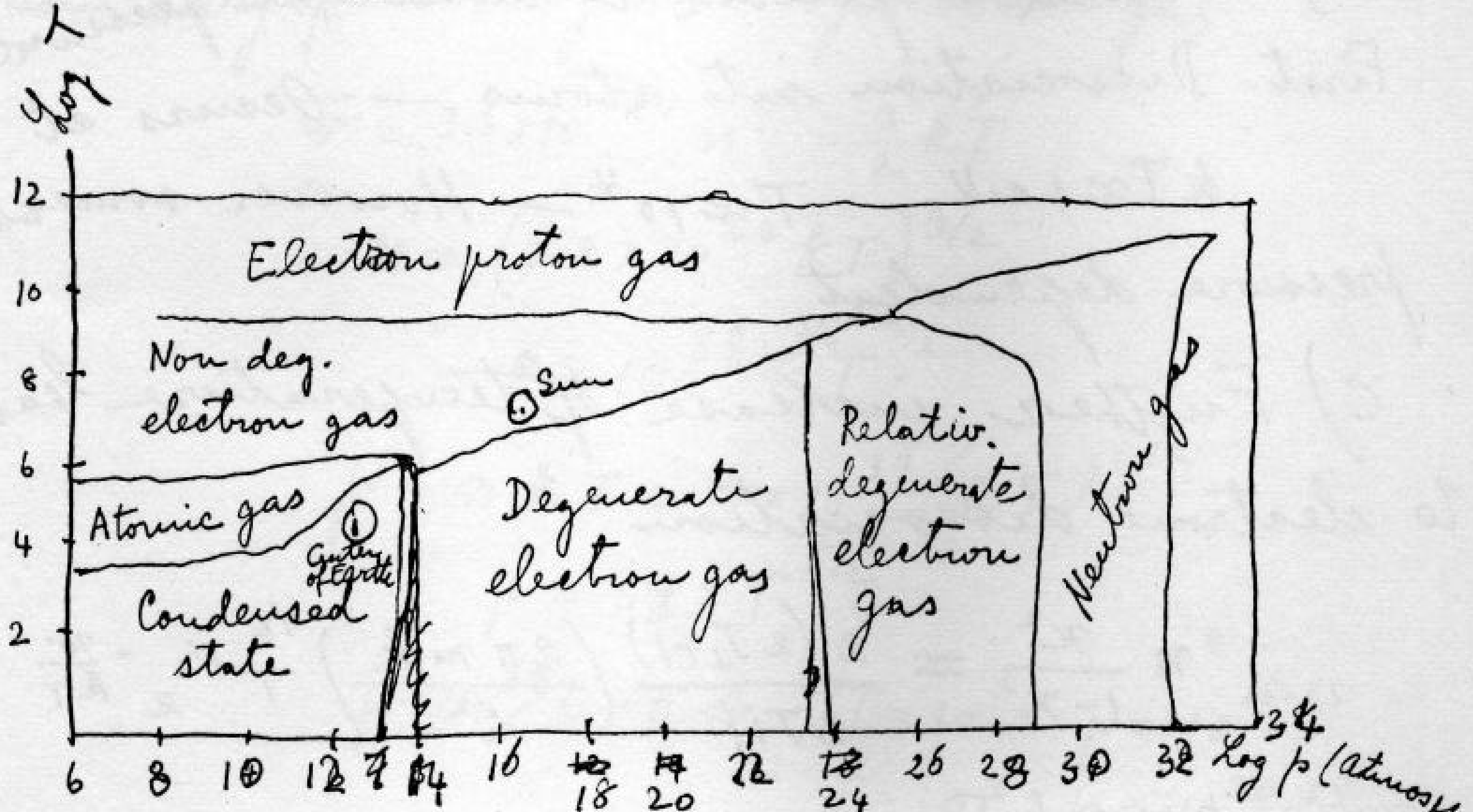
E. Fermi: *Notes on Thermodynamics and Statistics* (1953)

70 – Matter in unusual conditions 70 a



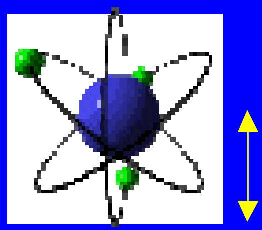
Start from ordinary condensed matter with ~~[illegible]~~ equation of state controlled by ordinary chemical forces.

Neutron stars below the surface

Surface is liquid. Expect primarily ^{56}Fe with some ^{4}He

$T \approx 10^7$ K $\simeq$ 1 KeV >> $T_{melting}$ (^{56}Fe)

Ionization:



$r_{Thomas\text{-}Fermi} = 0.8853\ a_0/Z^{1/3}$

Matter begins to ionize when $r_c = (3/4\pi\ n_{atoms})^{1/3} > r_{TF}$

=> $\rho > m_n AZ/\ a_0^3 \simeq 10\ AZ$ g/cm^3 $\approx 10^4$ g/cm^3 for ^{56}Fe; A = atomic no.

Electron degeneracy:

$T << T_e$ = electron degeneracy temperature = $p_e^2/2m_e = 2.5 \times 10^9\ (\rho/\rho_s)^{2/3}$

p_e = electron Fermi momentum;

$\rho_s = ((m_e c)^3/3\pi^2\ \hbar^3) m_n A/Z \simeq 3 \times 10^5$ g/cm^3

$\rho >> \rho_s$ => electrons are relativistic.

At T= 10^8 K, degeneracy sets in at $\rho > 3 \times 10^4$ g/cm^3

The dark inside: no photons

Photon dispersion relation in matter

$$\omega_{\text{photon}}(k) = \sqrt{c^2k^2 + \omega_{\text{plasma}}^2}$$

where plasma frequency is given by

$$\omega_{\text{p}}^2 = \frac{4\pi n_e e^2}{m_e}$$

or

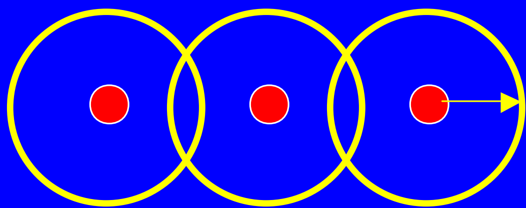
$$(\hbar\omega_{\text{p}})^2 = \frac{4}{3\pi}\frac{e^2}{\hbar c}\left(\frac{p_e}{m_e c}\right)^3 (m_e c^2)^2$$

At T = 10^8 K, **~** ω_p >> T for ρ >> 3 **£** 10^5 g/cm^3

Photon number $n_{\text{photons}} \sim e^{-\hbar\omega_p/T}$ greatly suppressed

Matter solidifies

$T_{melting}$ » $E_{binding}/\Gamma_m$ where Γ_m » 10^2



R_c

Wigner-Seitz cell containing one atom

$4\pi R_c^3/3 = 1/n_{atoms} = m_n A/\rho$

$$-E_b \simeq -\int_{\rm cell} d^3r \frac{Ze^2}{r} n_e + \frac{3}{5}\frac{Z^2e^2}{R_c} = -\frac{9}{10}\frac{Z^2e^2}{R_c}$$

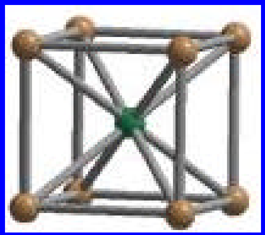
$$T_m \simeq \frac{9}{10}\frac{(Ze)^2}{\Gamma_m R_c} \simeq \frac{Z^{5/3}}{\Gamma_m}\frac{e^2}{\hbar c} m_e c^2 \left(\frac{\rho}{\rho_s}\right)^{1/3}$$

