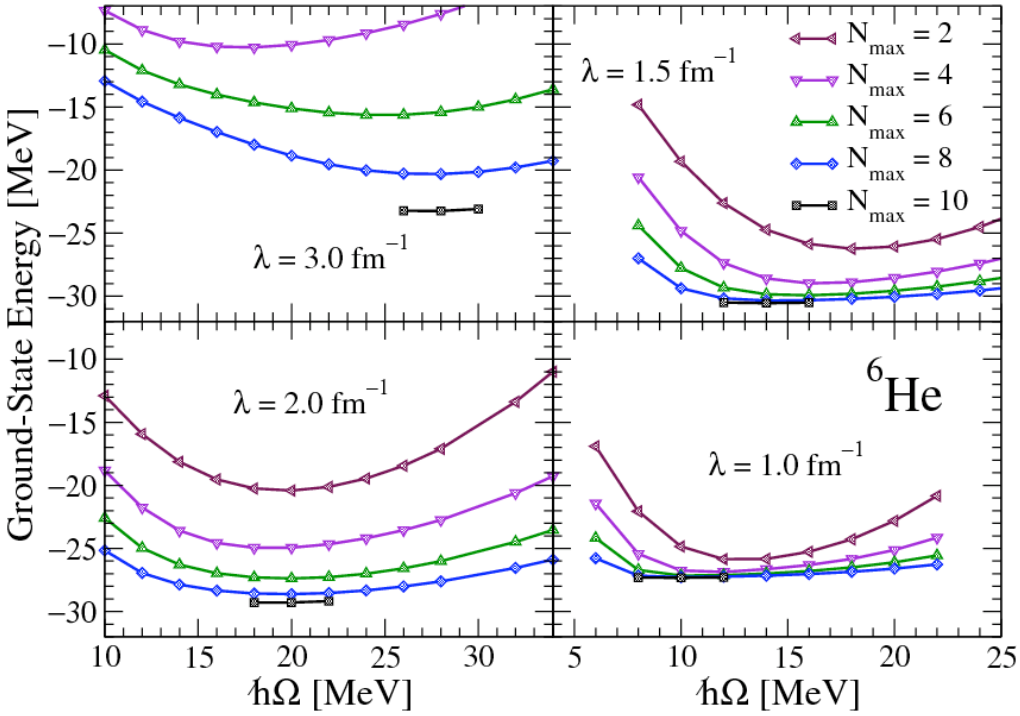
evolution towards band diagonal $V_{SRG}(\lambda)$



improved convergence for nuclei

10^3 states for N_{max}=2 vs.

10^7 states for N_{max}=10



Similarity RG interactions

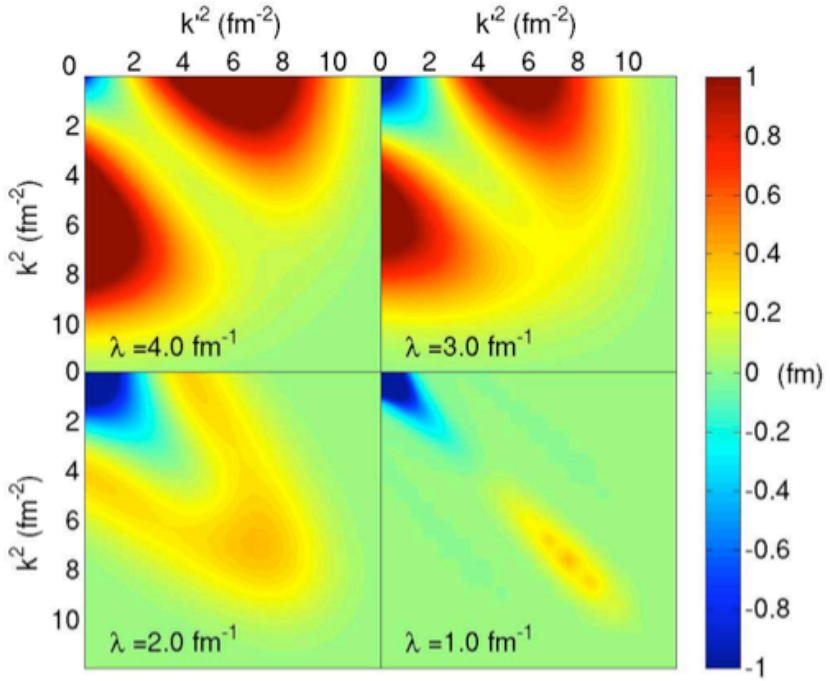
Unitary transformations to band-diagonal $V_{srg}(\lambda)$ from flow equations

Glazek, Wilson (1993), Wegner (1994)

$$\frac{dH_s}{ds} = [\eta_s, H_s] = [[G_s, H_s], H_s]$$

with flow operator $G_s = T_{rel}$ and resolution $\lambda = s^{-1/4}$ Bogner et al. (2007)

$$\frac{dV_s(k,k')}{ds} = -(k^2 - k'^2)^2\, V_s(k,k') + \frac{2}{\pi}\int_0^\infty q^2\, dq\, (k^2 + k'^2 - 2q^2)\, V_s(k,q)\, V_s(q,k')$$



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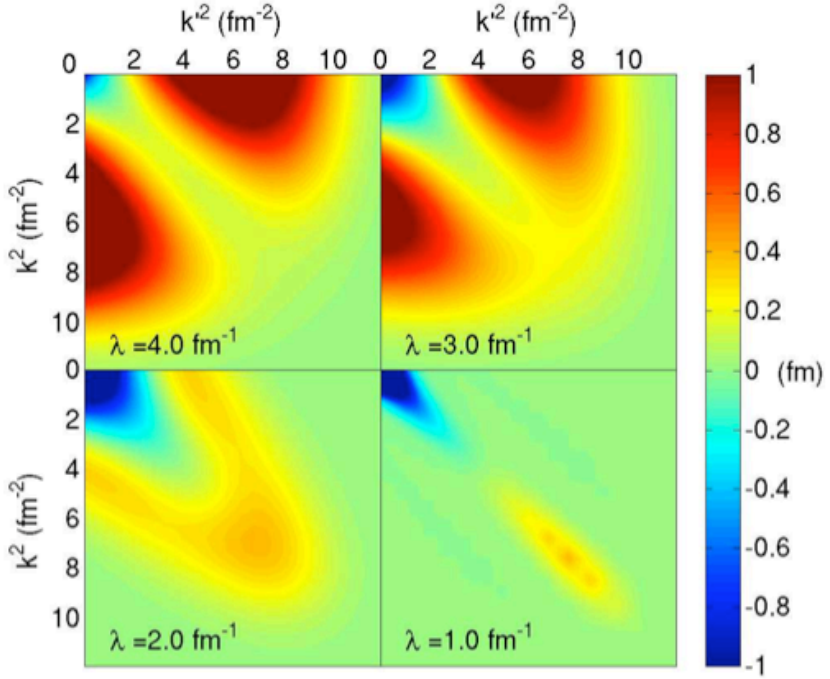
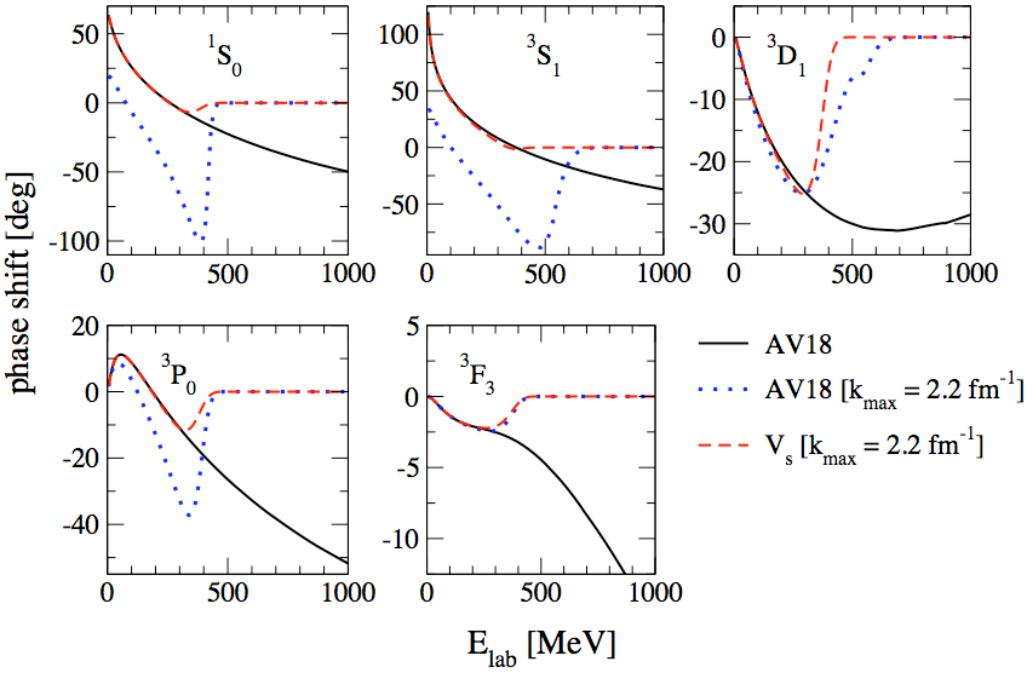
intermediate momenta $k > k_{max} \sim \lambda$ decouple for low energies



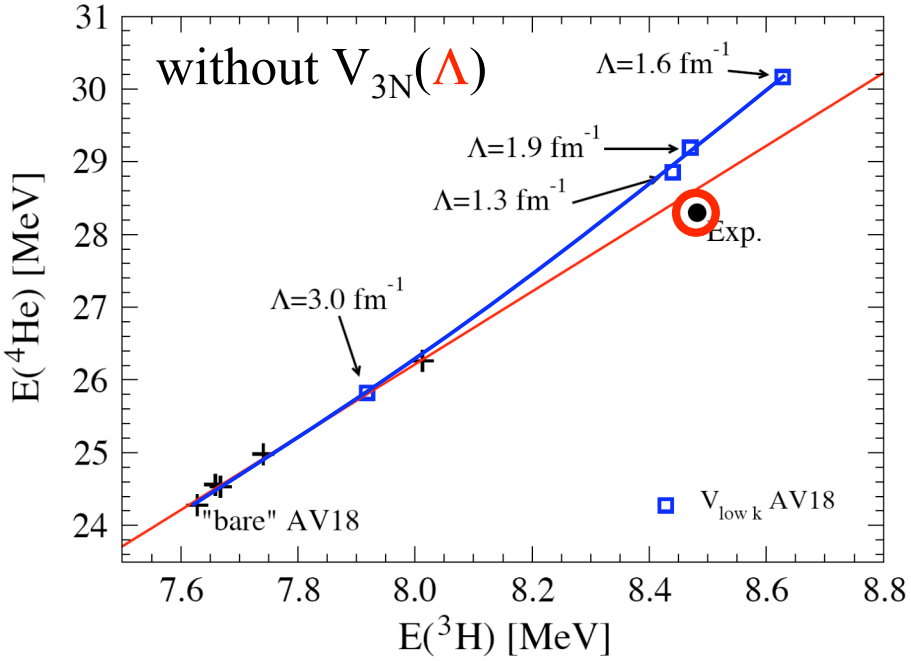


3N interactions required for renormalization

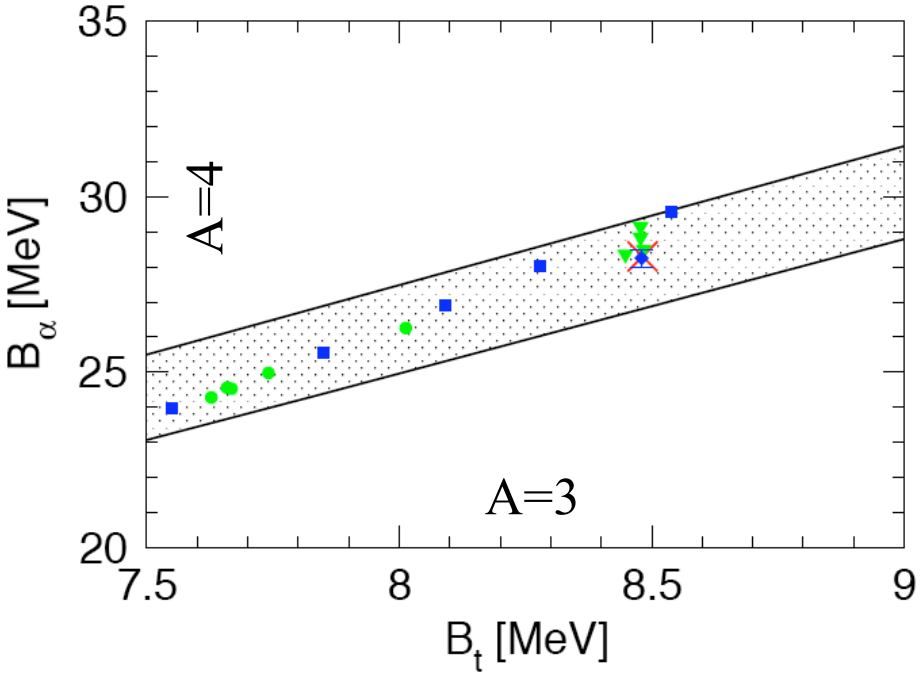
$V_{NN}(\Lambda)$ defines NN interactions with cutoff-independent NN observables

cutoff variation estimates errors due to neglected parts in $H(\Lambda)$

cutoff dependence explains "Tjon line" → 3N for renormalization



Nogga, Bogner, AS (2004)



pionless EFT: Platter, Hammer, Meissner (2005)

large scattering lengths drive correlation

Low-momentum 3N fits

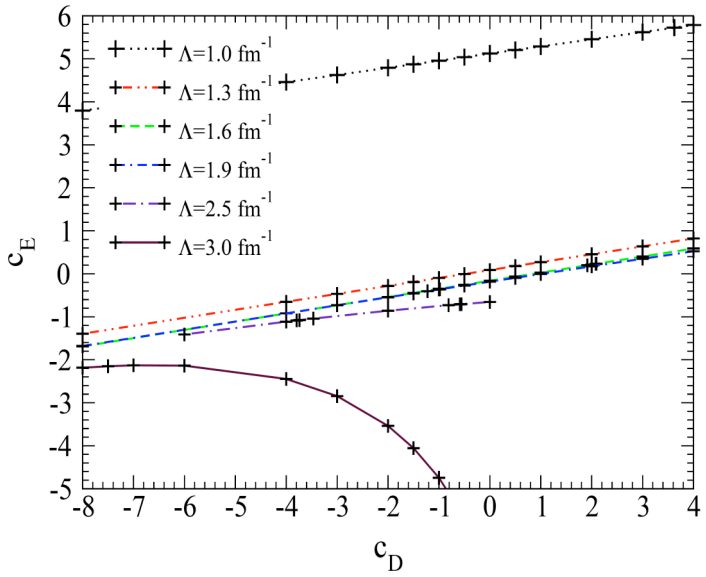
fit D,E couplings to A=3,4 binding energies for range of cutoffs

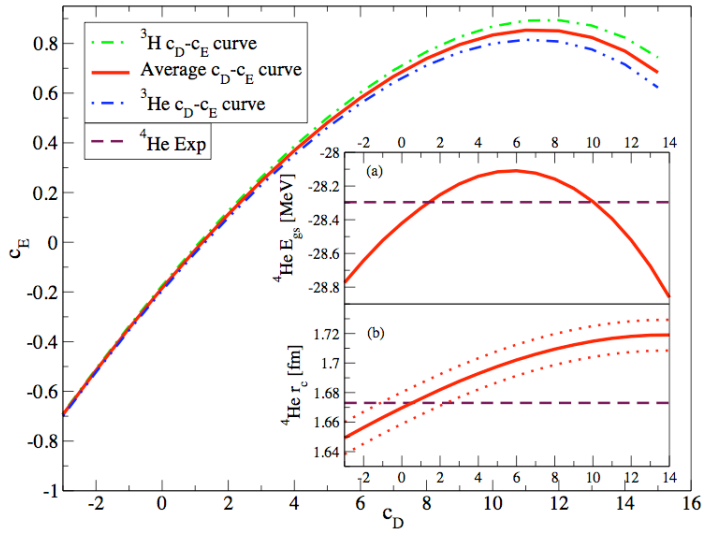
linear dependences in fits to triton binding

3N interactions perturbative for $\Lambda \lesssim 2\,\mathrm{fm}^{-1}$
Nogga, Bogner, AS (2004)

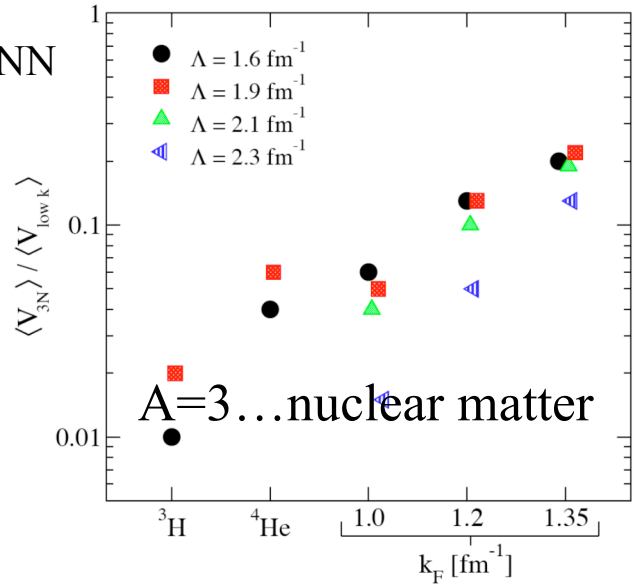
nonperturbative at larger cutoffs
cf. chiral EFT $\Lambda \approx 3$ fm^{-1}

3N exp. values natural
$\sim (Q/\Lambda)^3\, V_{NN} \sim 0.1\, V_{NN}$





Navratil et al. (2007)

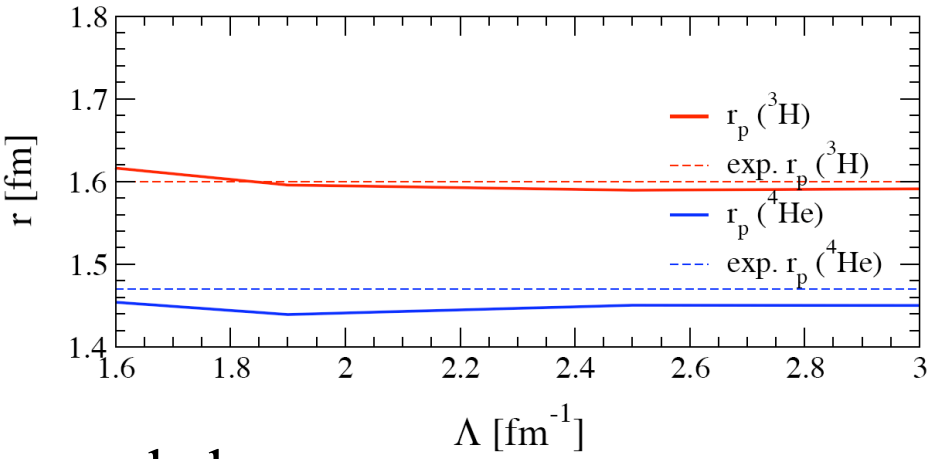


A=3…nuclear matter

Theoretical uncertainties

EFT/RG interactions: more accurate with higher orders and cutoff variation estimates theoretical uncertainties

Radii of light nuclei approximately cutoff-independent, agree with exp.

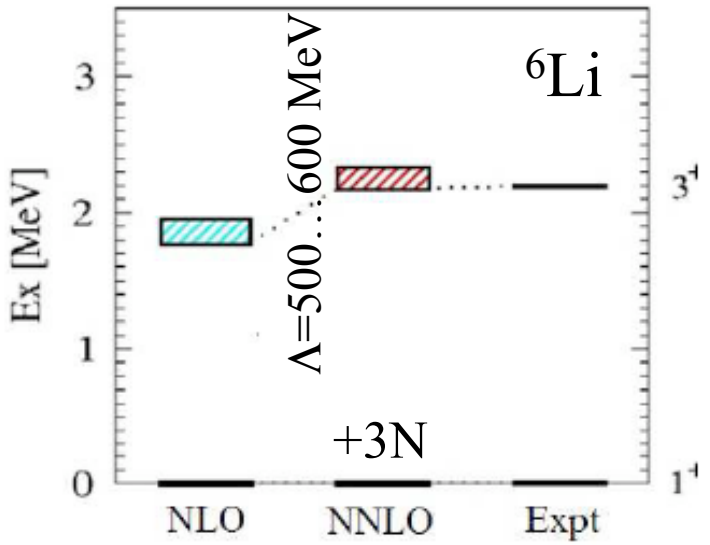


goal: apply to pivotal matrix elements needed to constrain beyond Standard Model physics

isospin-symmetry breaking corrections for superallowed beta decay
V_{ud}=0.97416(13) (14/18)theo.

nuclear matrix elements for 0νββ decay

octupole EDM enhancement

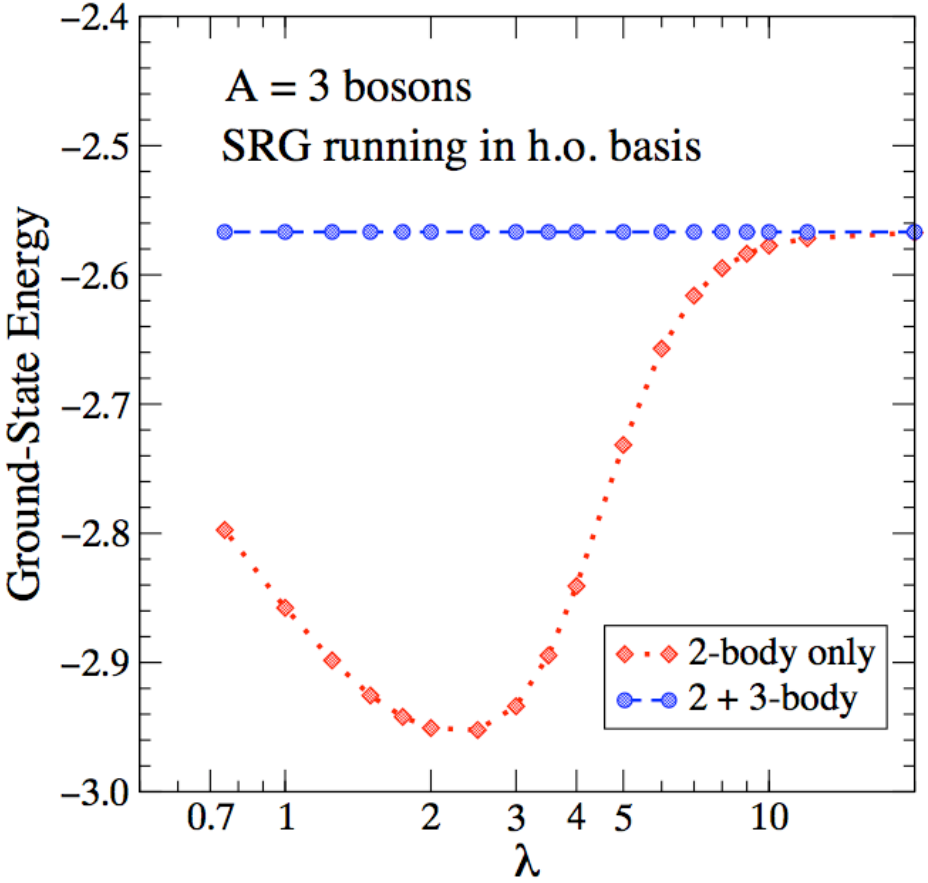


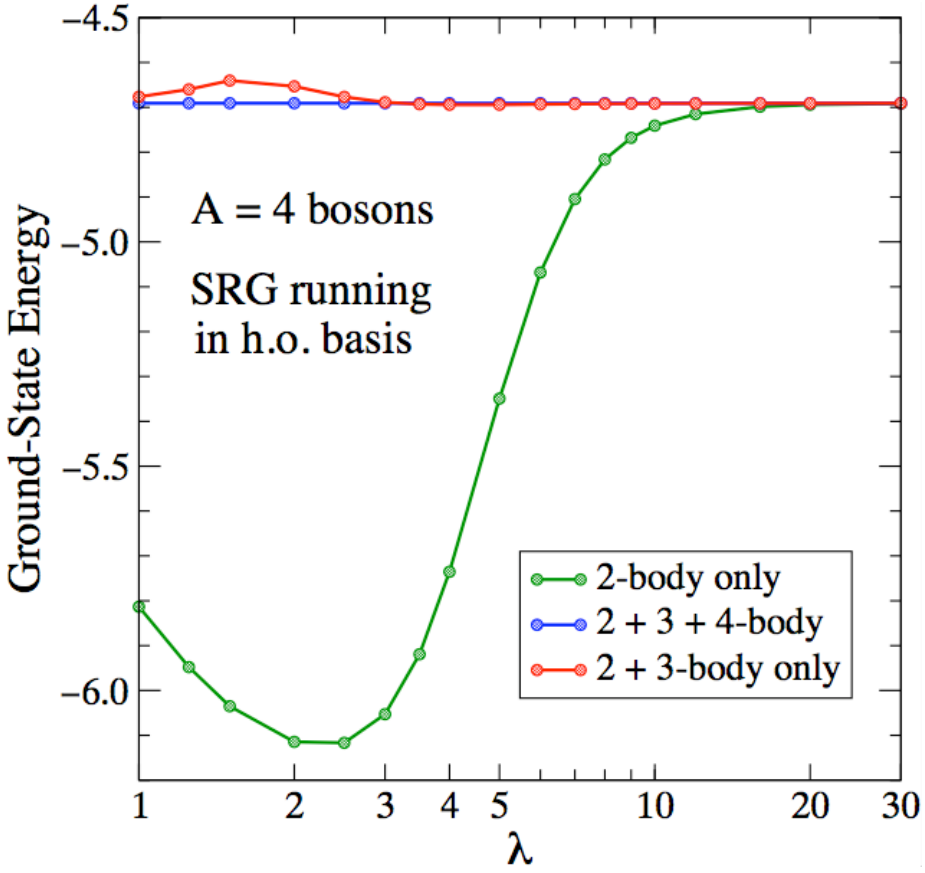
from A. Nogga

Towards evolving 3N interactions

Similarity RG evolution for 1d systems in harmonic oscillator basis

Jurgenson, Furnstahl (2008)



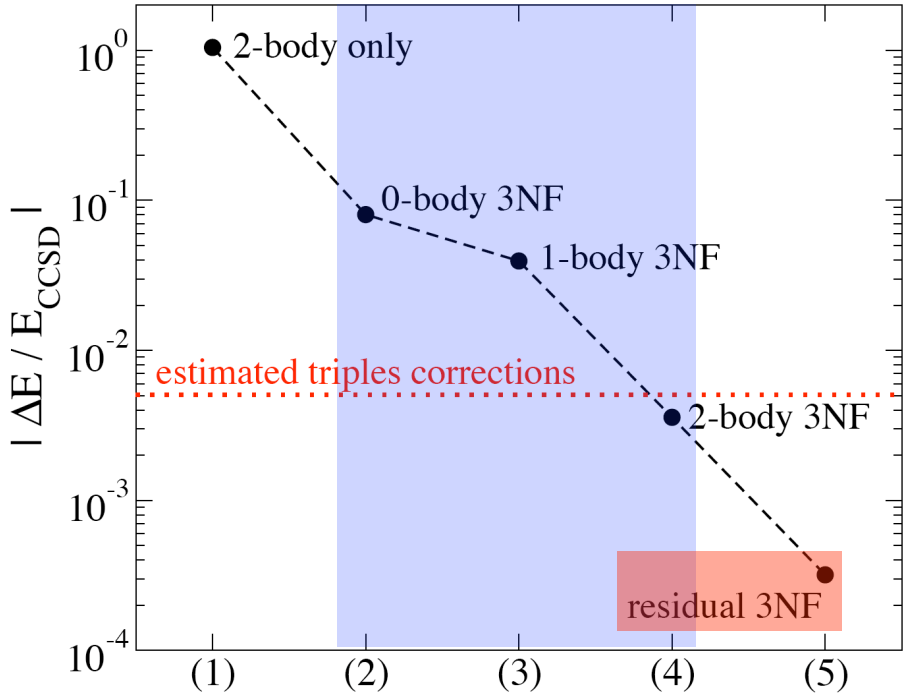


gearing up to evolve chiral 3N interactions

for now: use chiral EFT is complete basis, $V_{3N}(\Lambda)$ fits for lower cutoffs

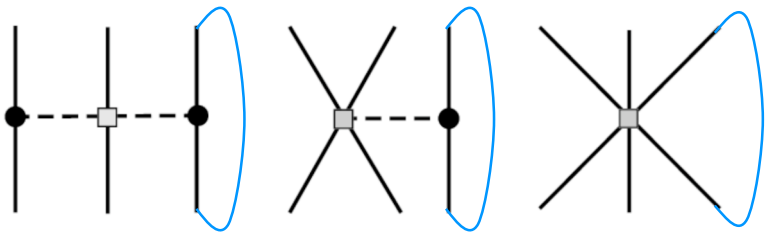
Towards 3N interactions in medium-mass nuclei

developed coupled-cluster theory with 3N interactions Hagen et al. (2007)
first benchmark for ^{4}He based on low-momentum interactions



show that 0-, 1- and 2-body parts of 3N interaction dominate

2-body part



occupied orbits

residual 3N interaction can be neglected: very promising

Outline

1. Introduction to effective field theory and the renormalization group

2. Chiral effective field theory

3. Renormalization group for nuclear forces

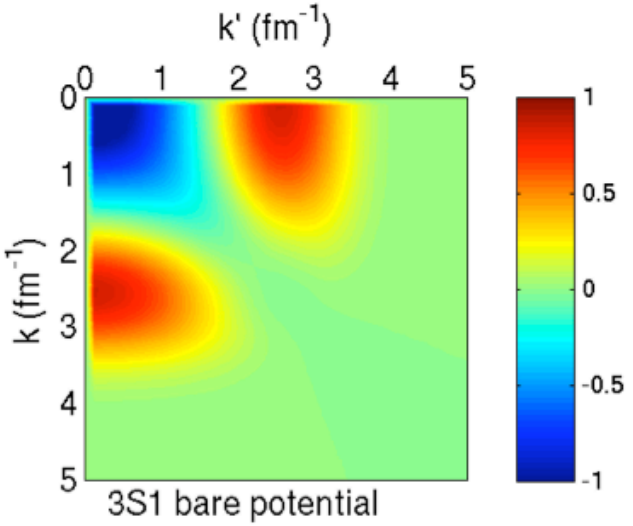
4. EFT and RG for nuclear matter

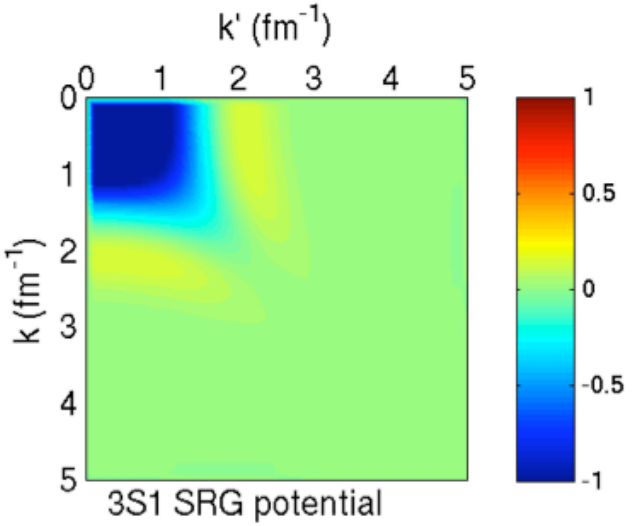
5. Nuclear matter in astrophysics

6. Neutrino processes in supernovae from chiral EFT

Advantages of low-momentum interactions for nuclei

conventional G matrix approach does not solve off-diagonal coupling, renders Bethe-Brueckner-Goldstone expansion necessarily nonperturb.

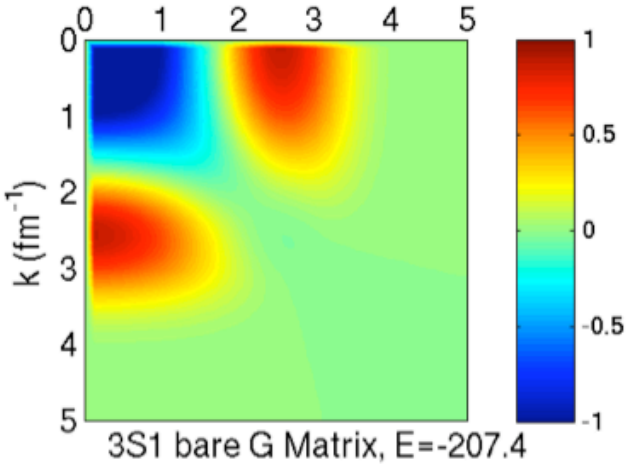


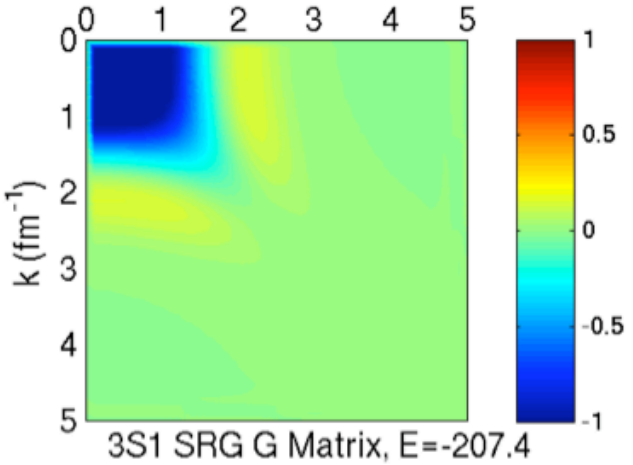


chiral N^{3}LO potential

SRG evolved

G matrix

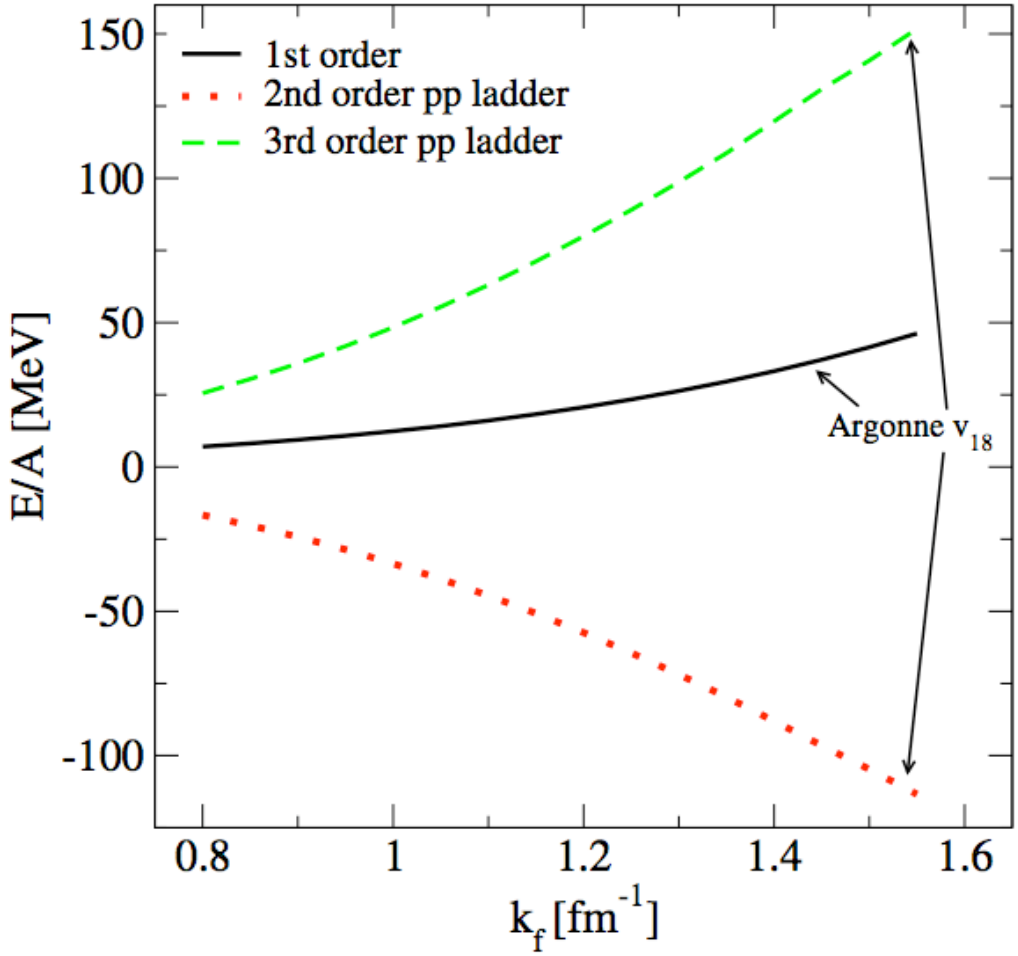




Is nuclear matter perturbative with chiral EFT and RG?