For Z = 26, $\Gamma_m = 10^2$, T_m » 10^8 K

Melt at ρ » 5 £ 10^7 g/cm^3,

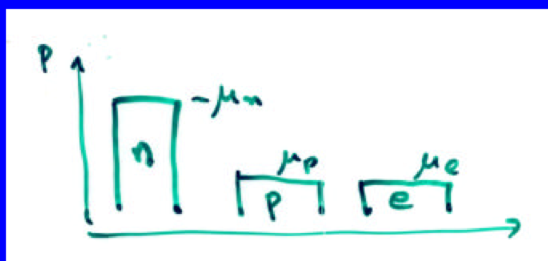
about 10m below surface

Form BCC lattice

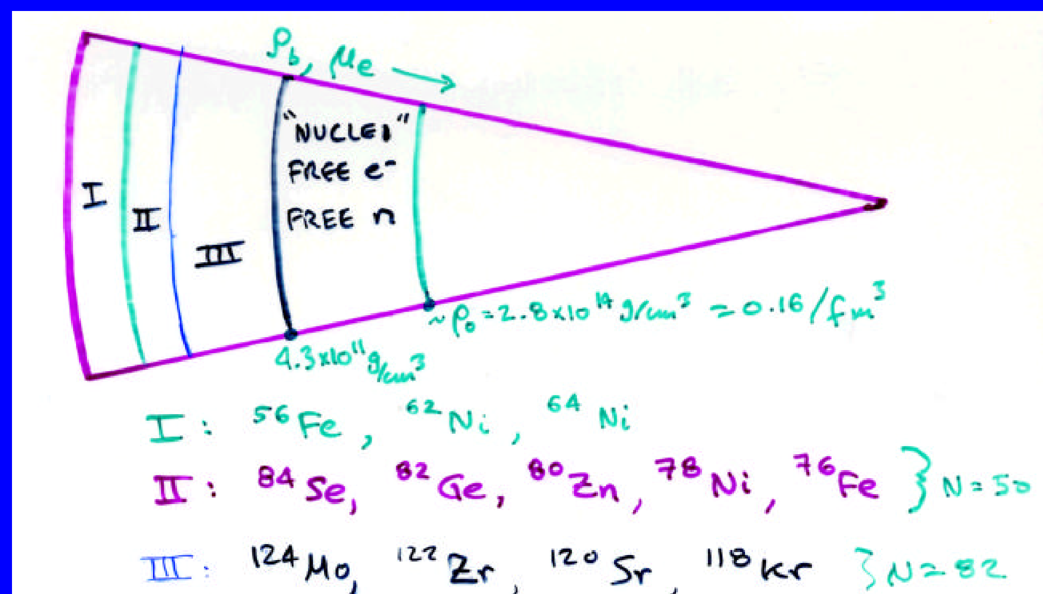


Nuclei before neutron drip

$e^- + p \longrightarrow n + \nu$: makes nuclei neutron rich
as electron Fermi energy increases with depth
$n \longrightarrow p + e^- + \bar{\nu}$: not allowed if e^- state already occupied



Beta equilibrium: $\mu_n = \mu_p + \mu_e$



Shell structure (spin-orbit forces) for very neutron rich nuclei?
Do N=50, 82 remain magic numbers?
Will be explored at rare isotope accelerators, RIA, RIKEN, GSI

Nuclear sizes: minimize energy

1) At fixed Z/A = x, balance nuclear Coulomb and surface energies

$E_{surface}$ » R_n^2 » $A^{2/3}$ R_n = nuclear radius

$E_{Coulomb}$ » Z^2/R_n » $Z^2/A^{1/3}$ » $x^2\ A^{5/3}$

Minimize surface + Coulomb per nucleon: $\frac{\partial}{\partial A}\left(\cdots A^{-1/3} + \cdots x^2 A^{2/3}\right) = 0$

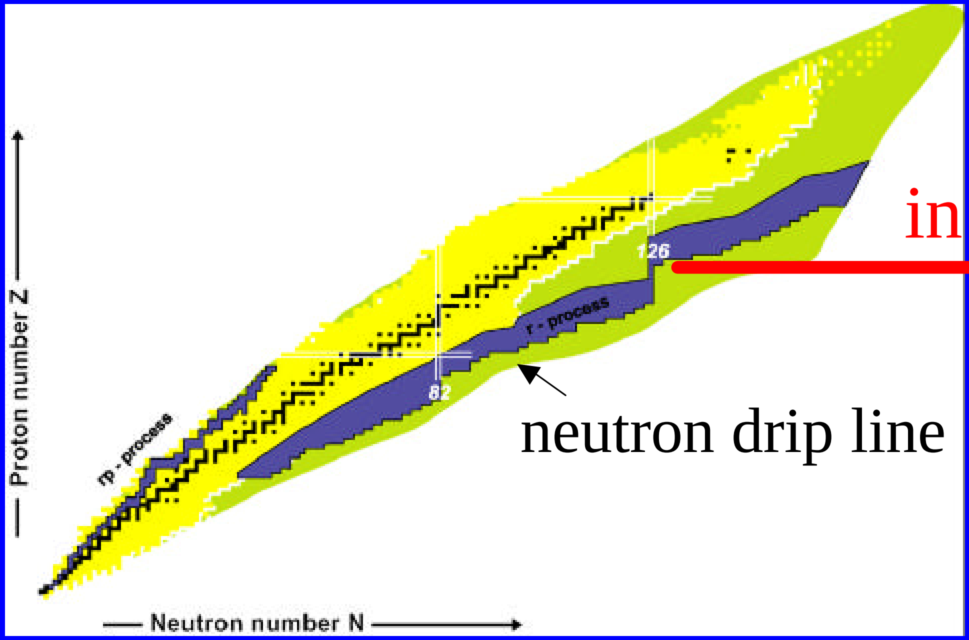
=> $E_{surface} = 2\ E_{Coul}$ A ' $12/x^2$ (cf. ^{56}Fe at x ' 1/2)

2) Best Z/A: No energy cost to convert n to $p+e^-$ (+neutrino)