conventional Bethe-Brueckner-Goldstone expansion:
no, due to nonpert. cores (flipped-V bound states) and off-diag coupling

start from chiral EFT and RG evolution:
nuclear matter converged at ≈ 2nd order



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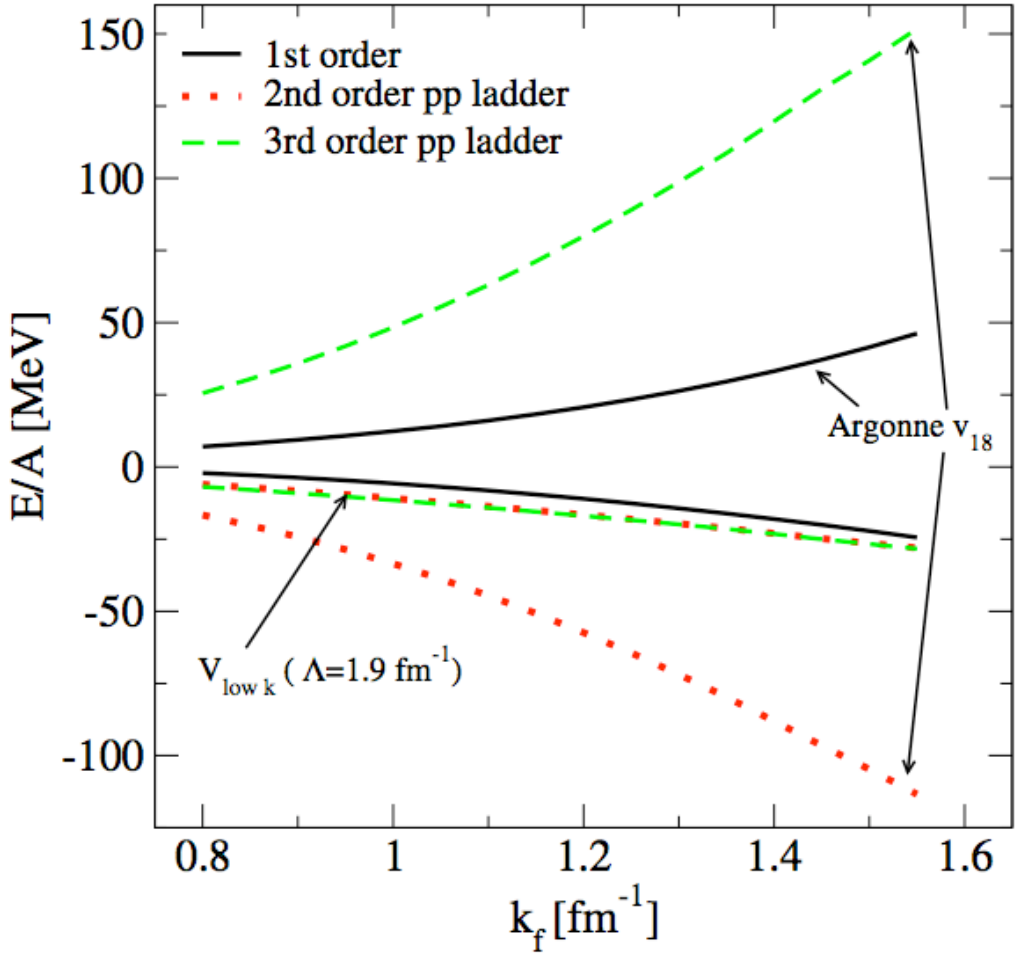


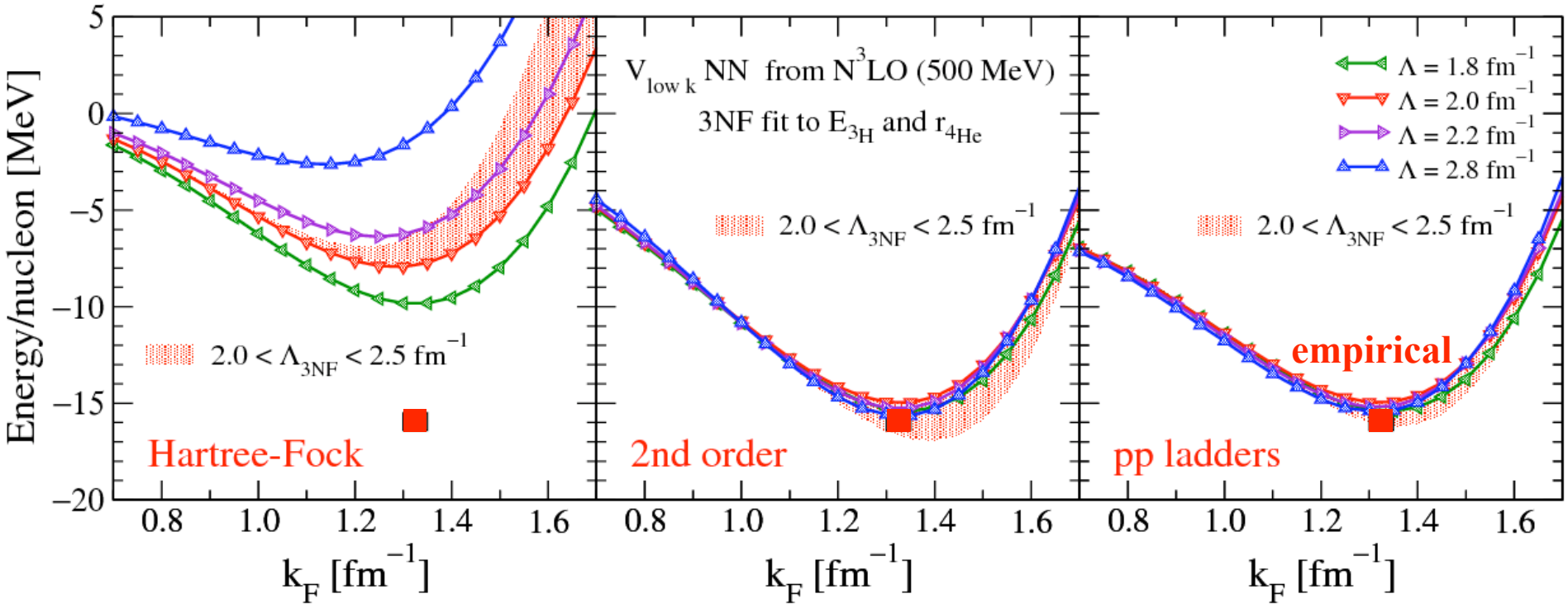
Is nuclear matter perturbative with chiral EFT and RG?

conventional Bethe-Brueckner-Goldstone expansion:
no, due to nonpert. cores (flipped-V bound states) and off-diag coupling

start from chiral EFT and RG evolution:
nuclear matter converged at ≈ 2nd order, 3N drives saturation

weak cutoff dependence, improved by 3N fits to ^{4}He radius

exciting: empirical saturation within theoretical uncertainties

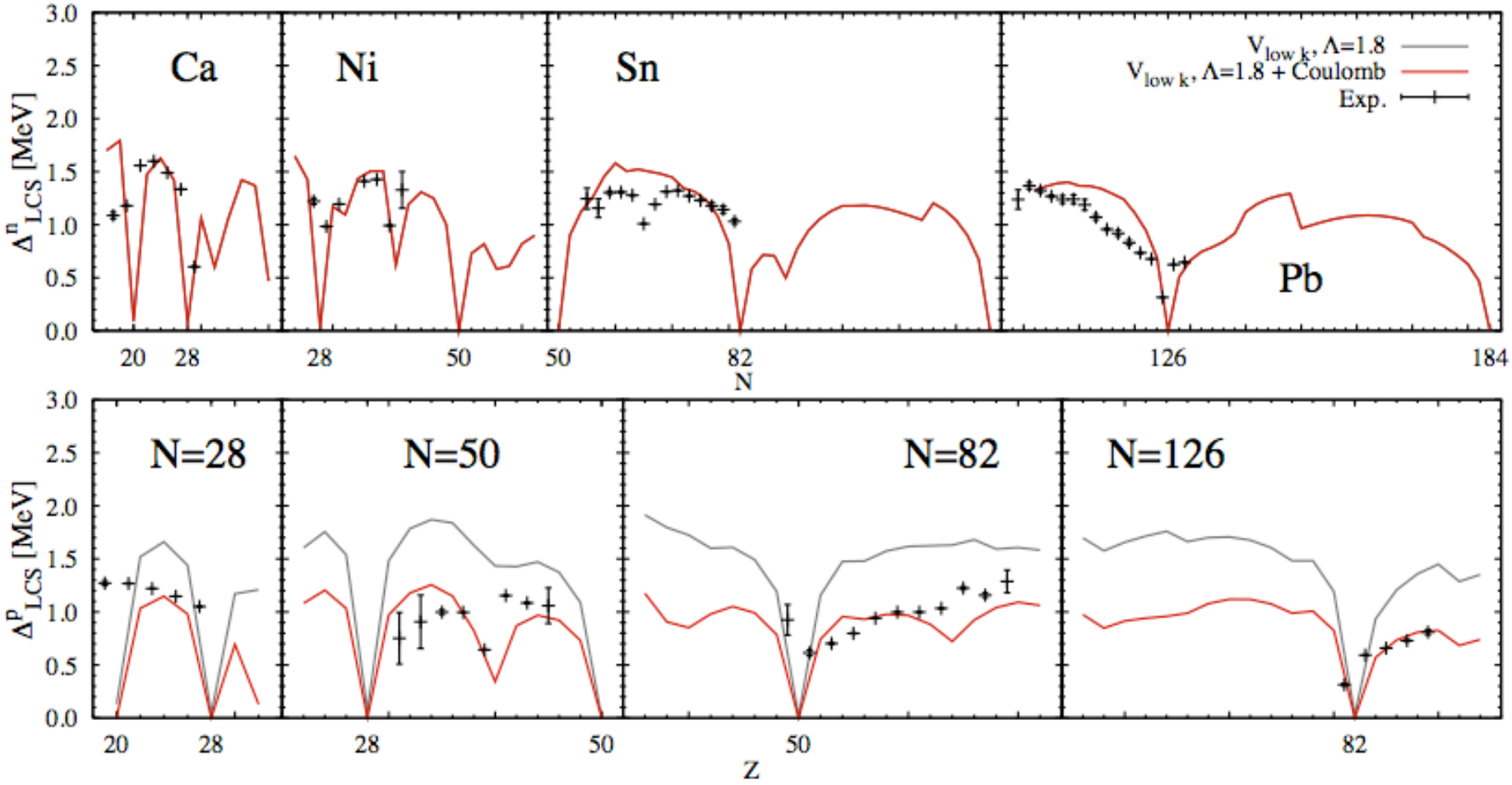


Bogner, AS, Furnstahl, Nogga (2005) + improvements, in prep.

Pairing gaps

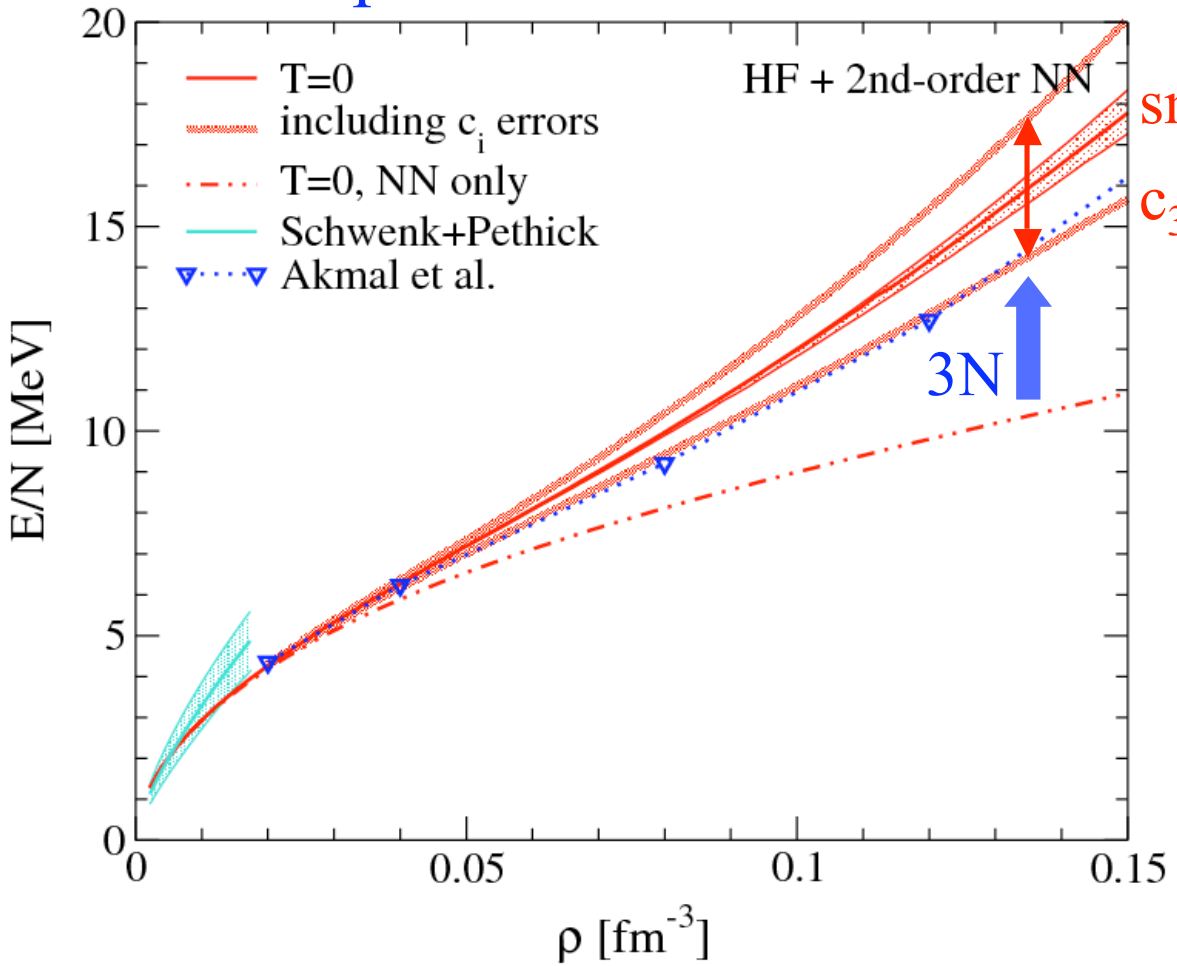
first density functional theory results with microscopic pairing functional from $V_{low\ k}$ + Coulomb interaction

Lesinski, Duguet et al. (2008)



current limitations: no 3N forces, no density/spin/isospin fluctuations

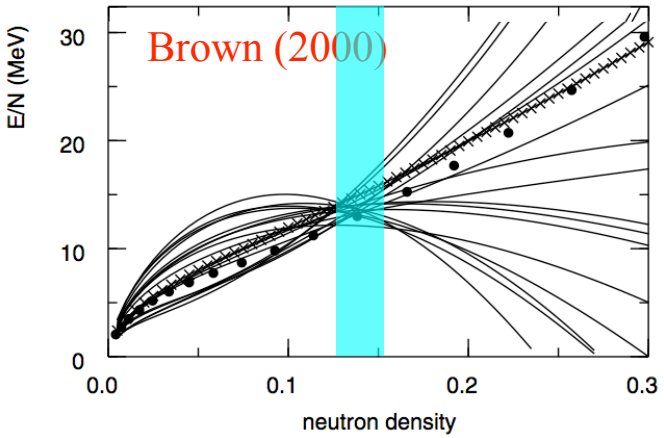
Impact of 3N interactions on neutron matter



small cutoff variation

c_3 uncertainty

Tolos et al. (2007)



uncertainties from c_i overwhelm errors due to cutoff variation, mainly c_3

combine with knowledge of nuclear properties

important for dense matter in astrophysics

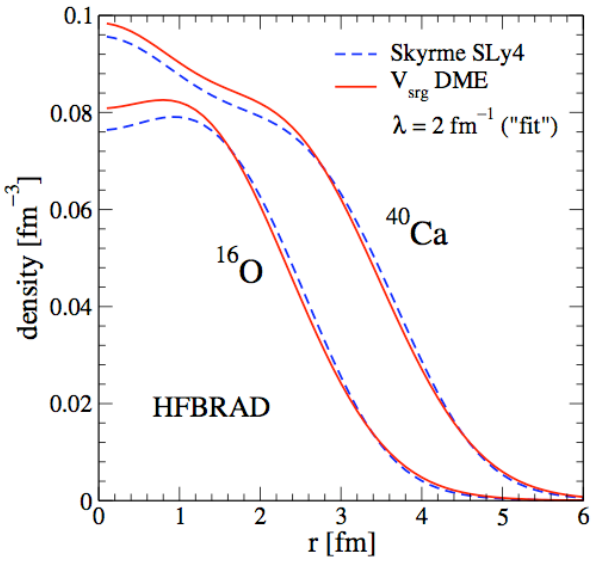
Towards a universal nuclear energy density functional

low-momentum interactions
provide key input to SciDAC UNEDF

using density matrix expansion

Bogner, Furnstahl, Platter (2008), Finelli, Kaiser, Weise,…

$$\mathcal{E} = \frac{\tau}{2M} + A[\rho] + B[\rho]\tau + C[\rho]|\nabla\rho|^2 + \cdots$$

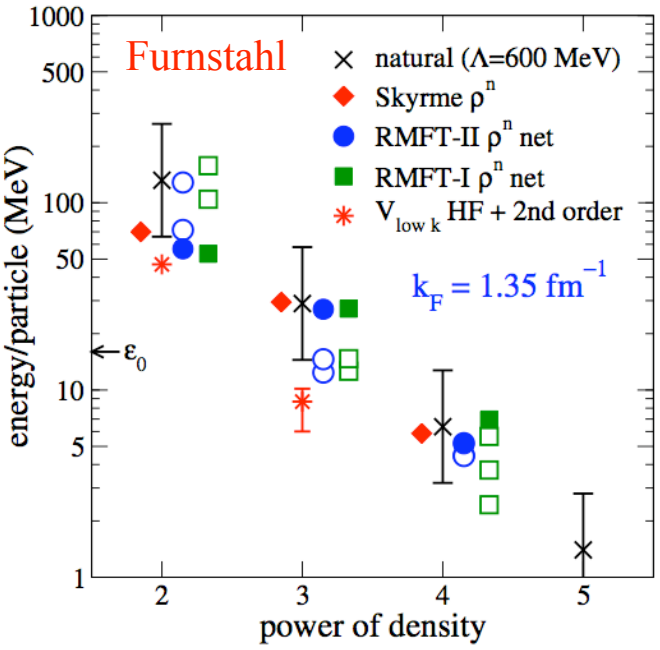


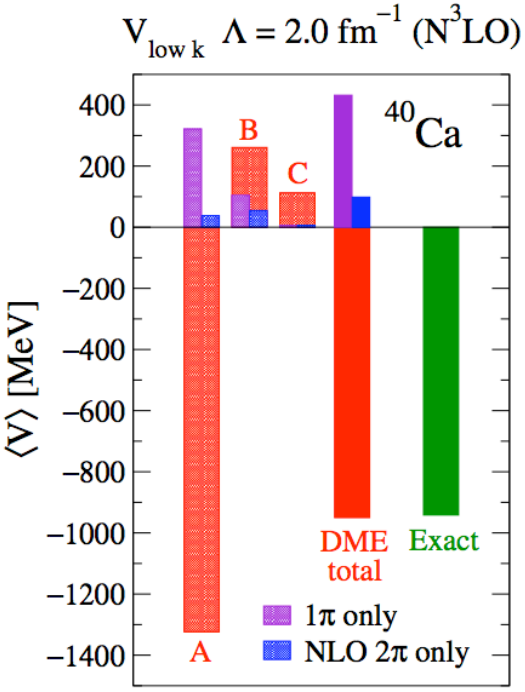
use EFT/RG interactions:

to identify new terms
in density functional

to quantify theoretical
errors for extrapolations

to benchmark with
ab-initio methods





Three-nucleon interactions

3N interactions crucial for:

binding energies

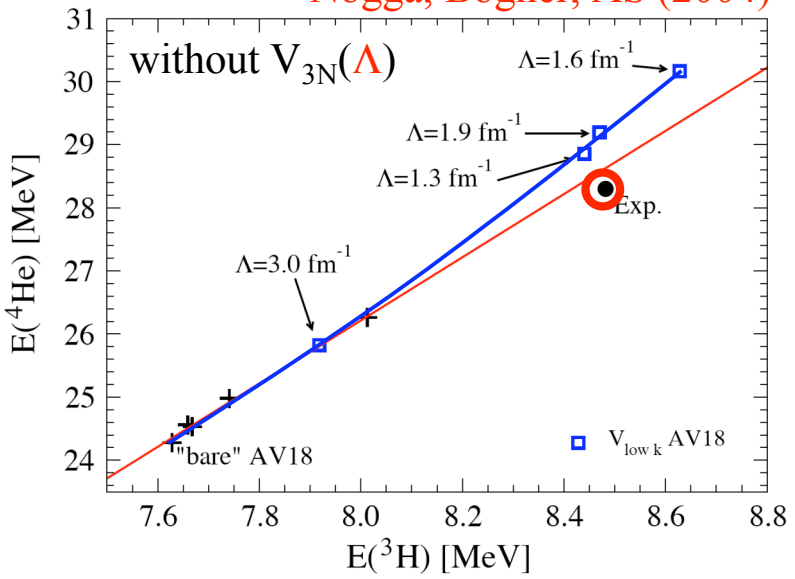
cutoff dependence explains Tjon line due to neglected parts in H(Λ), 3N required