=> beta equilibrium condition on chemical potentials: $\mu_n = \mu_p + \mu_e$

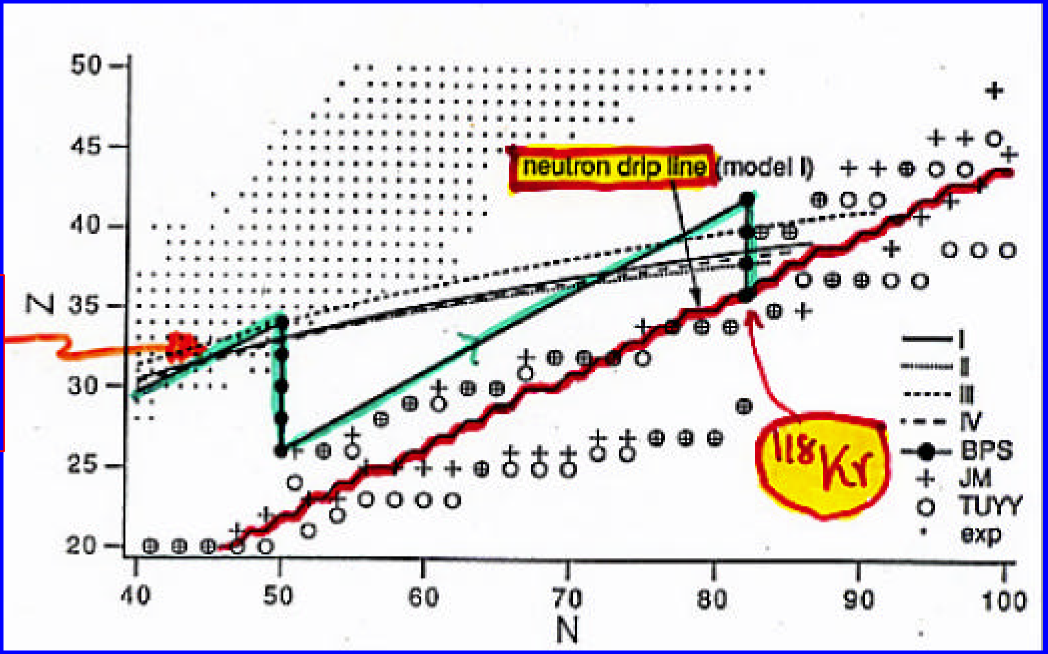
Valley of b stability

in neutron stars

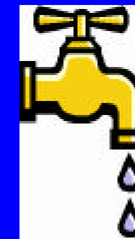


neutron drip line

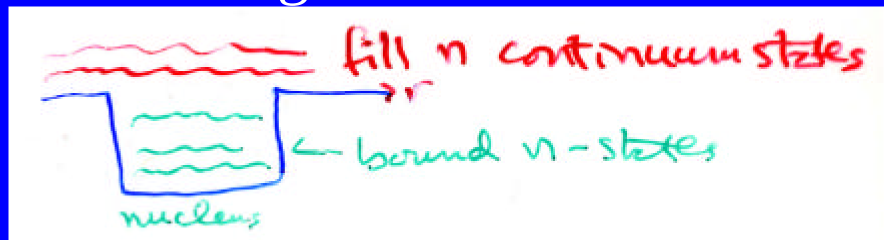
nuclei
before drip



Neutron drip

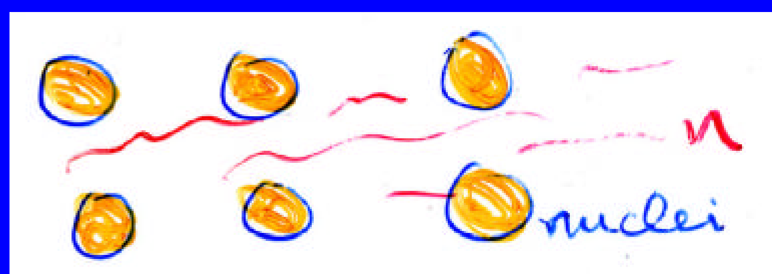


Beyond density ρ_{drip} » 4.3 £ 10^{11} g/cm^3 neutron bound states in nuclei become filled. Further neutrons must go into continuum states. Form degenerate neutron Fermi sea.



At drip point, $\mu_n = m_n c^2$. Beta equilibrium and $E_{surf} = 2\,E_{coul}$ =>
A ' 122, Z ' 39 =>
x ' 0.32, μ_e = 24.6 MeV, and ρ_{drip} = 4.3£10^{11}g/cm^3.

Neutrons in neutron sea are in equilibrium with those inside nucleus (common μ_n)



Protons never drip, but remain in bound states until nuclei merge into interior liquid.

Hartree-Fock nuclear density profiles

J. Negele and D. Vautherin, Nucl. Phys. A207 (1973) 298

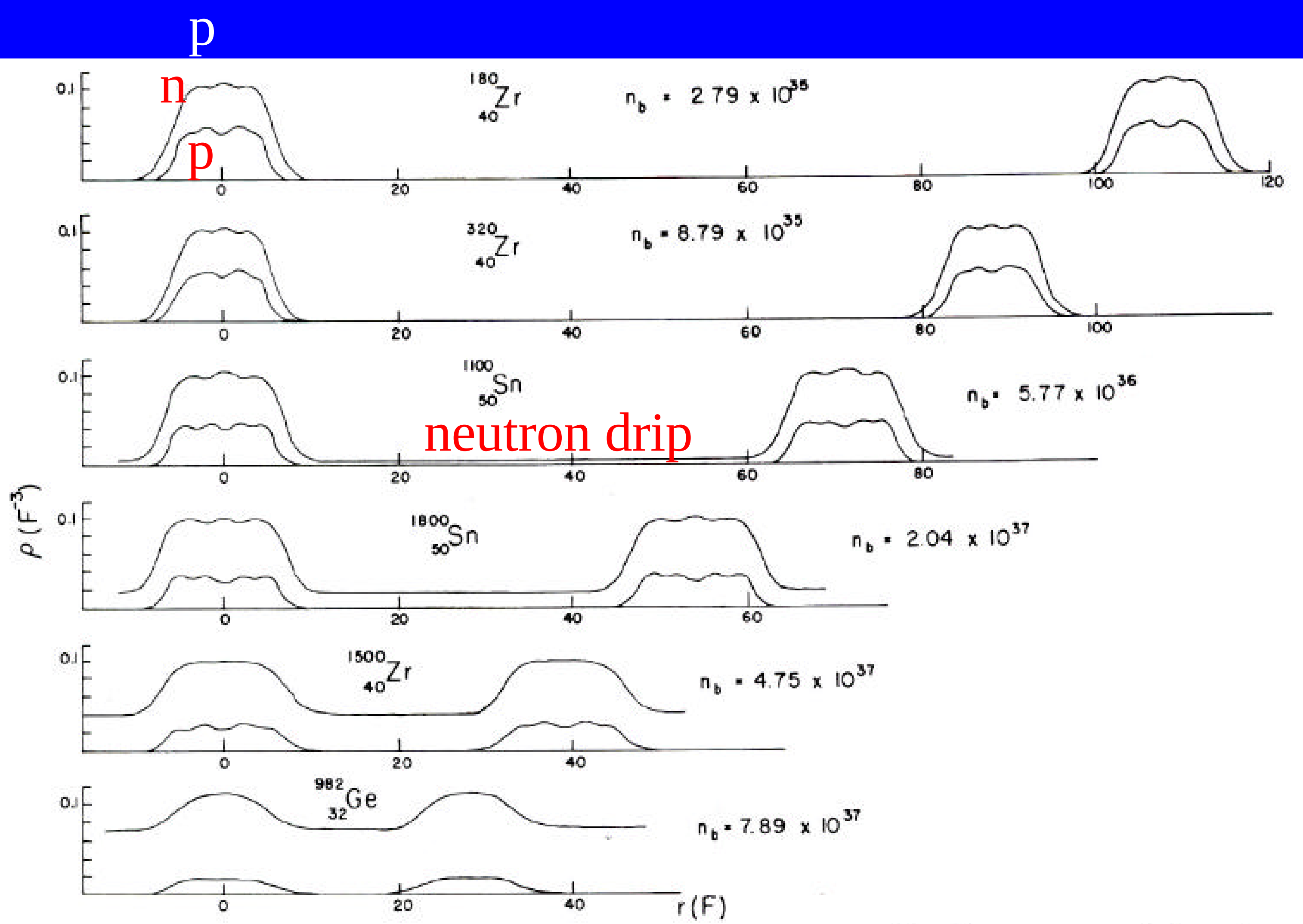
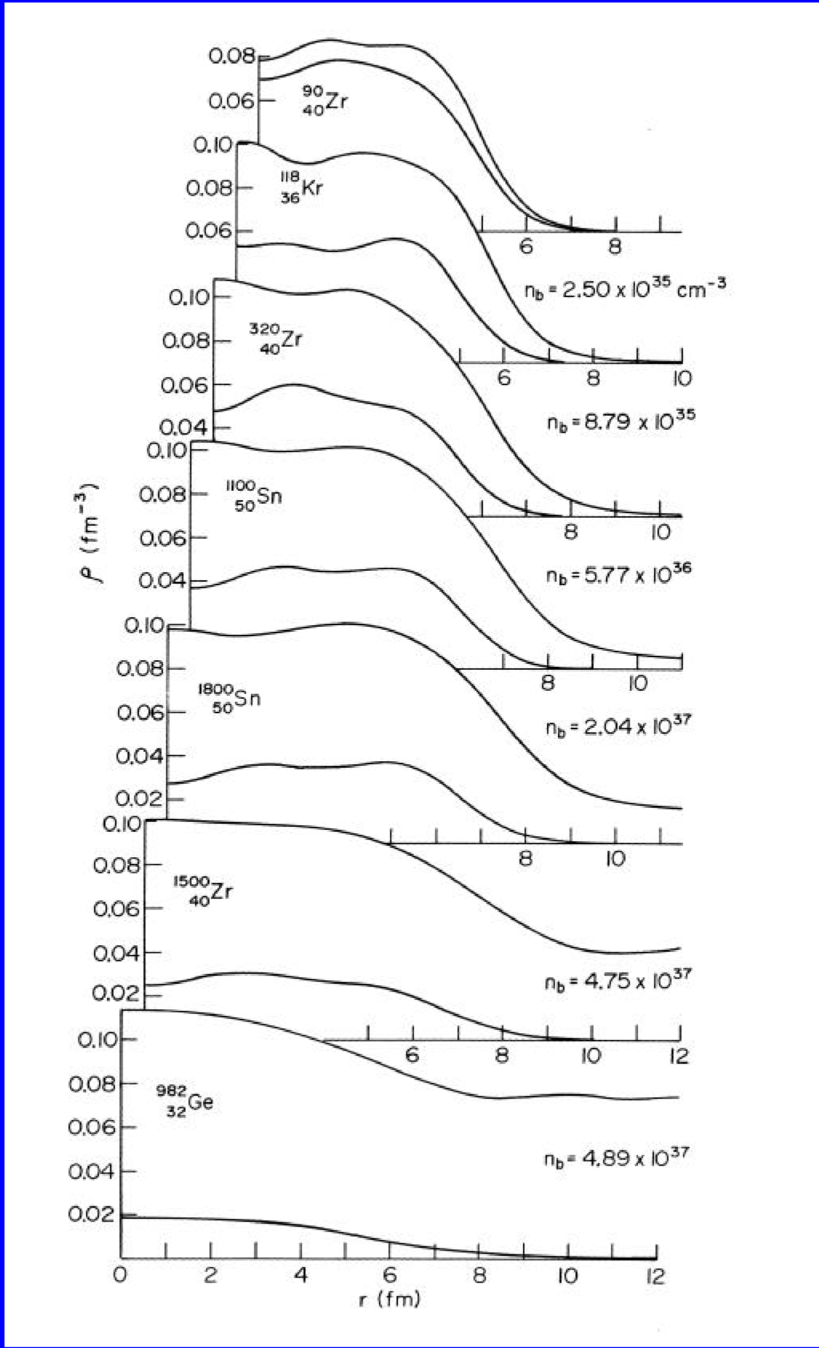