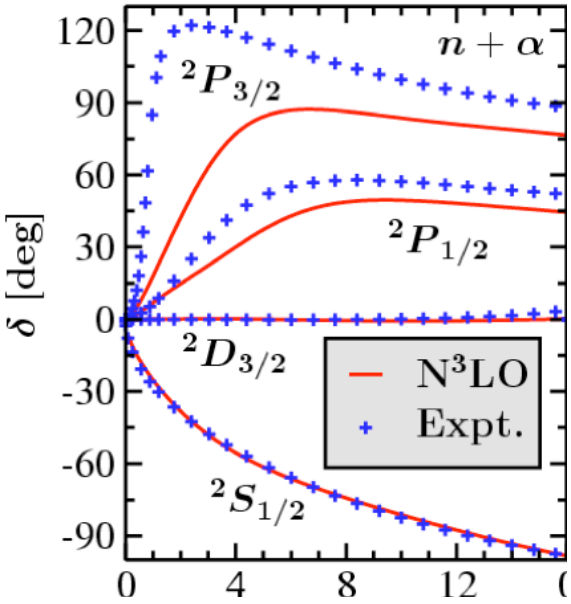
spin-orbit effects

isospin dependence

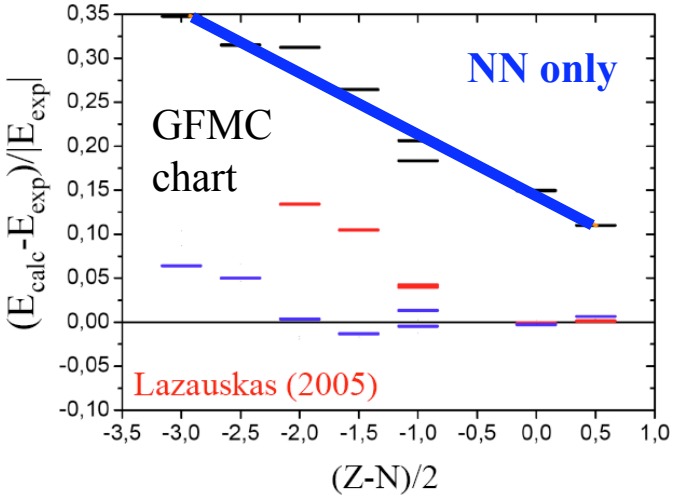
density dependence

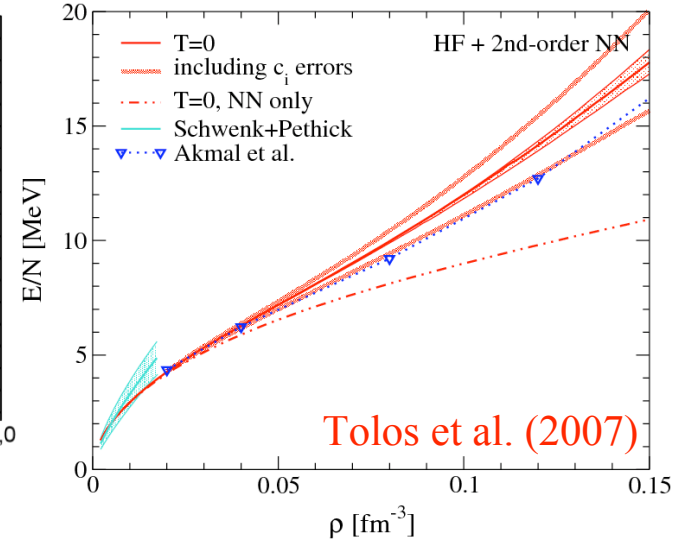
Nogga, Bogner, AS (2004)



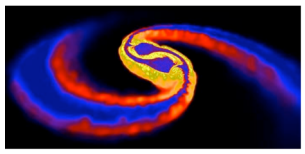


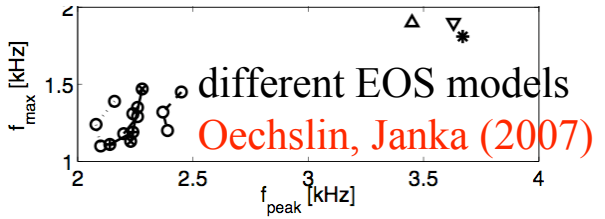
Quaglioni, Navratil (2008)





neutron star mergers → gravitational waves





Outline

1. Introduction to effective field theory and the renormalization group

2. Chiral effective field theory

3. Renormalization group for nuclear forces

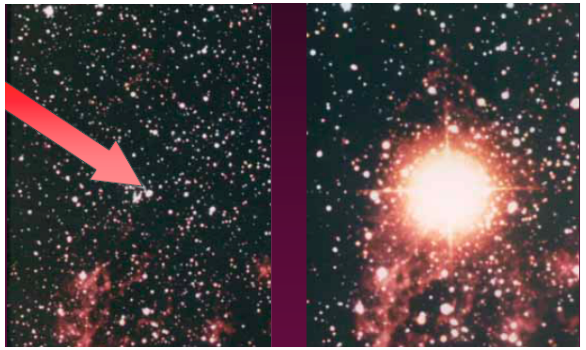
4. EFT and RG for nuclear matter

5. The virial equation of state and light nuclei

6. Neutrino processes in supernovae from chiral EFT

Motivation

Core-collapse supernovae most sensitive to low-density nucleonic matter

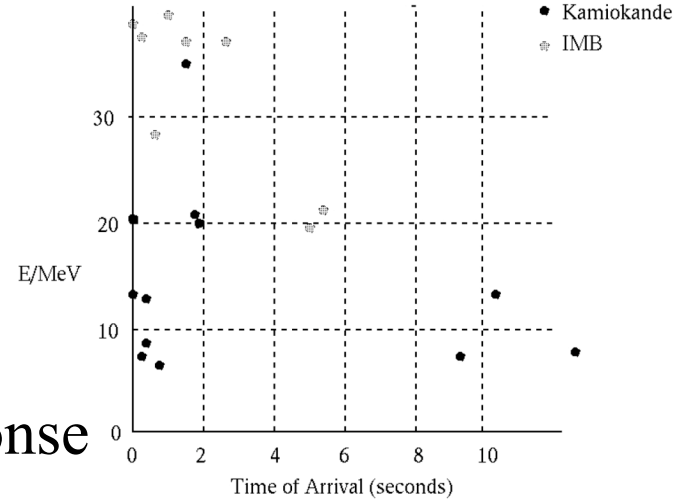


Before and after SN1987A

Conditions at neutrinosphere (surface of last scattering of neutrinos):

T ~ 4 MeV from ~20 SN1987a events

n ~ 10^{11}-10^{12} g/cm^3 from $n\sigma \sim n(G_F E_\nu)^2 \sim R^{-1}$



What is the equation of state and neutrino response of nuclear matter near the neutrinosphere?

Fugacity small $z = e^{\mu/T} \lesssim 0.5$ for $n \lesssim 4 \cdot 10^{11}\,(T/\mathrm{MeV})^{3/2}\,\mathrm{g/cm^3}$

Virial exansion gives model-independent answers for SN neutrinosphere

Horowitz, AS (2006)

Virial expansion: general formalism for low n, high T

assumptions: gas phase, T > any T_{crit}, fugacity $z = e^{\mu/T}$ small

Neutron matter

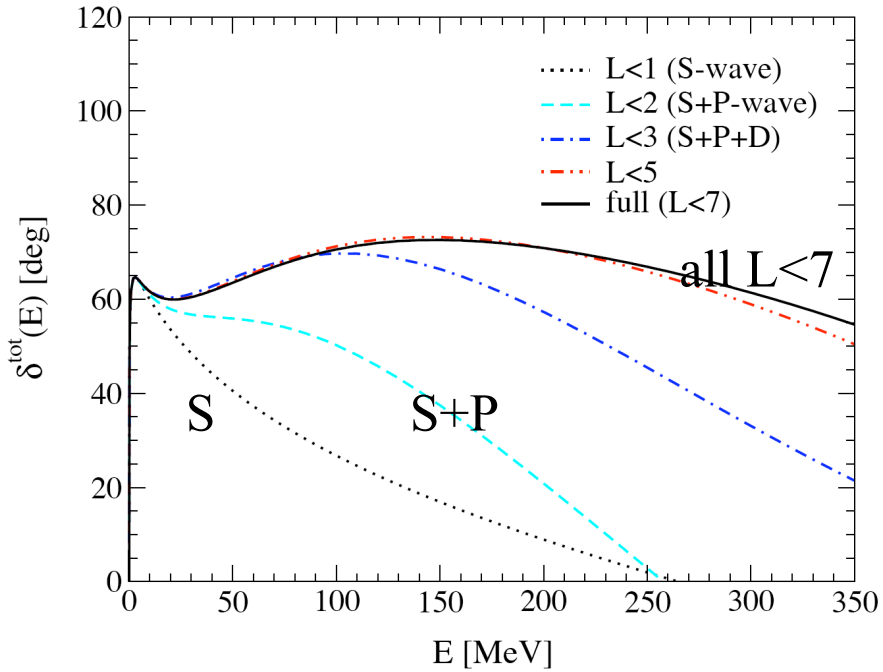
$$P = \frac{2T}{\lambda^3}\left(z + z^2 b_n + z^3 b_n^{(3)} + \mathcal{O}(z^4)\right) \qquad n = \frac{2}{\lambda^3}\left(z + 2z^2 b_n + 3z^3 b_n^{(3)} + \mathcal{O}(z^4)\right)$$

Second virial coefficient ~ 2-particle partition fn

$$b_n(T) = \frac{1}{2^{1/2}\,\pi T}\int_0^\infty dE\; e^{-E/2T}\,\delta^{\rm tot}(E) - 2^{-5/2}$$

For infinite scattering length $a=\pm\infty$ $b_n = 3/2^{5/2} = 0.53$, not $k_F a$ expansion, tested in cold atoms Ho, Mueller (2004); Thomas et al. (2005)

Second virial coefficient for neutrons approx T independent $b_n=0.30$, leads to scaling $E/E_{free}=P/P_{free}=\xi(T/T_F)$

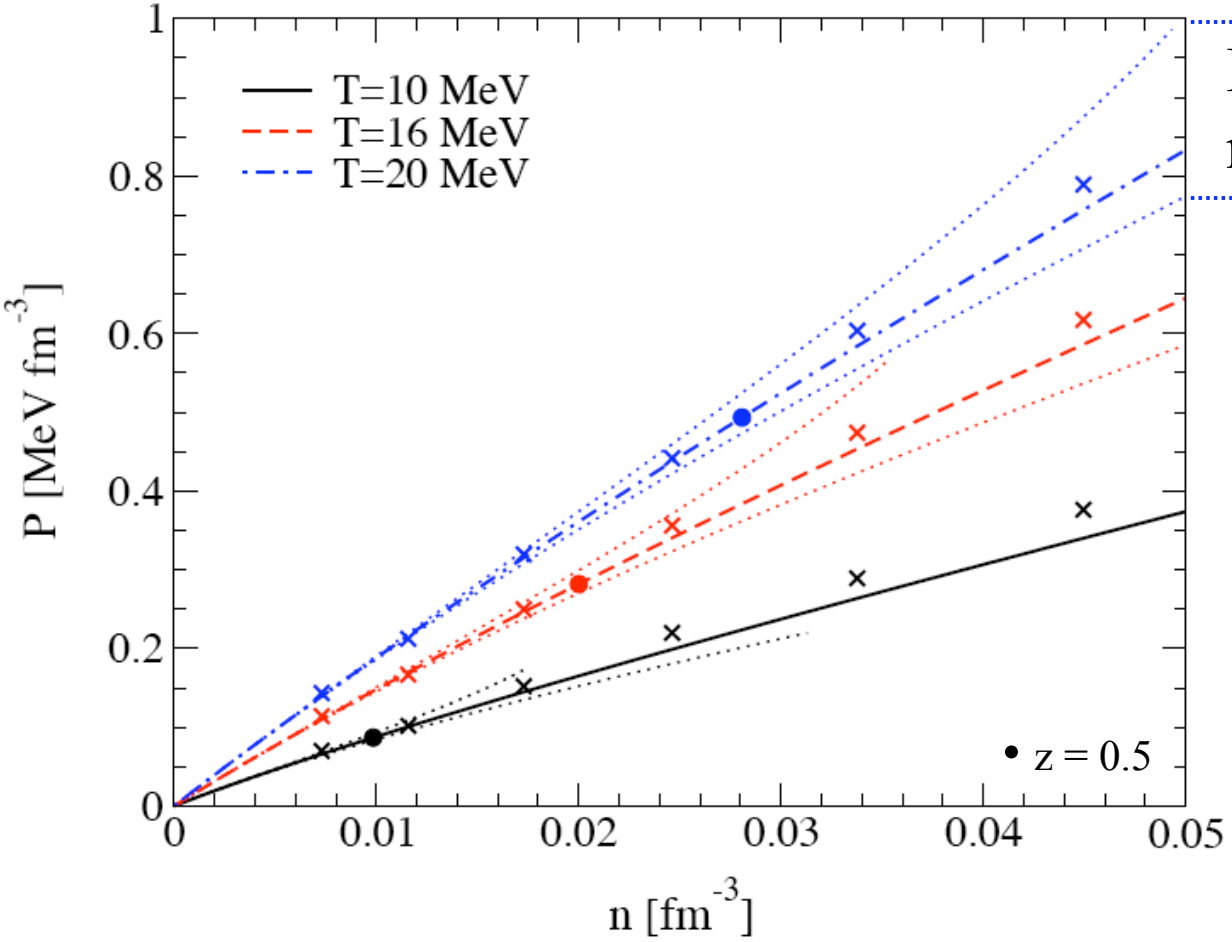


Previous work Buchler, Coon (1977); Pratt et al. (1987); Venugopalan, Prakash (1992); Roepke at al.

Neutron matter equation of state

Fugacity small for $n \lesssim 4 \cdot 10^{11}\,(T/\mathrm{MeV})^{3/2}\,\mathrm{g/cm^3}$

Comparison to Friedman, Pandharipande (×)



Error estimates due to neglected $b_n^{(3)} = \pm b_n/2$

Nuclear matter

deuterons enter as bound state contribution to $b_2 \sim e^{E_d/T}$
nuclei as bound state contributions to b_A, limits nucleon virial expansion

at low densities, nuclear matter mainly composed of n,p and α particles, include α particles explicity, to second-order in fugacities z_n, z_p, z_α

$$\frac{P}{T} = \frac{2}{\lambda^3}(z_n + z_p + (z_n^2 + z_p^2)b_n + 2z_p z_n b_{pn}) + \frac{1}{\lambda_\alpha^3}(z_\alpha + z_\alpha^2 b_\alpha + 2z_\alpha(z_n + z_p)b_{\alpha n})$$

second virial coefficients directly from NN, Nα, αα phase shifts and E_d

model-independent description of matter in thermal equilibrium

consider chemical equilibrium $z_\alpha = z_p^2\, z_n^2\, e^{E_\alpha/T}$
adjust z_n, z_p to reproduce desired baryon density and proton fraction

can include heavy nuclei at higher densities with z_A
virial b_{NA},… correct NSE models for strong interactions between nuclei

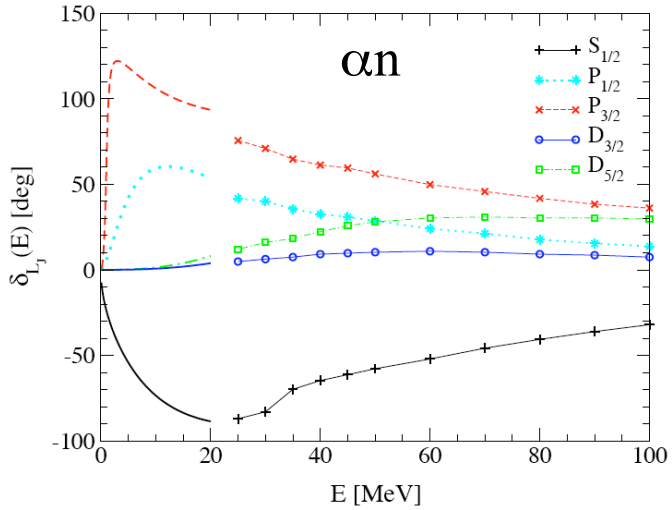
Virial coefficients

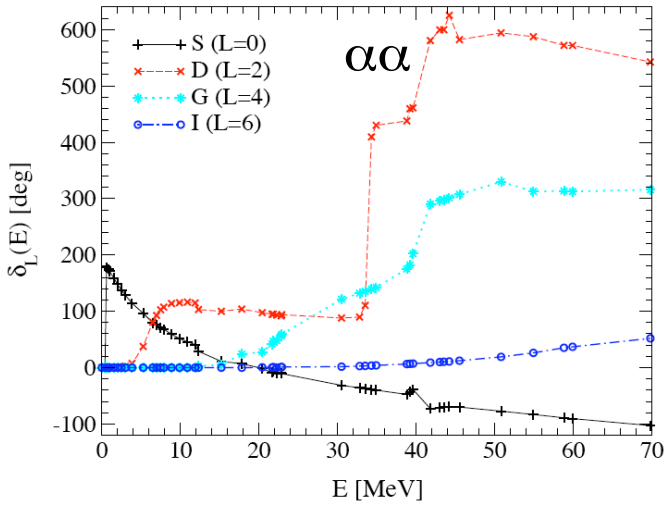
neglected Coulomb (use np,nα phase shifts; b_2 for plane wave bc), mixing parameters and inelasticities in scattering, can improve this

for b_{NN}: all L≤6 from Nijmegen PWA93, includes deuteron and large $^1S_0$ scattering lengths on equal footing