Figure 5 Density profiles of lattice unit cells in the crust for various average densities. The upper curve is the neutron number density; the lower is the proton number density; n_b denotes the average nucleon density, measured in nucleons/cm^3. The horizontal axis is distance in fm. From (27). (The density of nuclear matter is 1.7×10^{39} nucleons/cm^3.)

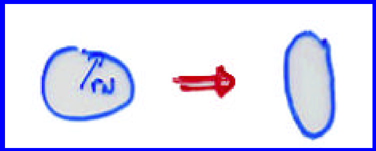
Cross sections of nuclei in crust



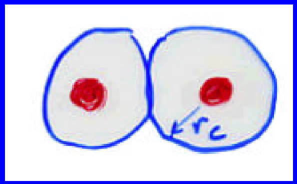
neutron drip

New states of nuclei in depths of neutron star crust

Bohr-Wheeler fission:
nuclei distort if $E^0_{coul} = 3Z^2e^2/5r_N > 2\,E_{surface}$
$\Rightarrow Z^2/A \gtrsim 50$



Above neutron drip: $E_{coul} = E^0_{coul}\{1-(3/2)(r_N/r_c)+(1/2)(r_N/r_c)^3\}$,
taking lattice Coulomb energy into account



r_N = radius of nucleus, r_c = radius of Wigner-Seitz cell

Surface vs. Coulomb energies:

$E_{surf} = 2E_{Coul} \Rightarrow$ fission if $E^0_{coul} > 2E_{surf} = 4E_{Coul}$

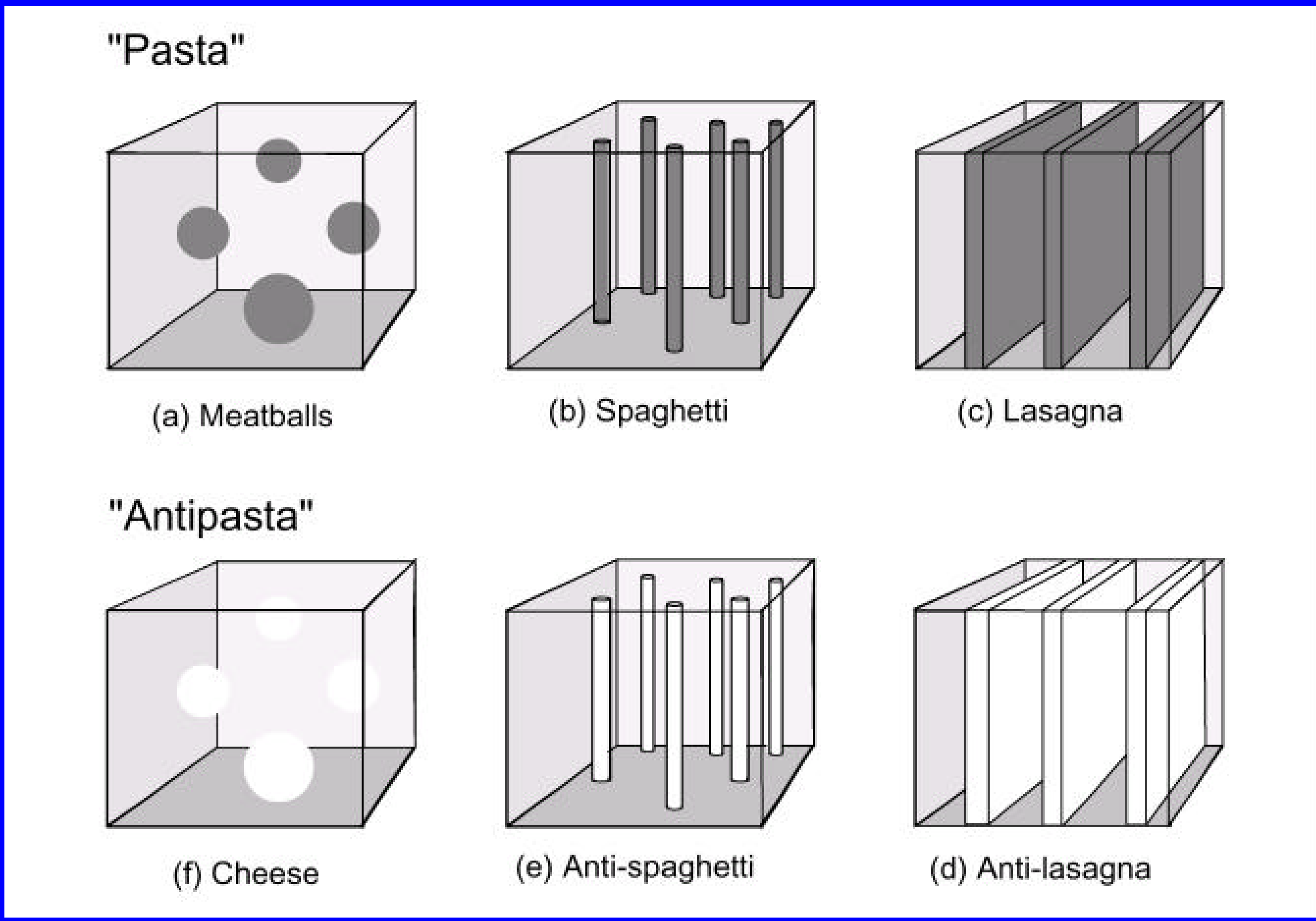
$\{1-(3/2)(r_N/r_c)+(1/2)(r_N/r_c)^3\} < 1/4 \quad \Rightarrow \quad r_N/r_c \gtrsim 1/2$

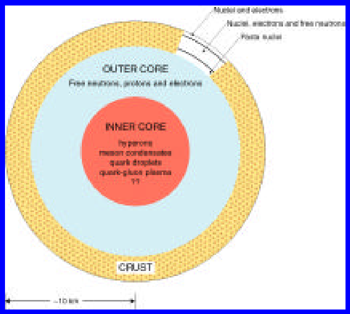
When nuclei fill » 1/8 of space => fission. Onset of non-spherical shapes.

Pasta Nuclei:

Lorentz, Pethick and Ravenhall
PRL 70 (1993) 379

over half the mass of the crust !!





Important effects on crust bremsstrahlung of neutrinos,
pinning of n vortices, ...

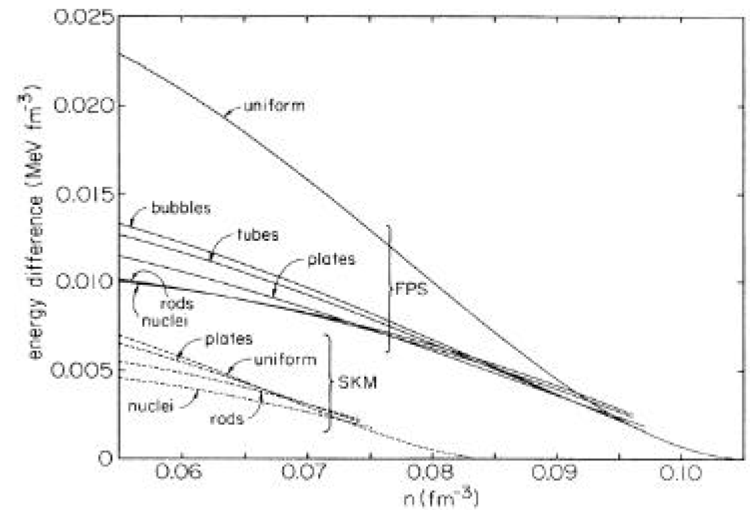


FIG. 1. Energies per unit volume as a function of density for the one-fluid phase, and the three-, two-, and one-dimensional nucleus phases, with (for FPS) the bubble (inverted structure) versions of the first two, after subtraction of the energy of the two-fluid phase, neglecting Coulomb and interface effects. The two nuclear interactions illustrated are SKM [6] and the version of FPS [8, 9] described in the text.

Energies of pasta phases

Density profile of neutron star

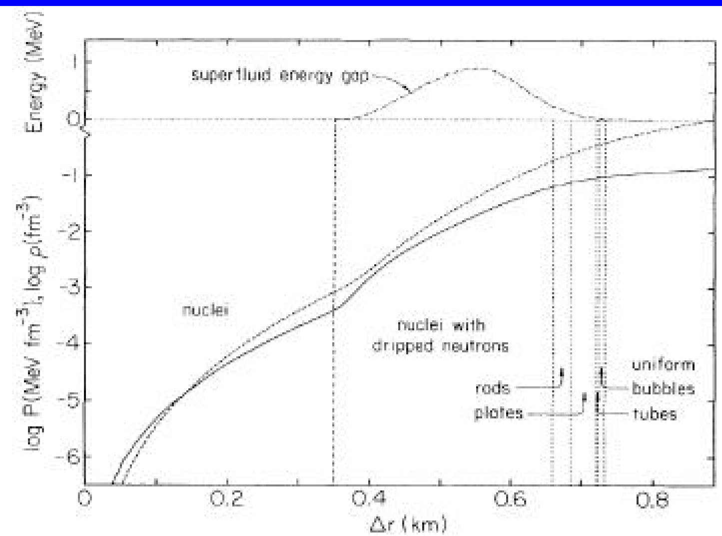


FIG. 2. Profile of a neutron star crust as given by FPS [8, 9]. The distance (in km) is measured from the surface. The solid line is density ρ/m_n, in fm^{-3}, and the dashed line is pressure, in MeV fm^{-3}, plotted logarithmically. Vertical lines indicate the phase boundaries described in the text. At the top is shown the superfluid energy gap [22].

Lorentz, Pethick and Ravenhall, PRL 70 (1993) 379

Simulations of pasta phase

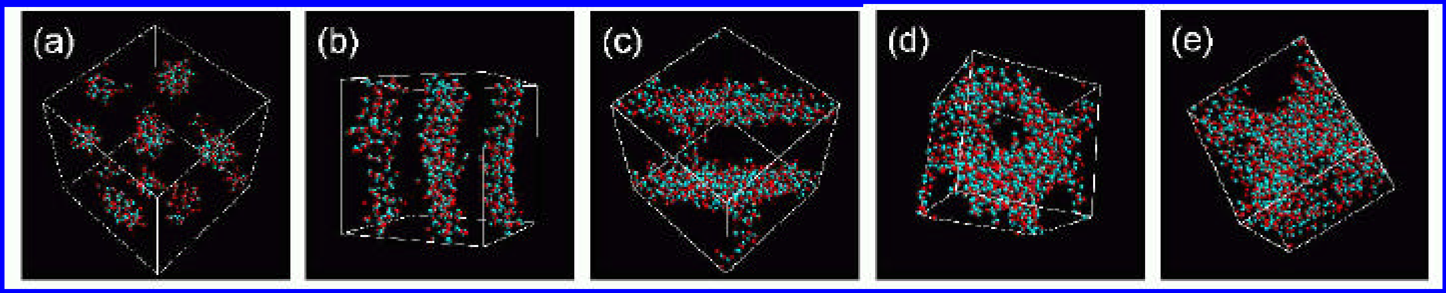
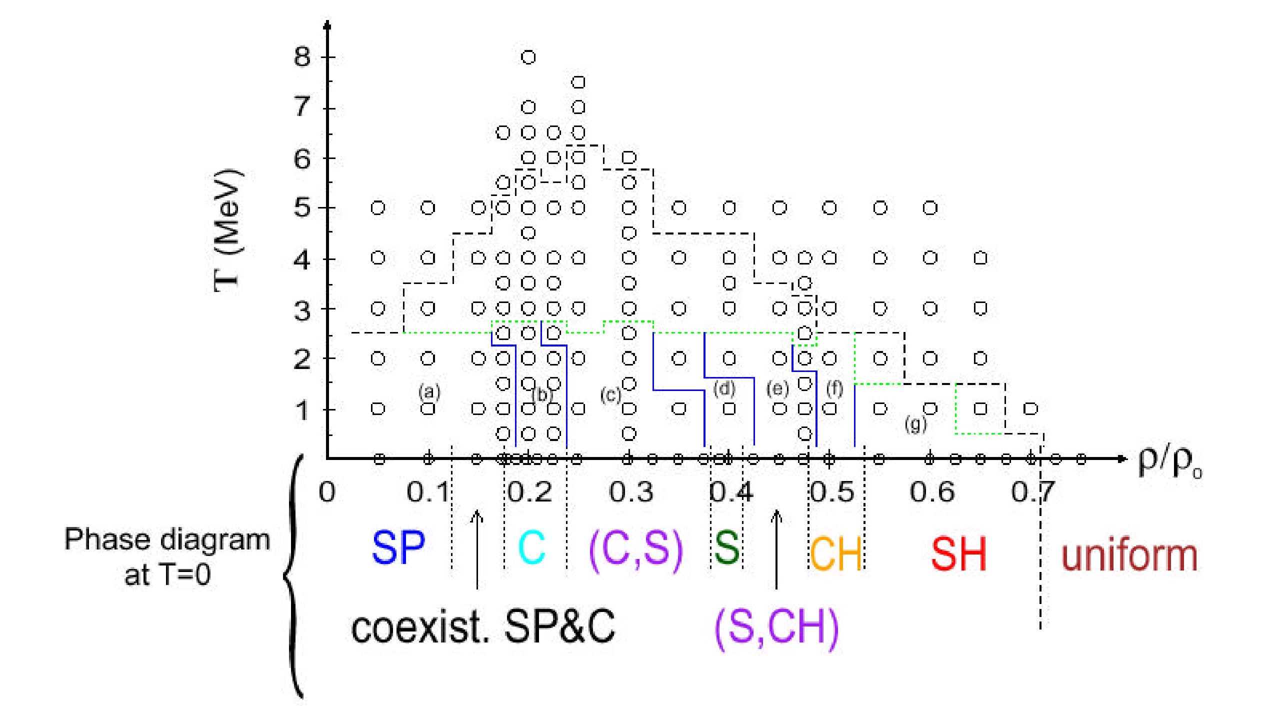