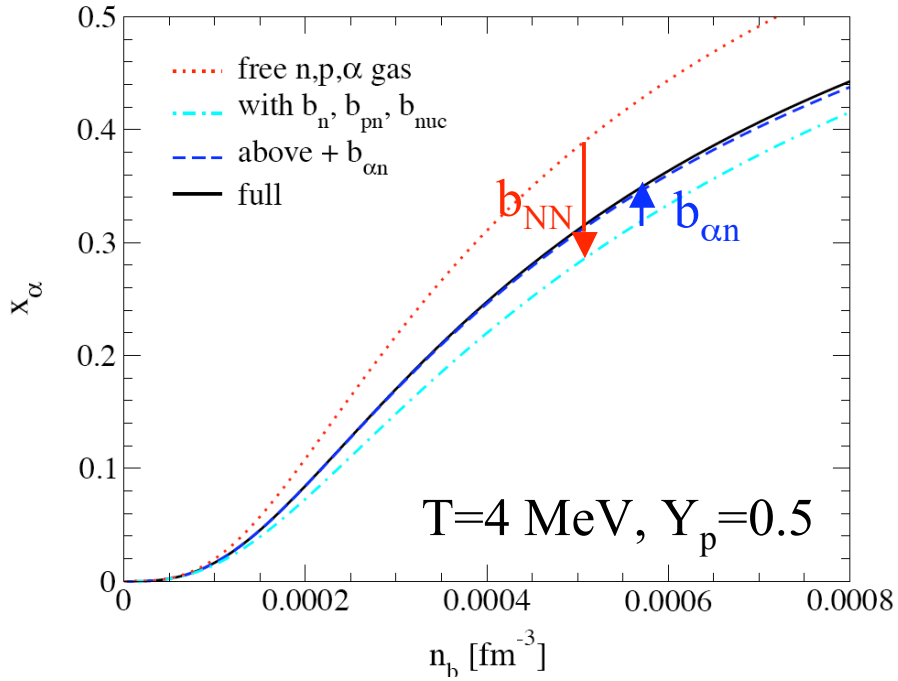
for $b_{\alpha n}$: all L≤3 from Arndt, Roper (1970) for E<20 MeV, Amos, Karataglidis (2005) optical model for higher E, includes $P_{3/2}$ resonance

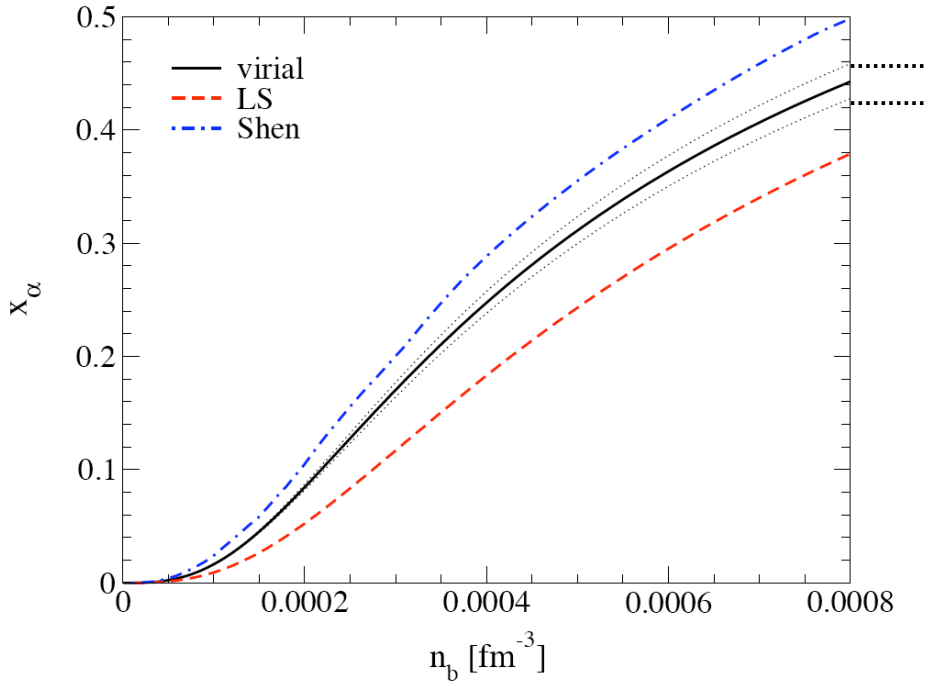
for $b_{\alpha\alpha}$: all L≤6 from Afzal et al. (1969) for E<30 MeV, Bacher et al. (1972) for 30<E<70 MeV includes $0^+,2^+$ resonances





virial coefficients dominated by resonant (large a) interactions





Composition: α mass fraction

Hierarchy of virial contributions:

b_{NN} more important than $b_{\alpha n}$, b_α

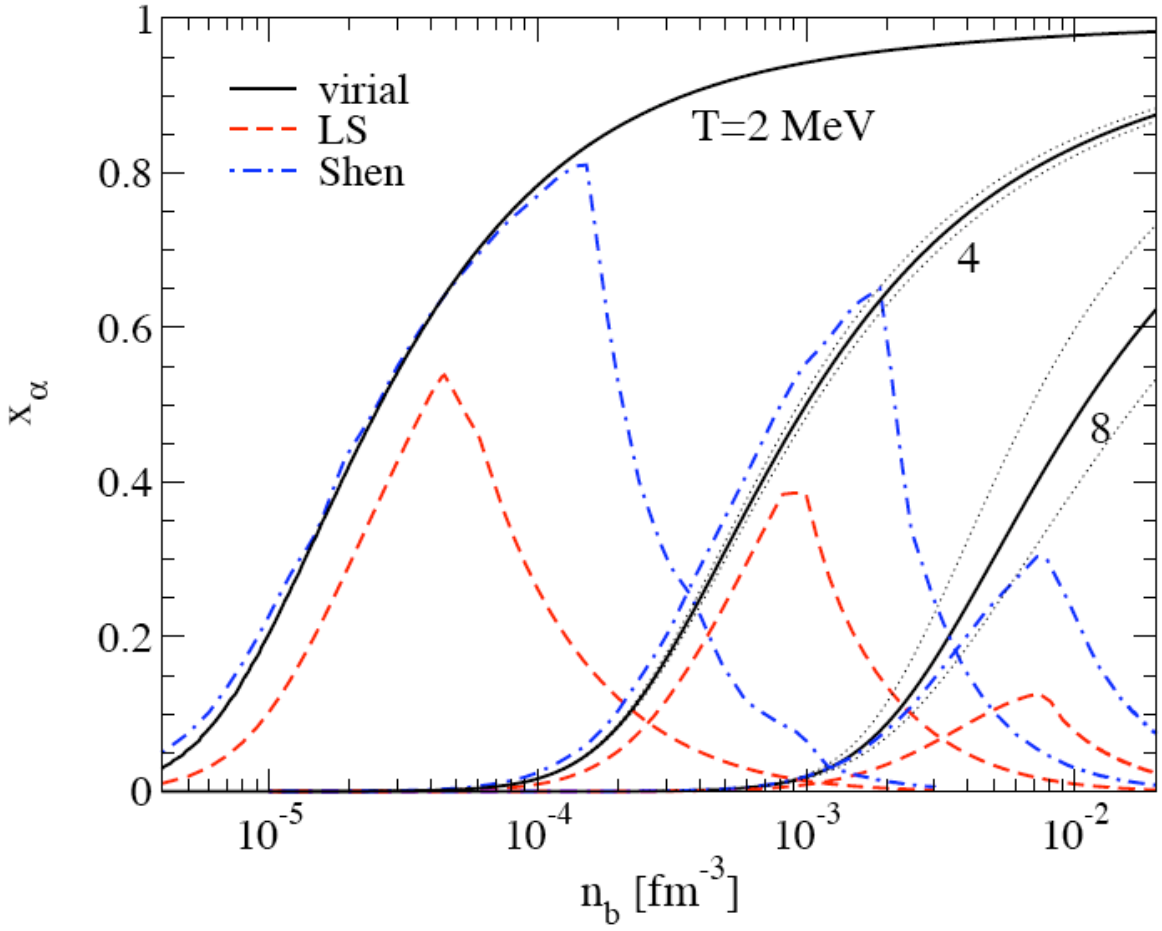
$b_{\alpha n}$ attractive due to $P_{3/2}$ resonance

Estimate errors due to neglected third virial coefficient $b_3 \sim \pm 10$

α mass fraction differs from LS=Lattimer-Swesty, Shen et al. EOS used in SN simulations

LS models nα interaction with repulsive excluded volume

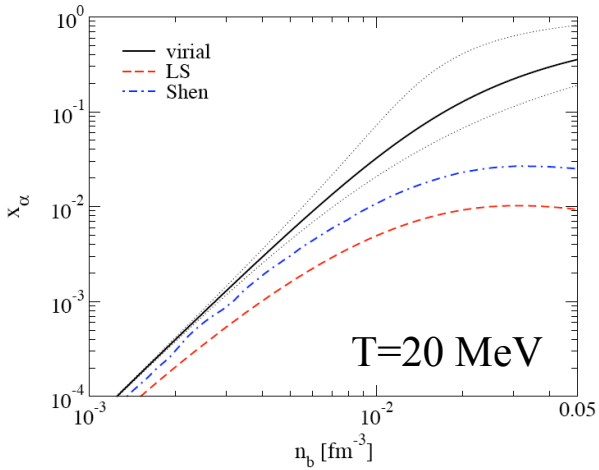
α mass fraction for various T



α fractions drop in LS/ Shen at high density due to formation of heavy A

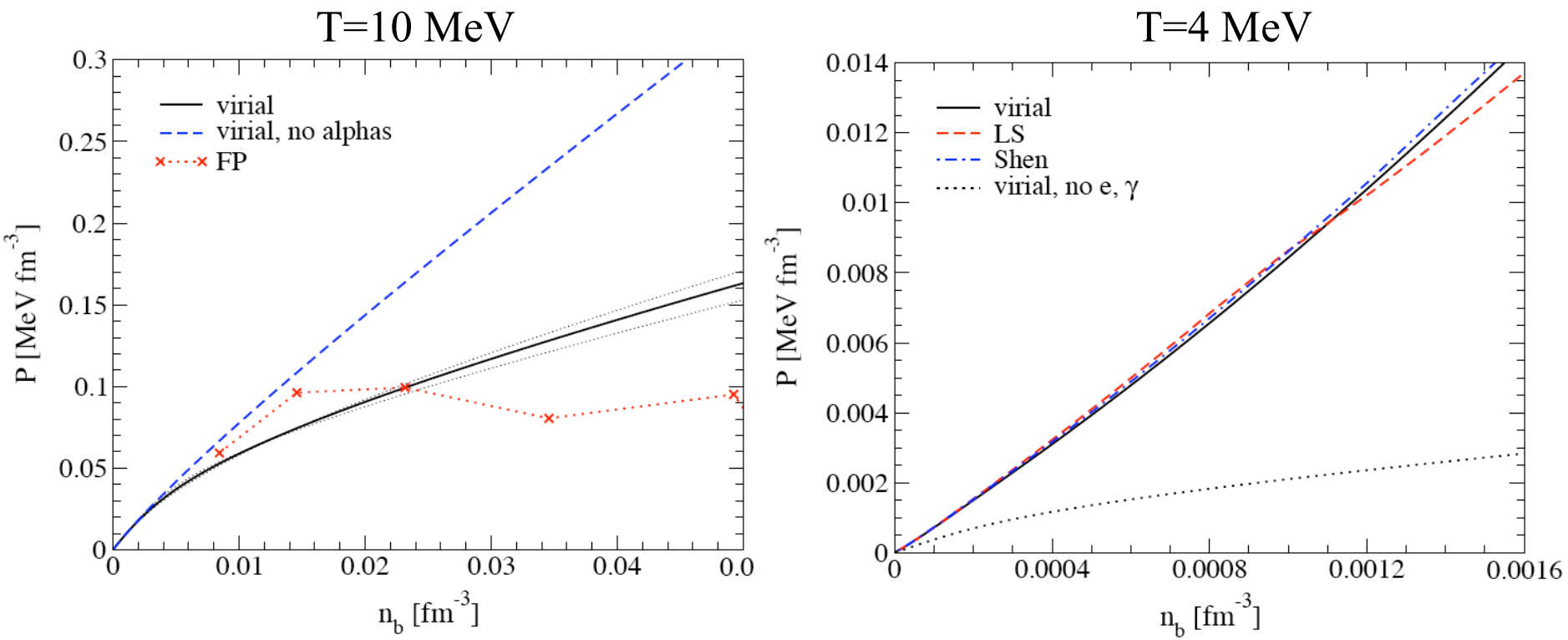
limits validity of n,p,α virial EOS

for T>10 MeV, models underestimate α fraction



x_α important for spin/neutrino response, since α particles have J=0

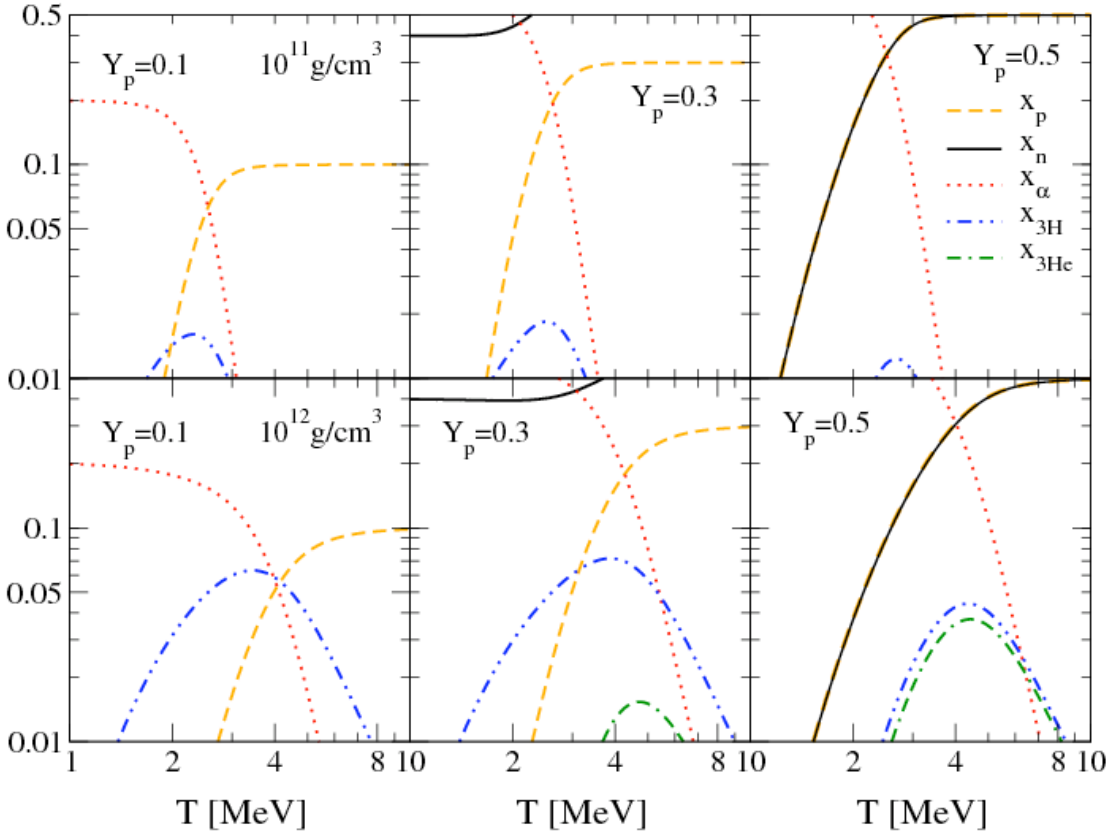
Pressure

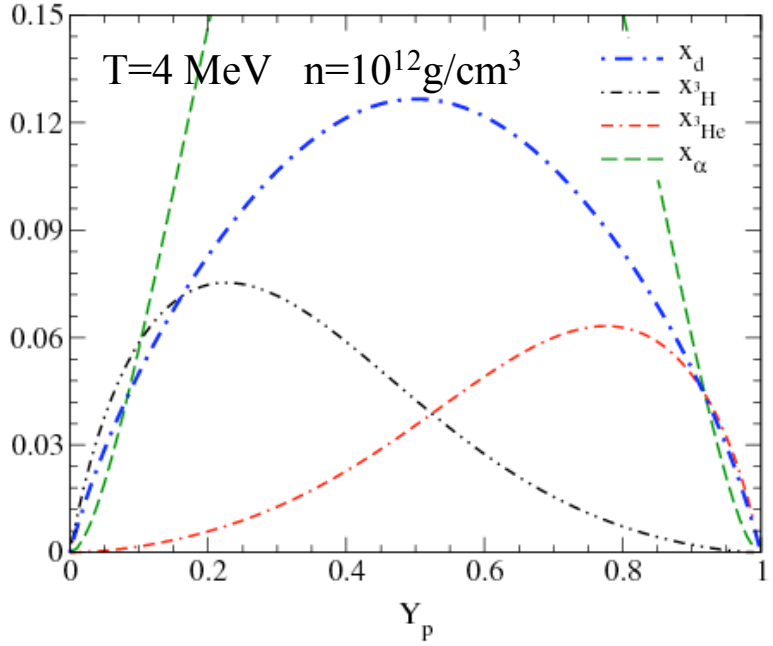


Variational calculations Friedman, Pandharipande fail to describe α contributions

Pressure agrees well with LS, Shen et al. EOS

Virial equation of state with light nuclei





O'Connor et al. (2007)

included A=3 nuclei and nucleon-A=3 virial coefficients

A=3 nuclei decrease alpha mass fraction, small effects of $b_{N\text{-}A=3}$

near neutrinosphere ~10% in A=3

d, ^{3}H, ^{4}He mass fractions can be comparable for neutron-rich matter

Neutrino breakup of A=3

T_ν [MeV]	^{3}H		^{3}He	
1	1.97×10^{-6}	1.68×10^{-5}	3.49×10^{-6}	2.76×10^{-5}
2	4.62×10^{-4}	4.73×10^{-3}	6.15×10^{-4}	5.94×10^{-3}
3	5.53×10^{-3}	6.38×10^{-2}	6.77×10^{-3}	7.41×10^{-2}
4	2.68×10^{-2}	3.37×10^{-1}	3.14×10^{-2}	3.77×10^{-1}
5	8.48×10^{-2}	1.14	9.70×10^{-2}	1.25
6	2.09×10^{-1}	2.99	2.35×10^{-1}	3.21
7	4.38×10^{-1}	6.61	4.87×10^{-1}	7.03
8	8.20×10^{-1}	13.0	9.03×10^{-1}	13.7
9	1.41	23.4	1.54	24.6
10	2.27	39.3	2.47	41.2