FIGURE 3. (Color) Nucleon distributions of the pasta phases in cold matter at $x = 0.5$; (a) sphere phase, $0.1\rho_0$ ($L_{\rm box}$ = 43.65 fm, $N = 1372$); (b) cylinder phase, $0.225\rho_0$ ($L_{\rm box} = 38.07$ fm, $N = 2048$); (c) slab phase, $0.4\rho_0$ ($L_{\rm box} = 31.42$ fm, $N = 2048$); (d) cylindrical hole phase, $0.5\rho_0$ ($L_{\rm box} = 29.17$ fm, $N = 2048$) and (e) spherical hole phase, $0.6\rho_0$ ($L_{\rm box} = 27.45$ fm, $N = 2048$), where $L_{\rm box}$ is the box size and N is the total number of nucleons. The whole simulation box is shown in this figure. The red particles represent protons and the green ones neutrons. Taken from Ref. [39].

G. Watanabe,
T. Maruyama,
K. Sato, K. Yasuoka,
T. Ebisuzaka.
PRL. 94 (2005) 031101



The Liquid Interior

Neutrons (likely superfluid) » 95% Non-relativistic
Protons (likely superconducting) » 5% Non-relativistic
Electrons (normal, T_c » $T_f\,e^{-137}$) » 5% Fully relativistic

Eventually muons, hyperons, and possibly exotica:

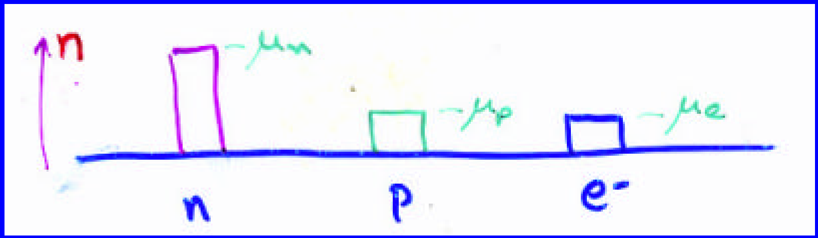
- pion condensation
- kaon condensation
- quark droplets
- bulk quark matter

n_0 = baryon density
in large nuclei ' 0.16 fm^{-3}
1fm = 10^{13}cm

Phase transition from crust to liquid at n_b ' 0.7 n_0 ' 0.09 fm^{-3}
or ρ = mass density » 2 £10^{14}g/cm^3

Why “neutron” star?

β equilibrium: $\mu_n = \mu_p + \mu_e$.
Charge neutrality: $n_p = n_e$



Non-interacting matter:

$\mu_n = p_n^2/2m_n$, $\mu_p = p_p^2/2m_p$, $\mu_e = cp_e = cp_p$

=> p_e/p_n ' $p_n/2m_n c$ => n_p/n_n ' $(p_n/2m_n c)^3$ ' 0.03

Mean field effects: $(p_n^2/2m_n) + V_n = (p_p^2/2m_p) + V_p + p_p c$

$V_p < V_n$ => n_p/n_n ' 0.05

Matter is primarily neutron liquid

Estimate the value of V_n-V_p to get n_p/n_n ' 0.05

Properties of matter near nuclear matter density

Determine N-N potentials from

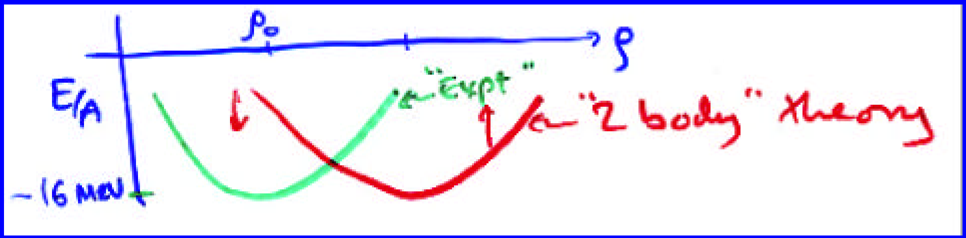
- scattering experiments E<300 MeV
- deuteron, 3 body nuclei (^3He, ^3H)

ex., Paris, Argonne, Urbana 2 body potentials

Solve Schrödinger equation by variational techniques

Large theoretical extrapolation from low energy laboratory nuclear physics at near nuclear matter density

Two body potential alone:



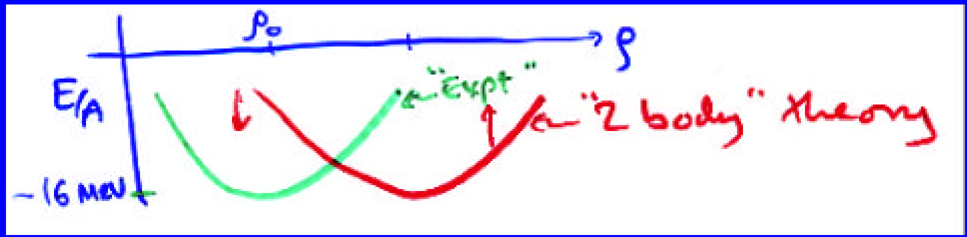
Underbind ^3H: Exp = -8.48 MeV, Theory = -7.5 MeV