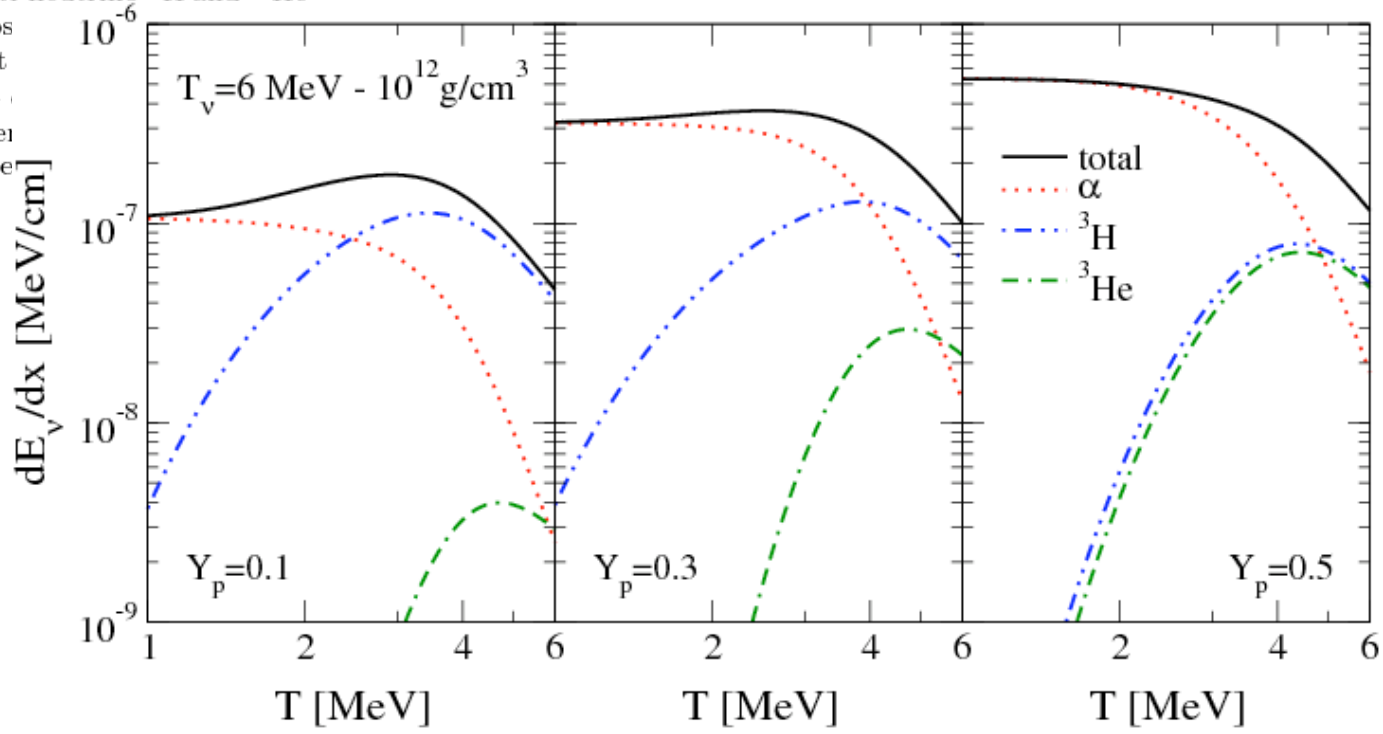
TABLE II: Averaged neutrino- and anti-neutrino-^{3}H and -^{3}He neutral-current inclusive inelastic cros (A=3), $\langle\sigma\rangle_{T_\nu} = \frac{1}{2A}\langle\sigma_\nu + \sigma_{\overline{\nu}}\rangle_{T_\nu}$ (left transfer cross-sections, $\langle\omega\sigma\rangle_{T_\nu} = \frac{1}{2A}$ columns), as a function of neutrino te of 10^{-42} cm^2 and 10^{-42} MeVcm^2 respe

O'Connor et al. (2007)

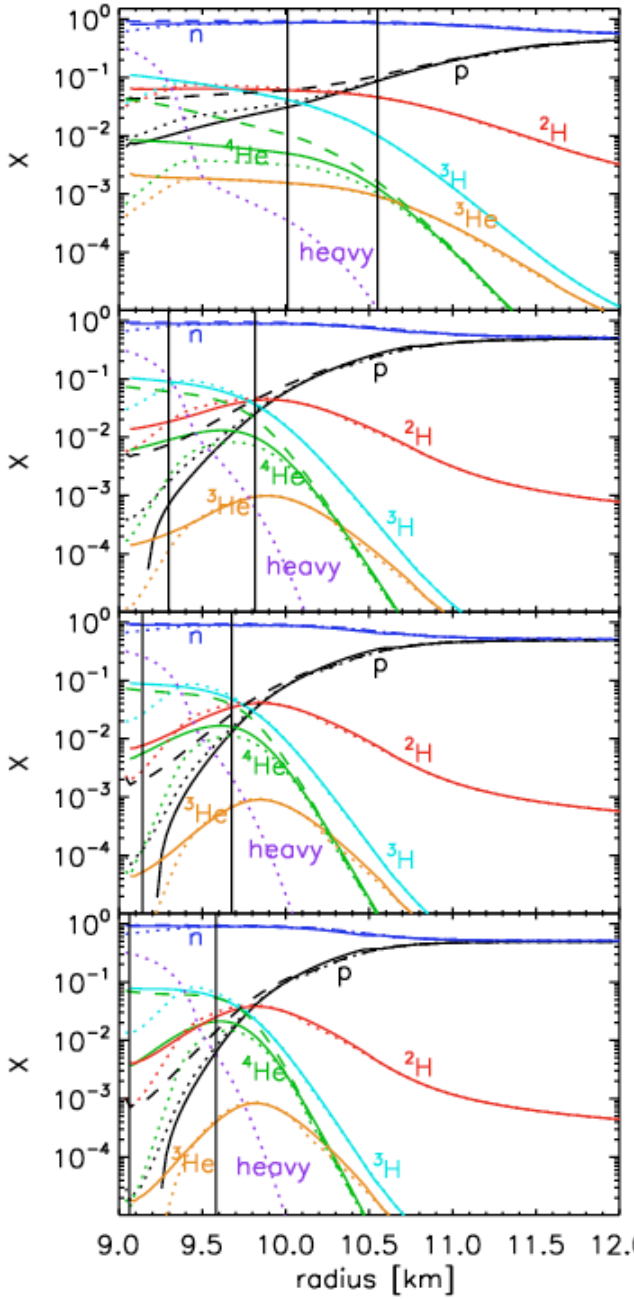
energy transfer

$$\frac{dE_\nu}{dx} = n_b \sum_{i=^3\mathrm{H},\,^3\mathrm{He},\,^4\mathrm{He}} x_i \langle\omega\sigma\rangle_{i,\,T_\nu}$$

can be dominated by breakup of loosely-bound A=3 nuclei



Light nuclei and neutrino-driven supernova outflows



t=2s Arcones et al. (2008)
dashed=simulation →
dotted=NSE
solid=virial

t=5s

t=7s

t=10s

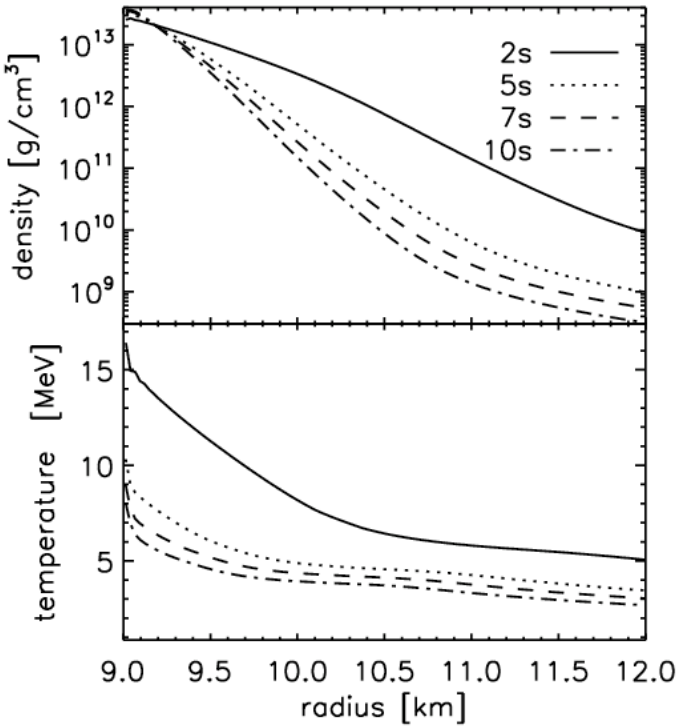
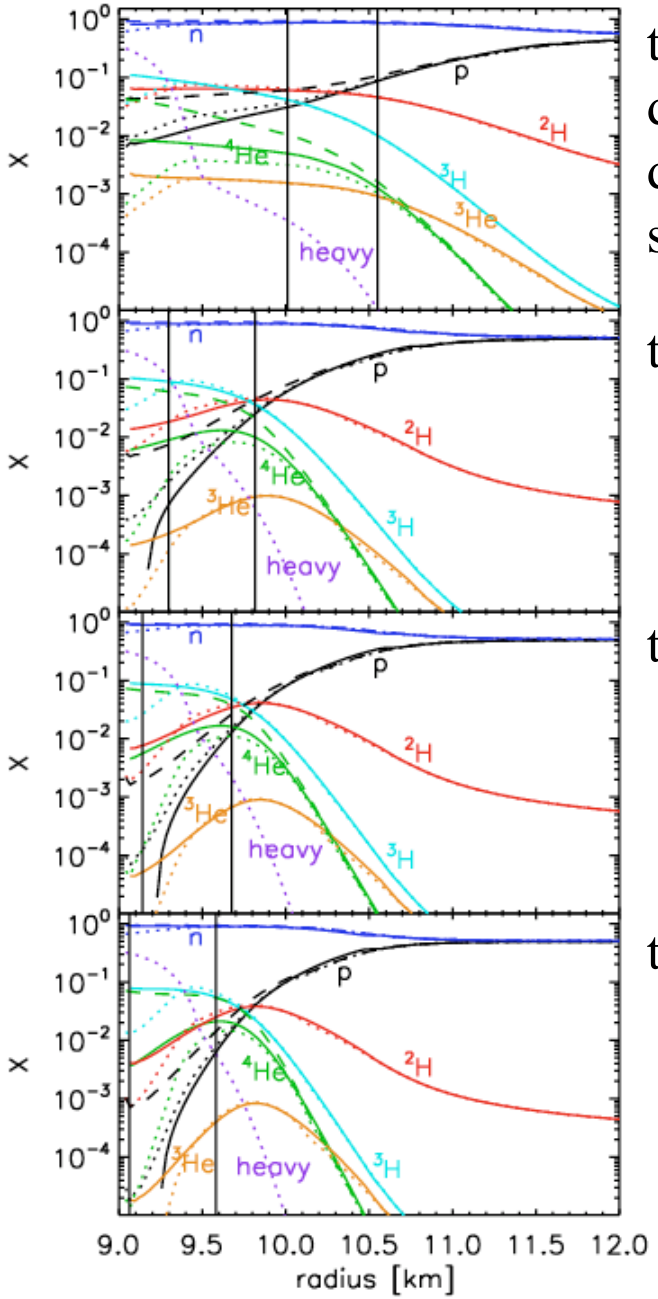


FIG. 1: Neutron star atmosphere profiles of density and temperature corresponding to model M15-l1-r1 of Ref. [7] for times $t = 2, 5, 7$ and 10 s post bounce.

Light nuclei present in substantial amounts in outer layers of protoneutron star

dominated by d, ^{3}H (and ^{4}He)

Light nuclei and neutrino-driven supernova outflows



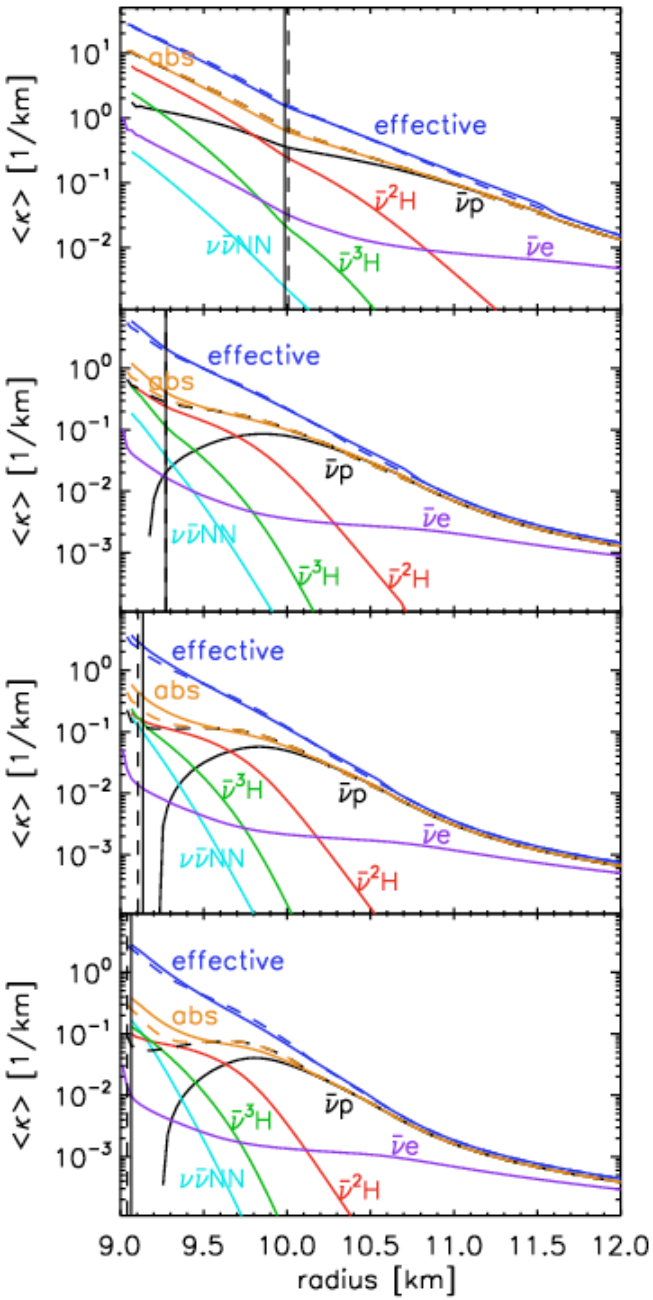
t=2s Arcones et al. (2008)
dashed=simulation
dotted=NSE
solid=virial

t=5s

t=7s

t=10s

absorption of electron antineutrinos on d, ^{3}H dominates over p



Light nuclei and neutrino-driven supernova outflows

TABLE II: Different cases explored in Sect. III.

Case	Y_e determined from	EOS and composition
A	beta equilibrium	NSE (n, p, ^{4}He)
B	case A	NSE (nucleons and nuclei)
C	beta equilibrium	NSE (nucleons and nuclei)
D	beta equilibrium	virial (n, p, $A \leqslant 4$ nuclei)

Arcones et al. (2008)

impact of light nuclei on electron antineutrino emission and wind Y_e compared to reference case A

TABLE III: Neutrinosphere radii $R_{\bar{\nu}_e,\nu_e}$, neutrino spectral temperatures $T_{\bar{\nu}_e,\nu_e}$, and average energies $\langle\epsilon_{\bar{\nu}_e,\nu_e}\rangle$, as well as number luminosities L_n, spectral parameter η_{ν_e}, and wind electron fractions $Y_e^{\rm w}$ at four different times post bounce.

	$R_{\bar{\nu}_e}$ [km]	$T_{\bar{\nu}_e}$ [MeV]	$\langle\epsilon_{\bar{\nu}_e}\rangle$ [MeV]	L_n [10^{56} s^{-1}]	η_{ν_e}	R_{ν_e} [km]	T_{ν_e} [MeV]	$\langle\epsilon_{\nu_e}\rangle$ [MeV]	$Y_e^{\rm w}$
				$t = 2$ s					
A	10.01	8.14	25.64	6.05	0.72	10.55	6.34	20.71	0.514
B	9.977	8.30	26.16	6.38	0.79	10.55	6.34	20.80	0.507
C	10.00	8.17	25.73	6.10	0.73	10.55	6.35	20.75	0.513
D	9.979	8.29	26.12	6.36	0.77	10.53	6.37	20.87	0.509
				$t = 5$ s					
A	9.272	7.17	22.60	3.55	1.01	9.821	5.14	17.10	0.478
B	9.260	7.24	22.83	3.65	1.04	9.819	5.15	17.16	0.475
C	9.295	7.04	22.17	3.37	0.94	9.814	5.16	17.07	0.487
D	9.272	7.17	22.60	3.55	1.00	9.813	5.16	17.15	0.480
				$t = 7$ s					
A	9.107	6.88	21.69	3.03	1.15	9.683	4.73	15.90	0.462
B	9.095	6.97	21.95	3.13	1.19	9.681	4.74	15.96	0.458
C	9.139	6.68	21.04	2.78	1.04	9.676	4.75	15.82	0.475
D	9.134	6.71	21.14	2.82	1.05	9.675	4.75	15.85	0.473
				$t = 10$ s					
A	9.041	6.94	21.86	3.06	1.49	9.592	4.37	15.05	0.431
B	9.039	7.02	22.12	3.17	1.53	9.590	4.37	15.12	0.427
C	9.063	6.49	20.44	2.51	1.23	9.582	4.39	14.82	0.456
D	9.065	6.45	20.32	2.47	1.20	9.581	4.39	14.80	0.458

Summary

virial equation of state provides model-independent constraints for low-density nuclear matter and neutrino response

based directly on scattering phase shifts, includes bound states and resonant interactions on equal footing

important for supernova neutrinosphere

light nuclei can be present in significant amounts,
d and ^{3}H favored for neutron-rich conditions

include light nuclei and interactions with neutrinos in supernova and neutrino-driven wind simulations

Outline

1. Introduction to effective field theory and the renormalization group

2. Chiral effective field theory

3. Renormalization group for nuclear forces

4. EFT and RG for nuclear matter

5. The virial equation of state and light nuclei

6. Neutrino processes in supernovae from chiral EFT

Motivation and basics

Neutrino processes in supernovae and neutron stars:
important for explosion, neutrino spectra, neutron star cooling,…

processes involving two nucleons play a special role

neutrino-pair bremsstrahlung and absorption: key for production of muon and tau neutrinos, and for equilibrating neutrino number densities

Suzuki, Nakamura, Raffelt, Janka, Keil, Seckel, Hannestad, Thompson, Burrows, Horvath,…

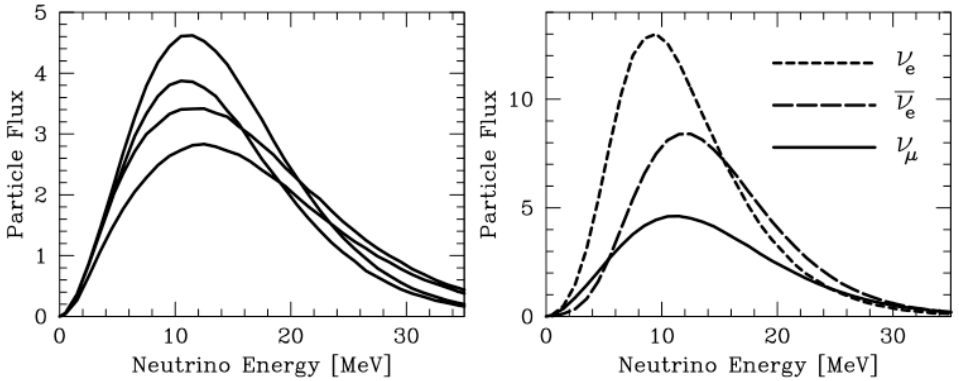
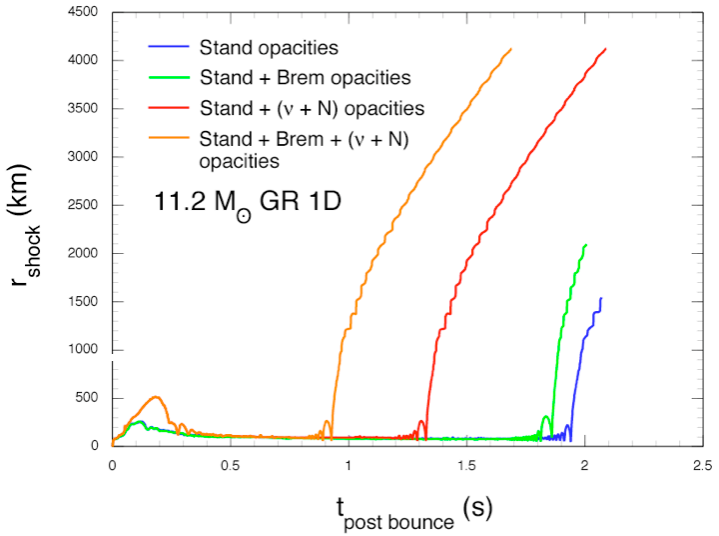