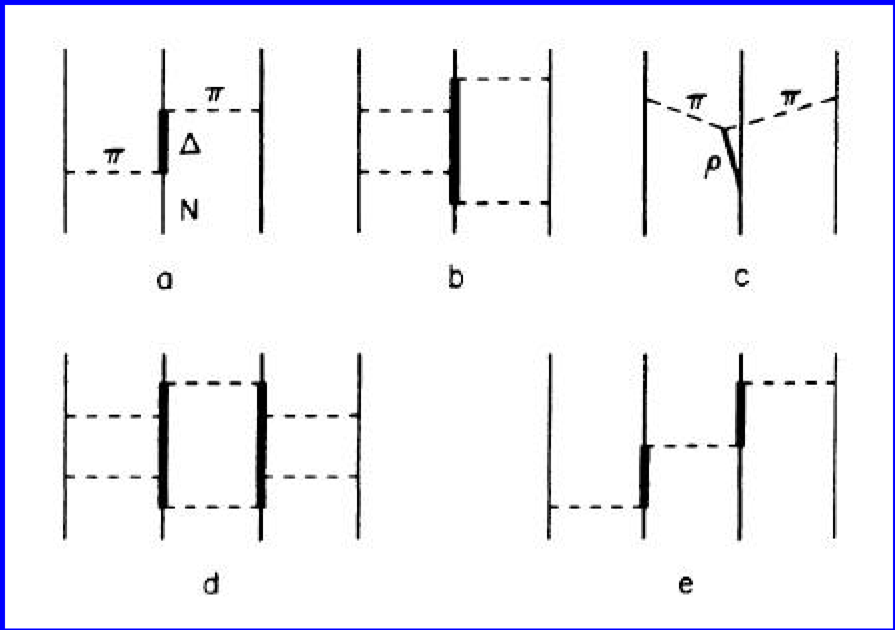
^4He: Exp = -28.3 MeV, Theory = -24.5 MeV

Importance of 3 body interactions

Attractive at low density

Repulsive at high density





Various processes that lead to three and higher body intrinsic interactions (not described by iterated nucleon-nucleon interactions.

Stiffens equation of state at high density

Large uncertainties

3-body forces in d+p elastic scattering: polarization transfer

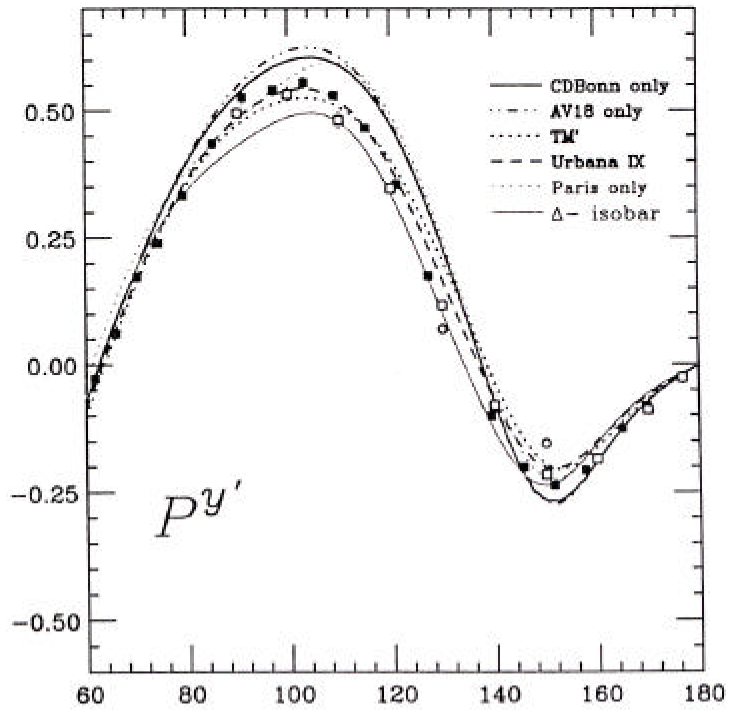


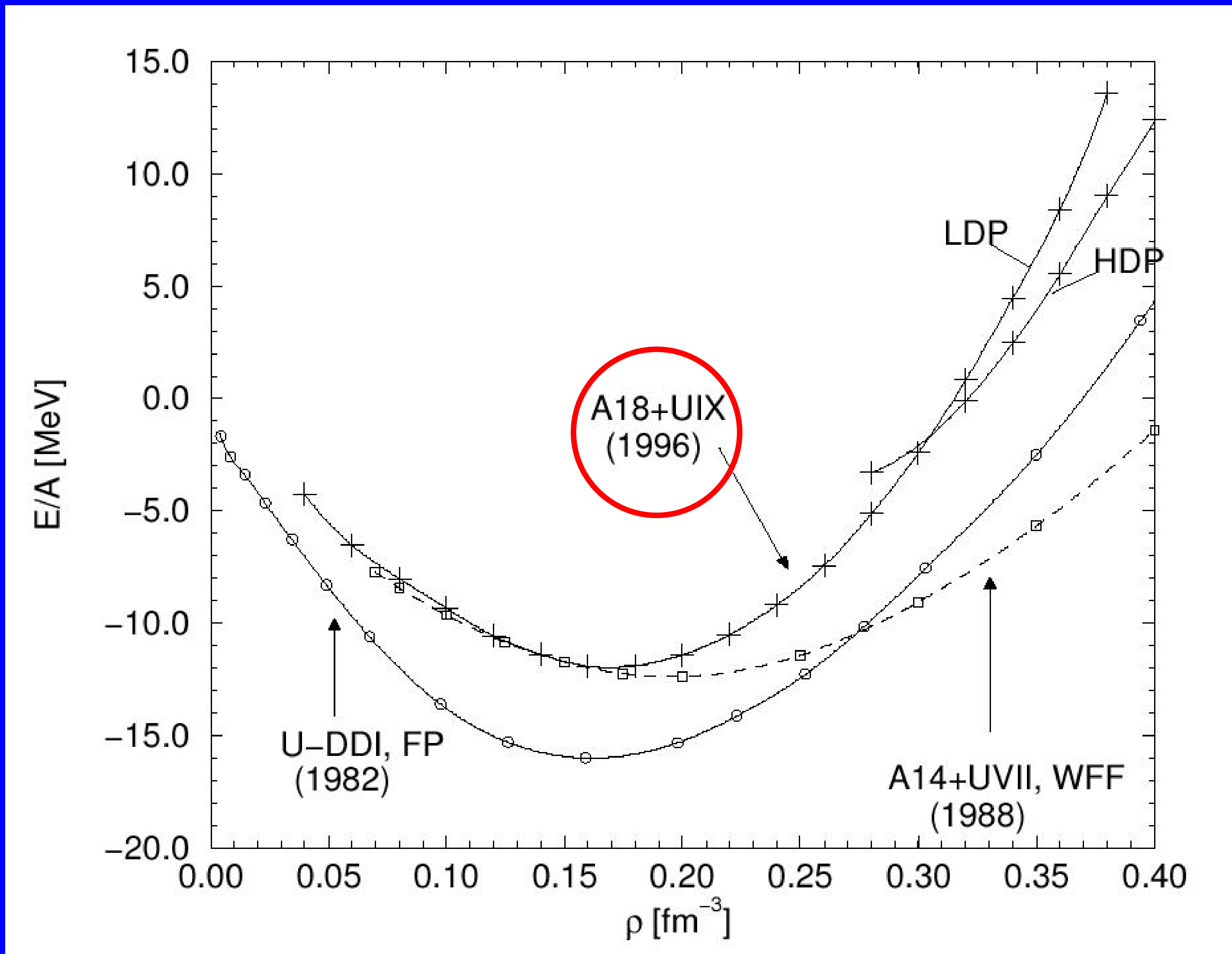
Figure 5.7: Induced Polarization $P^{y'}$. The predictions with and without Δ-isobar excitation are shown with red solid and dotted lines, respectively. A dotted line is the calculation with TM′ 3NF and the calculation with the Urbana IX 3NF is shown with a dashed line. Solid and dot-dot-dashed lines in the figure are the 2N force predictions using the CDBonn and AV18 potentials, respectively.