Figure 1. Neutrino fluxes for an accretion-phase model. *Left panel, curves from bottom to top:* Flux of ν_μ with traditional neutrino interaction channels, then adding nucleon bremsstrahlung, next adding nucleon recoils, and finally adding $\nu_e\bar{\nu}_e$ annihilation. *Right panel:* Fluxes for all flavors; the ν_μ curve includes all reaction channels.

from Raffelt et al. (2003)



from Mezzacappa (2008)

but standard rates based on one-pion exchange for NN interactions, nobody uses only OPE for nuclear structure

Single- and two-nucleon processes

elastic scattering from nucleons (space-like $\omega < q$)

initial and final state interactions, inelastic scattering $\nu nn \leftrightarrow \nu nn$
Landau-Pomeranchuk-Migdal effect

neutrino-pair bremsstrahlung/absorption $nn \leftrightarrow nn\nu\overline{\nu}$ (time-like $\omega > q$)

need collisions between nucleons for the latter processes

noncentral contributions, due to tensor forces from pion exchanges and spin-orbit forces, are essential for the two-nucleon response

follows from direct calculations Friman, Maxwell (1979)
and from conservation laws Olsson, Pethick (2002) (see later)

develop a unified treatment that consistently includes one- and two-quasiparticle-quasihole pair states Lykasov, Pethick, AS, PRC 78, 045803 (2008)

Neutrino processes and dynamical structure factors

neutrinos interact weakly → rates for neutrino scattering, emission and absorption determined by dynamical structure factors of nucleon matter

generally axial/spin response most important, ~ factor 3

rate for bremsstrahlung $\Gamma_{NN\leftrightarrow NN\nu\overline{\nu}} = 2\pi\, n\, G_{\rm F}^2\, C_{\rm A}^2\, (3-\cos\theta)\, S_{\rm A}(\omega,\mathbf{q})$

with spin dynamical structure factor $S_{\rm A}(\omega,\mathbf{q}) = \dfrac{1}{\pi n}\,\dfrac{1}{1-e^{-\omega/T}}\,{\rm Im}\,\chi_\sigma(\omega,\mathbf{q})$

response is diagonal in spin for long wavelengths

problem is to calculate structure factors of nucleon matter

not included:
reduction of axial coupling g_a for nucleon quasiparticles
by 5-10% in neutron matter Cowell, Pandharipande (2003)

beyond quasiparticle contributions (incoherent parts)

Relevant conditions

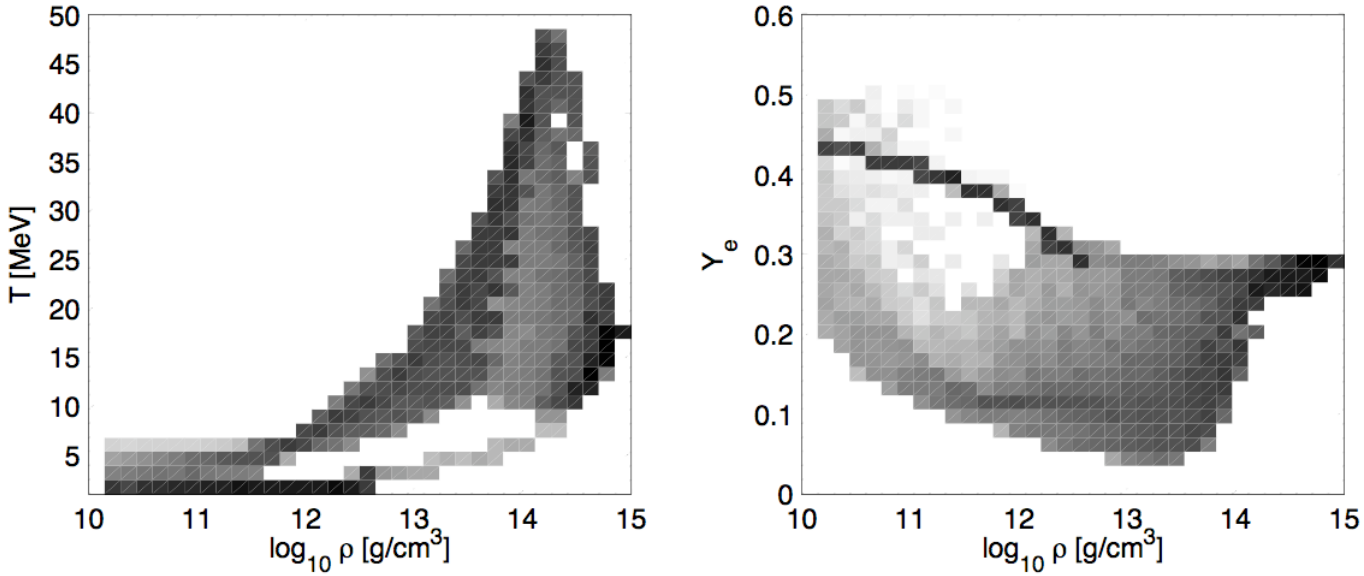
crucial densities seem to be below nuclear matter density $\rho \sim \rho_0/10$
(high densities: neutrinos trap; low densities: few interactions)

energy, momentum transfers ω, q small compared with Fermi momentum

temperatures ~ Fermi temperature or less

→ Landau Fermi liquid theory is a reasonable first approximation

conditions for muon and tau neutrino production: higher T and low Y_e



from Liebendoerfer (2008)

Unified approach to structure factors, qp transport equation

Linearized quasiparticle transport equation for the spin response $\delta\mathbf{s}_\mathbf{p}$

$$\left(\omega - \varepsilon_{\mathbf{p}+\mathbf{q}/2} + \varepsilon_{\mathbf{p}-\mathbf{q}/2}\right)\delta\mathbf{s}_\mathbf{p} + \left(n_{\mathbf{p}+\mathbf{q}/2} - n_{\mathbf{p}-\mathbf{q}/2}\right)\delta\mathbf{h}_\mathbf{p} = i\, I_\sigma[\mathbf{s}_{\mathbf{p}'}]$$

includes one-pair states through perturbation of quasiparticle energy

$$\delta\mathbf{h}_\mathbf{p} = \mathbf{U}_\sigma + 2\int \frac{d\mathbf{p}'}{(2\pi)^3}\, g_{\mathbf{p}\mathbf{p}'}\, \delta\mathbf{s}_{\mathbf{p}'}$$

rates involving two nucleons expressed in terms of the collision integral

solution to qp transport equation includes multiple-scattering effects, Landau-Pomeranchuk-Migdal effect

qp transport equation can be solved in a relaxation time approximation

Relaxation time approximation

approximate collision integral as $I_\sigma[\mathbf{s}_{\mathbf{p}'}] = -\dfrac{\delta\mathbf{s}_\mathbf{p} - \delta\mathbf{s}_\mathbf{p}\big|_{\rm le}}{\tau_\sigma}$

with spin relaxation time τ_σ = rate of change of nucleon spin through collisions with other nucleons

assumes all angular harmonics relax at the same rate

Spin relaxation rate

spin relaxation rate for frequencies $|\omega| \gg 1/\tau_\sigma$ and $q \ll k_{\rm F}$

$$\frac{1}{\tau_\sigma} = C_\sigma \left[T^2 + (\omega/2\pi)^2\right]$$

$$C_\sigma = \frac{\pi^3 m^*}{6k_{\rm F}^2}\left\langle \frac{1}{12}\sum_{k=1,2,3} {\rm Tr}\left[\mathcal{A}_{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2}(\mathbf{k},\mathbf{k}')\,\boldsymbol{\sigma}_1^k\left[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)^k\,,\,\mathcal{A}_{\boldsymbol{\sigma}_1,\boldsymbol{\sigma}_2}(-\mathbf{k},\mathbf{k}')\right]\right]\right\rangle$$

with qp scattering amplitude and average over the Fermi surface

commutator with 2-body spin: only noncentral interactions contribute

Resulting dynamical structure factors

solution to qp transport equation

$$\mathrm{Im}\chi_\sigma = N(0)\,\frac{\mathrm{Im}\,\widetilde{X}_\sigma}{|1+G_0\widetilde{X}_\sigma|^2}$$

isotropic Landau interaction G_0 dominates in neutron matter

in relaxation time approximation $\widetilde{X}_\sigma = 1 - \dfrac{\omega}{2v_\mathrm{F}q}\ln\left(\dfrac{\omega+i/\tau_\sigma+v_\mathrm{F}q}{\omega+i/\tau_\sigma-v_\mathrm{F}q}\right)$

generalizes earlier work to finite q and include mean-field effects

Limits

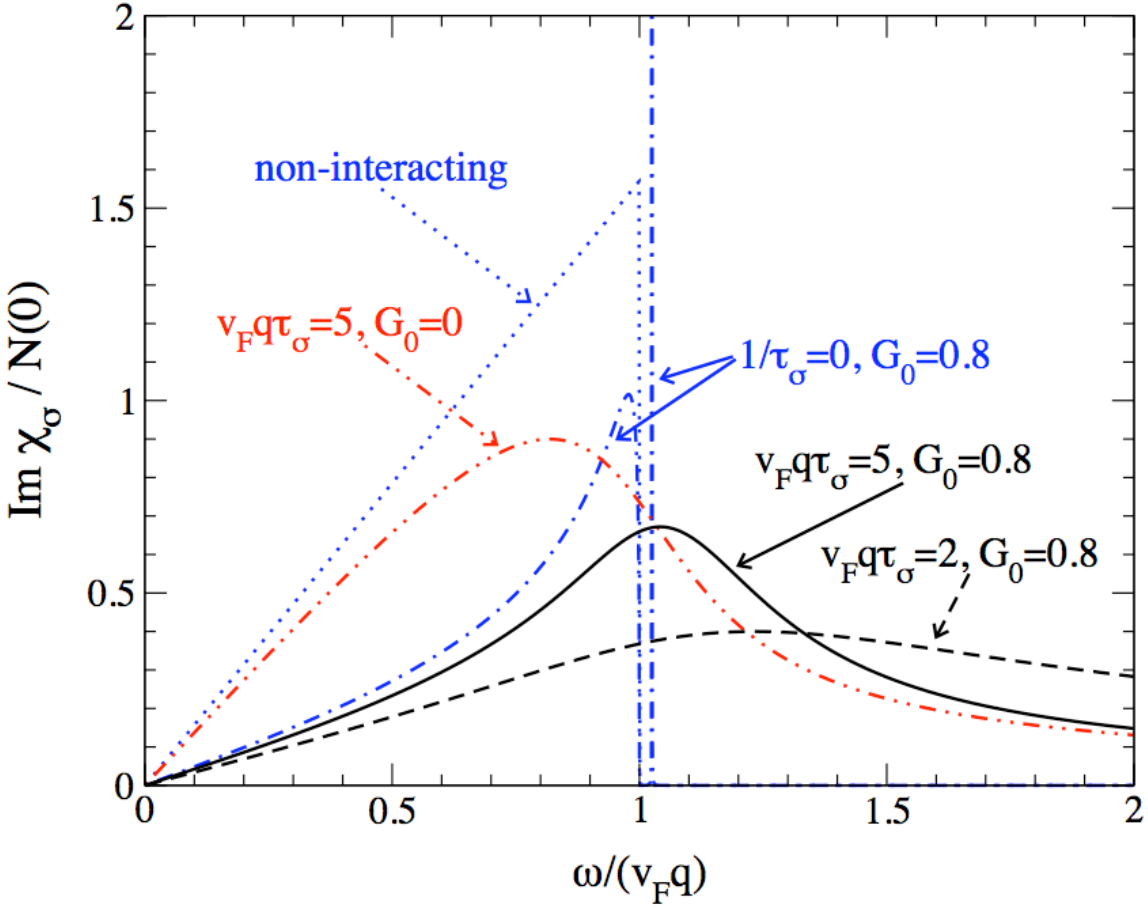
Long-wavelength limit:

$$\mathrm{Im}\chi_\sigma(\omega, q\to 0) = N(0)\,\frac{\omega\tau_\sigma}{(1+G_0)^2+(\omega\tau_\sigma)^2}$$

without mean-field effects (G_0=0): same form as Ansatz of Raffelt et al.
but multiple-scattering effects calculated not introduced as parameter

and for $|\omega|\tau_\sigma \gg 1$ reproduces standard rates with τ_σ based on OPE

Spin dynamical structure factor



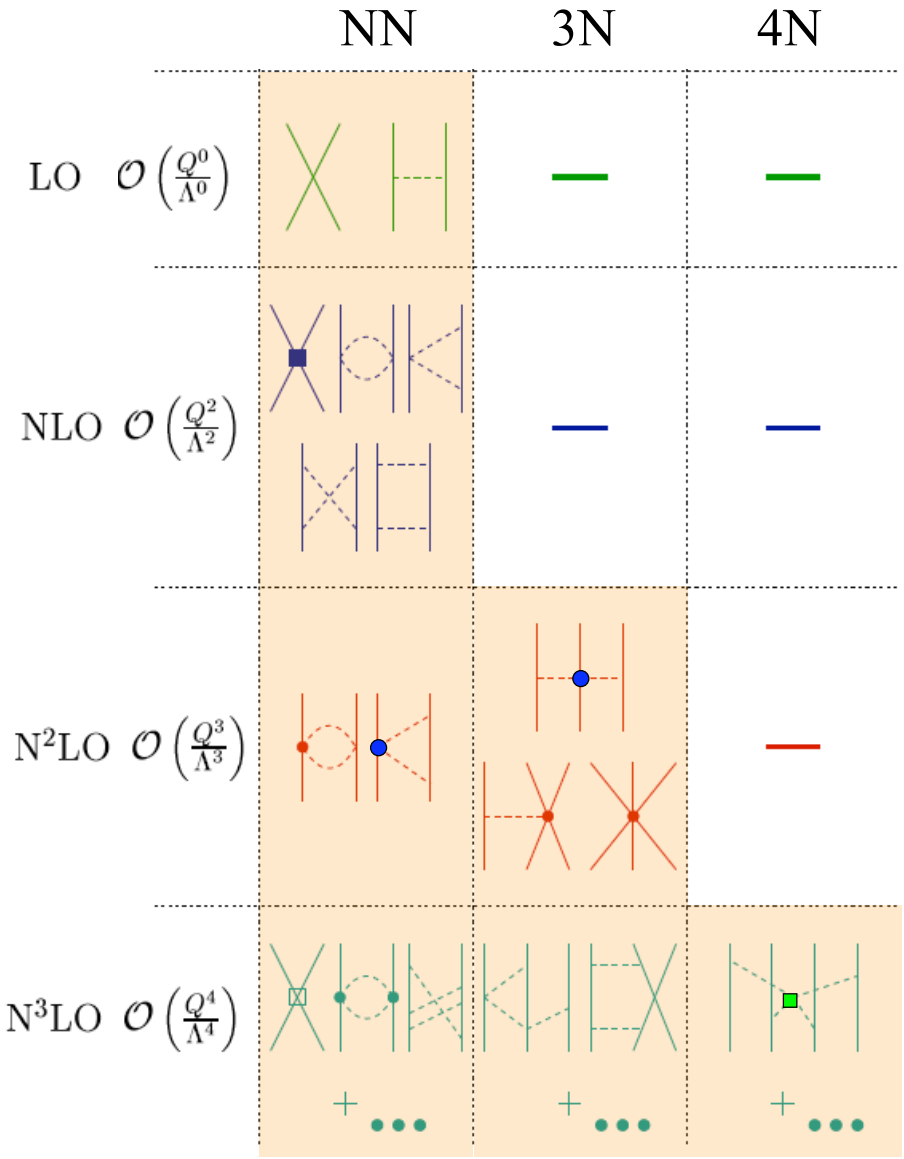
single-pair mean-field effects (G_0=0.8): collective spin-zero-sound pole

with increasing spin relaxation rate from $1/\tau_\sigma = 0$ to $v_F q/5$ and $v_F q/2$ spin-zero-sound peak disappears, response pushed to higher frequencies

significant for $1/\tau_\sigma$ comparable to $v_F q \rightarrow$ recoil effects may be important

Rates based on chiral effective field theory (EFT) for nuclear forces

Separation of scales: low momenta $Q \ll \Lambda_b$ breakdown scale ~500 MeV



limited resolution at low energies, can expand in powers Q/Λ_b

include long-range pion physics

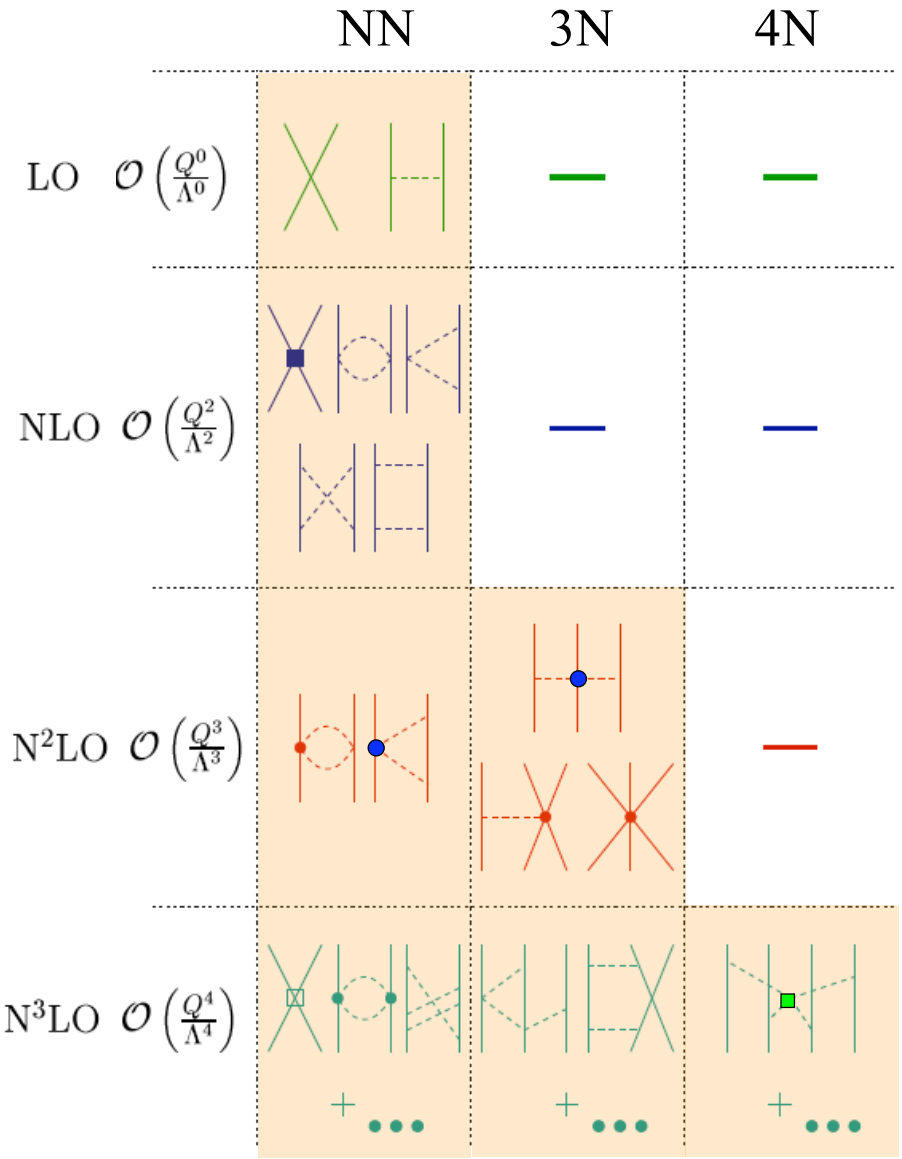
details of short-distance physics not resolved

capture in few short-range couplings, fit to experiment once

systematic: can work to desired accuracy and obtain error estimates

Weinberg, van Kolck, Kaplan, Savage, Wise, Epelbaum, Meissner, Nogga, Machleidt,…

Rates based on chiral effective field theory (EFT) for nuclear forces



standard rates for bremsstrahlung based on one-pion exchange (OPE)

reasonable starting point: long-range part and LO for neutrons

however: subleading contributions crucial for NN scattering at $k_F \sim 1.0\,\mathrm{fm}^{-1} \approx 200\,\mathrm{MeV}$

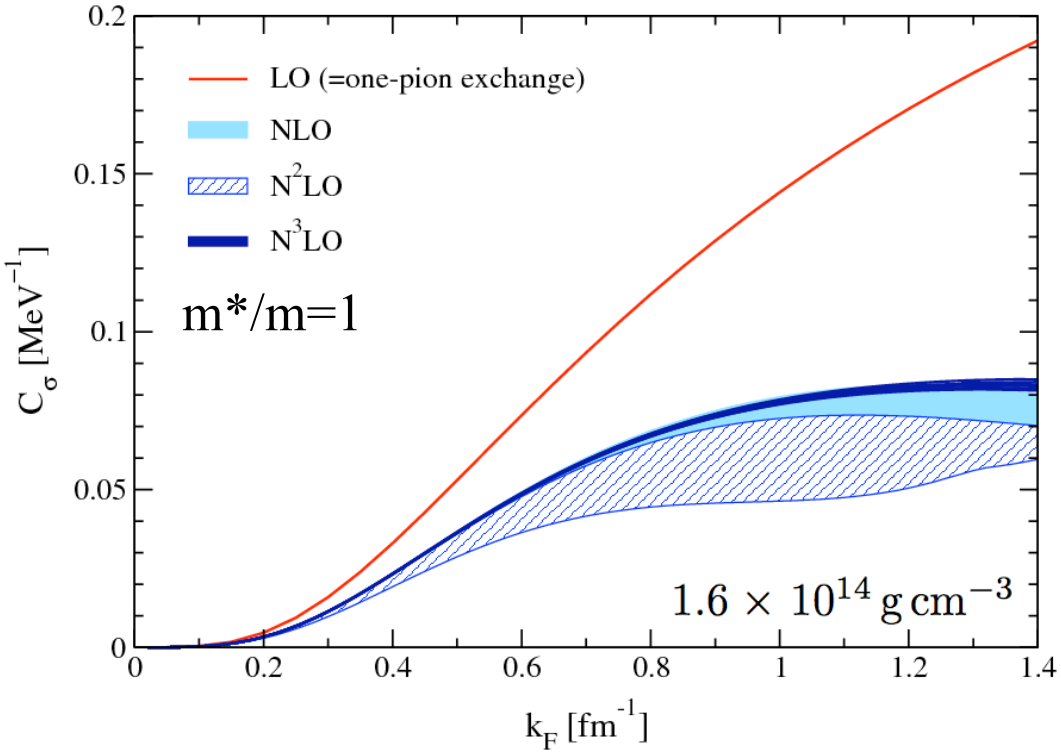
3N important for EOS for $k_F \gtrsim 1.5\,\mathrm{fm}^{-1}$

Tolos, Friman, AS (2008)

go beyond OPE approximation: NN contributions up to N^{3}LO at subnuclear densities

Bacca, Hally, Pethick, AS, arXiv:0812.0102.

Spin relaxation rate in neutron matter from chiral EFT



$$\frac{1}{\tau_\sigma} = C_\sigma \left[T^2 + (\omega/2\pi)^2\right]$$

bremsstrahlung rate ~ C_σ when $|\omega| \gg 1/\tau_\sigma$

chiral EFT interactions from Epelbaum et al. (2005) Entem, Machleidt (2003) used in Born approximation (will justify later)

error bands from cutoff variation

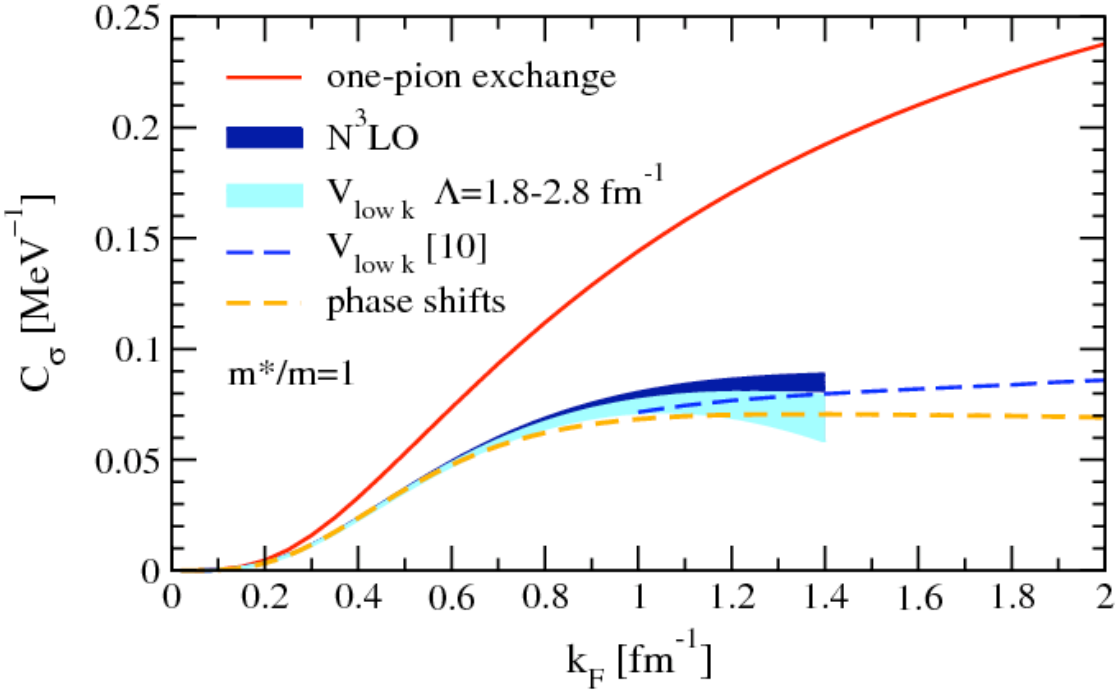
convergence with successively higher orders,
OPE significantly overestimates C_σ for all relevant densities

at NLO: leading two-pion exchange and noncentral contact interactions, constrained by NN scattering data

at N^3LO (accurately reproduces NN scattering): interaction independent

first model-independent results at subnuclear densities

Comparison to rates based on NN phase shifts



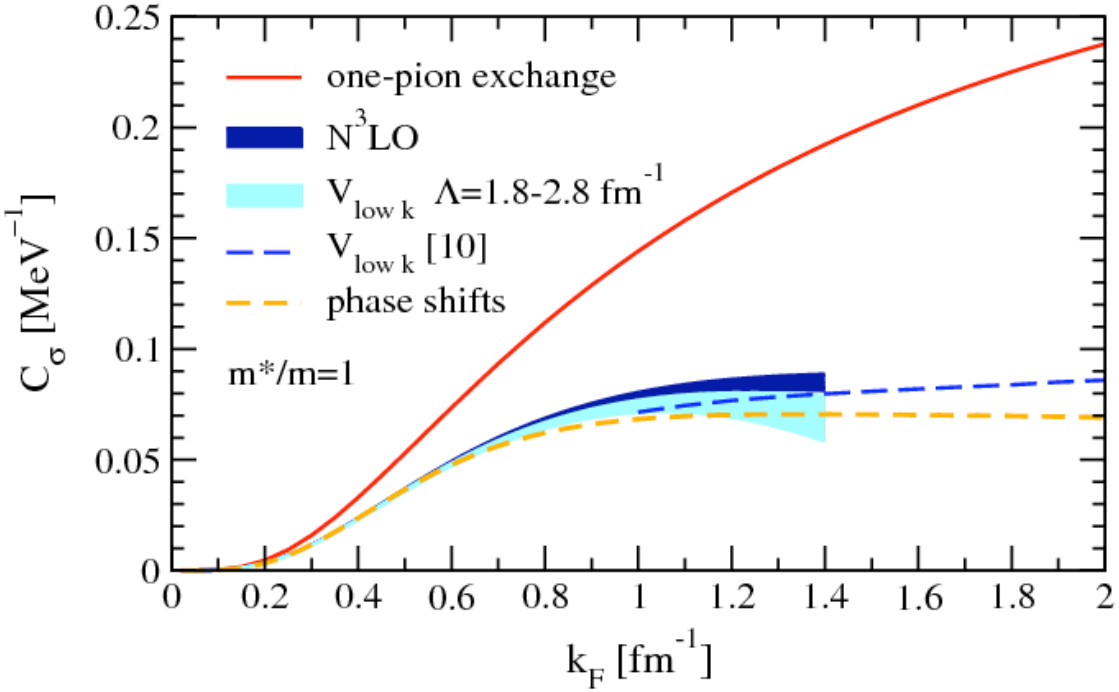
extended cutoff variation and estimate of uncertainty using RG-evolved low-momentum interactions $V_{low\ k}$

low densities: two-nucleon collisions dominate, model-independent result based on NN phase shifts from NN partial wave analysis (nn-online.org)
see also Hanhart, Phillips, Reddy (2001)

rates from chiral N^{3}LO interactions in Born approximation close to rates from phase shifts

noncentral part of the strong neutron-neutron amplitude is perturbative, rates based on phase shifts would otherwise be unreliable

Fit and many-body contributions



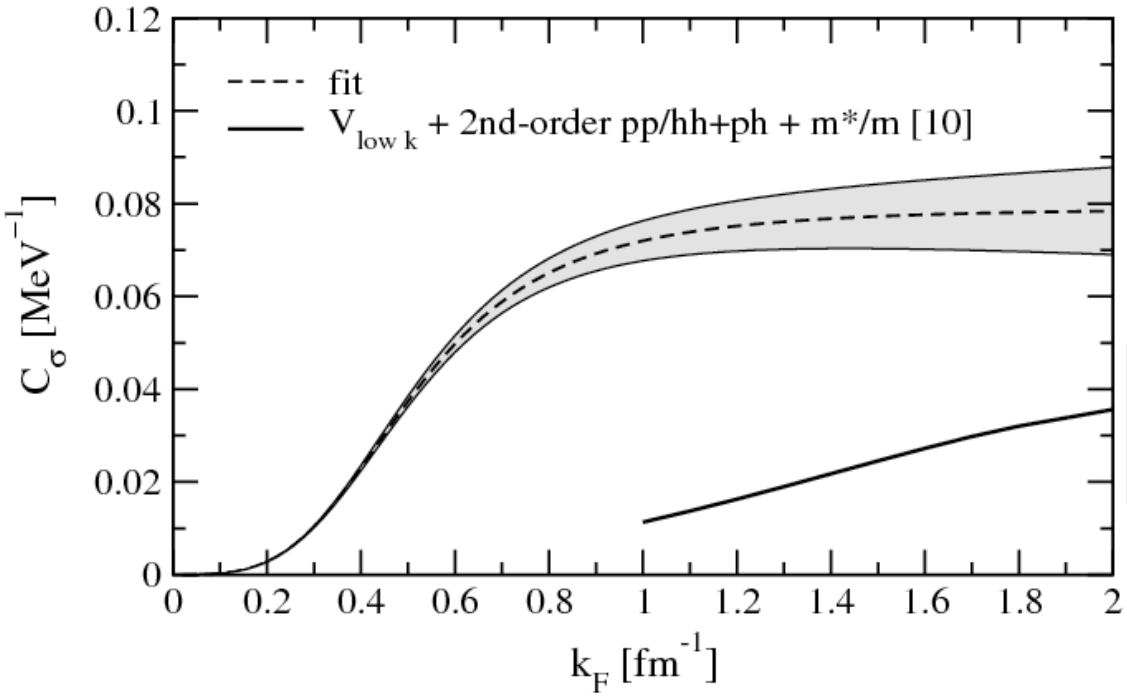
at subnuclear densities:

$\rho < 10^{14}\,\mathrm{g\,cm^{-3}}$ $(k_\mathrm{F} < 1.2\,\mathrm{fm}^{-1})$

the spin response is well constrained

all realistic results lie within a band

Fit and many-body contributions



at subnuclear densities:
$\rho < 10^{14}\,\mathrm{g\,cm^{-3}}$ $(k_\mathrm{F} < 1.2\,\mathrm{fm}^{-1})$
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simple fit function representing our results: $\dfrac{C_\sigma}{\mathrm{MeV}} = \dfrac{0.86\,(k_\mathrm{F}/\mathrm{fm}^{-1})^{3.6}}{1+10.9\,(k_\mathrm{F}/\mathrm{fm}^{-1})^{3.6}}$

further reduction from particle-hole and $(m^*/m)^3$ effects
(particle-hole mixing of tensor with strong central interactions)

sensitivity tests with C_σ times suppression over different density ranges
→ where to put many-body calculational effort

Mean-free paths and energy transfer

thermally-averaged inverse mf path due to neutrino-pair absorption

long-wavelength approximation $\langle \lambda^{-1} \rangle = \frac{C_{\rm A}^2 G_{\rm F}^2}{20\pi} \frac{n}{T^3} \int_0^\infty d\omega\, \omega^5\, e^{-\omega/T}\, S_\sigma(\omega)$

Hannestad, Raffelt (1998)

	G_0	0	0.8	0	0.8
$k_{\rm F}$ [fm^{-1}]	T [MeV]	C_σ from OPE		$V_{{\rm low}\,k}$	
1.0	5	0.0770	0.0697	0.0397	0.0386
	10	1.08	0.798	0.612	0.554
1.7	5	0.119	0.107	0.0476	0.0468
	10	1.66	1.21	0.744	0.700

in km^{-1}

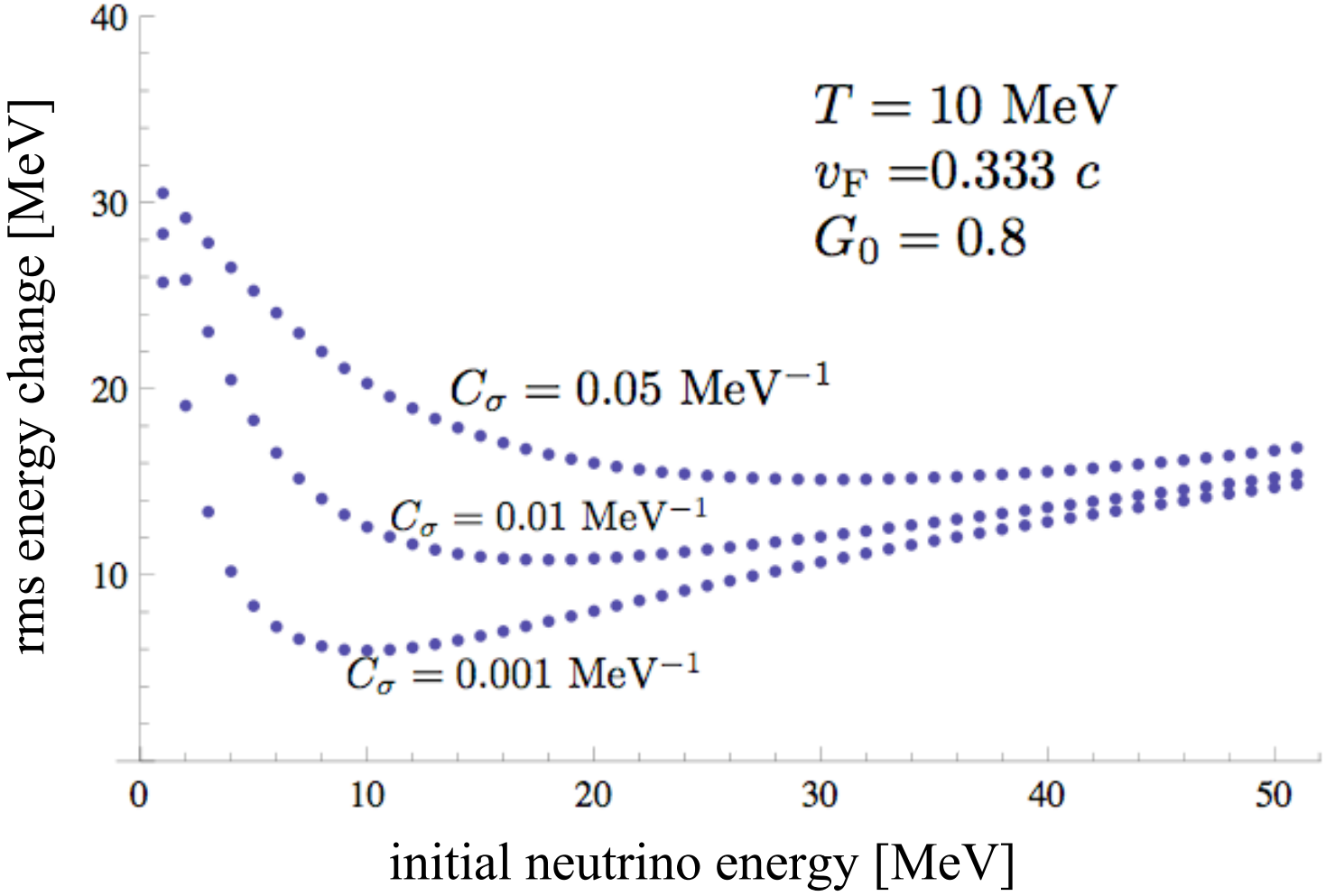
note: OPE based on solution to qp transport equation (not OPE in SN)

significantly longer mf paths compared to OPE,
consequently weaker mean-field effects (G_0=0.8 vs. G_0=0)

similar changes for energy loss due to neutrino-pair emission and energy transfer (inelastic scattering $\nu nn \leftrightarrow \nu nn$ important)

Energy transfer in neutrino scattering

rms energy change in neutrino scattering including recoil effects



significant dependence on spin relaxation rate

Summary and outlook

unified approach to neutrino processes based on Landau Fermi liquid theory, includes one- and two-quasiparticle-quasihole pair states

derived dynamical structure factors from qp transport equation in a relaxation time approximation

first neutrino rates for supernovae based on chiral EFT, to N^3LO, over wide density range and including theoretical uncertainties

two-pion exchange interactions and shorter-range noncentral forces reduce neutrino rates significantly

spin response is well constrained for densities $\rho < 10^{14}\,\mathrm{g\,cm^{-3}}$

Future: neutron-proton mixtures, extend to less degenerate conditions, two-body contributions to weak currents, charged current interactions, extension to superfluid phases,…

Summary

Exciting era with advances on many fronts

For the first time, approaches from light to heavy nuclei
and for astrophysics based on the same interactions

Three-nucleon interactions are a frontier

Exciting intersections with problems in many related areas

Major investments in new facilities and institutes worldwide


Thank you for listening, for your interest,
your warm hospitality and a terrific time!!