K. Sekiguchi
Riken

$\vec{d} + p \rightarrow \vec{p} + d$
at 270 MeV

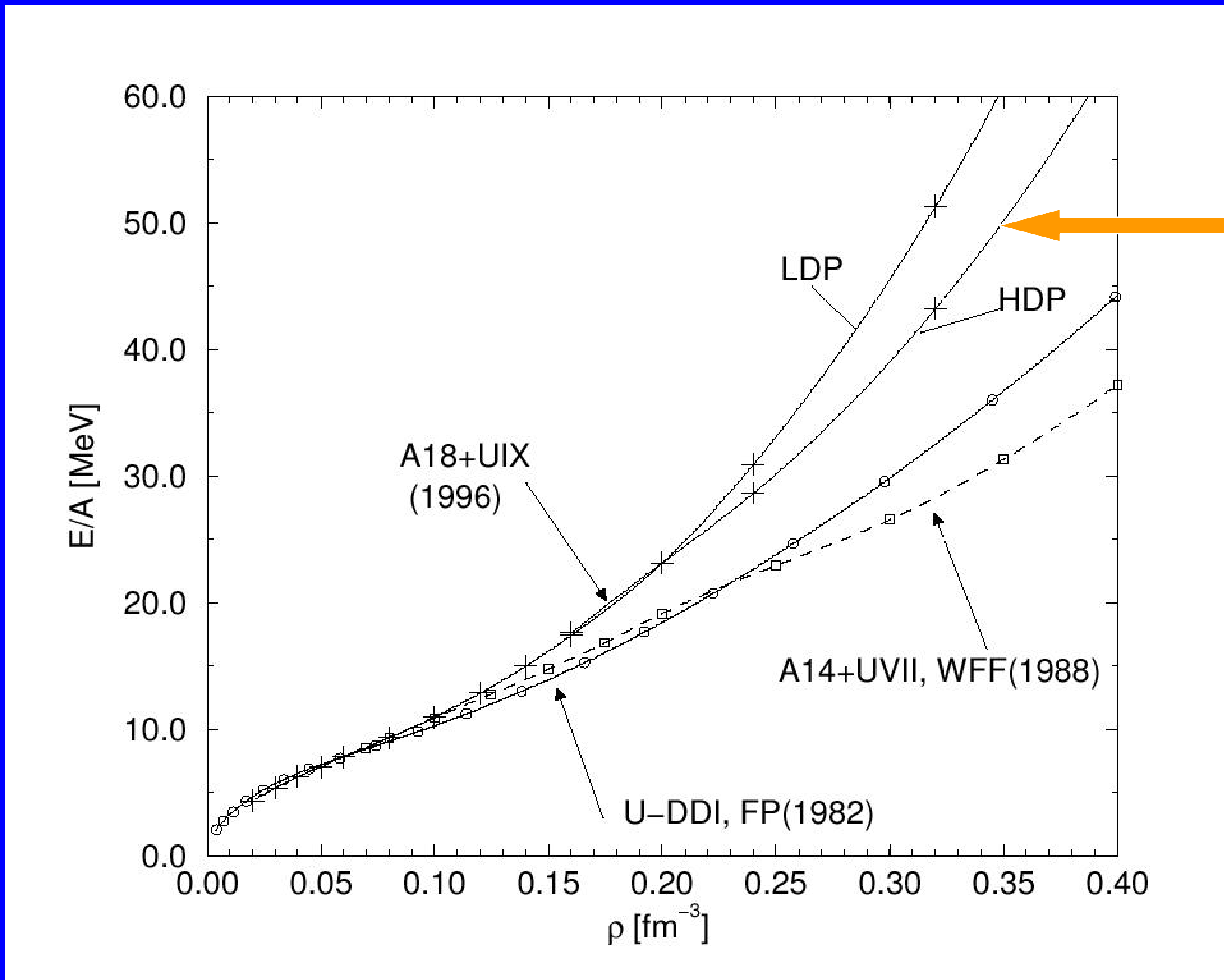
Energy per nucleon vs. baryon density in symmetric nuclear matter

Akmal, Pandharipande and Ravenhall, Phys. Rev. C58 (1998) 1804



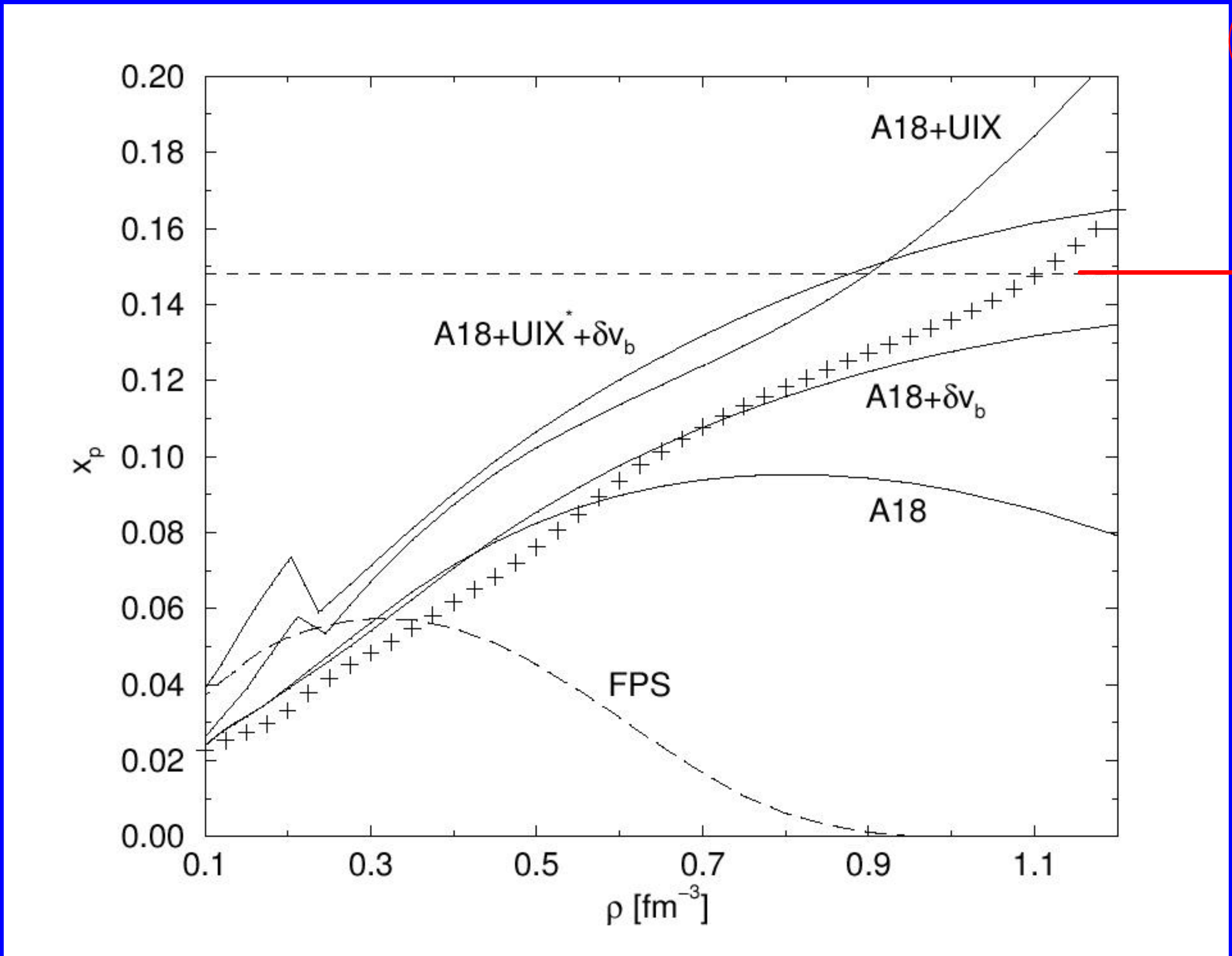
Energy per nucleon in pure neutron matter

Akmal, Pandharipande and Ravenhall, Phys. Rev. C58 (1998) 1804



h π^0 i
condensate

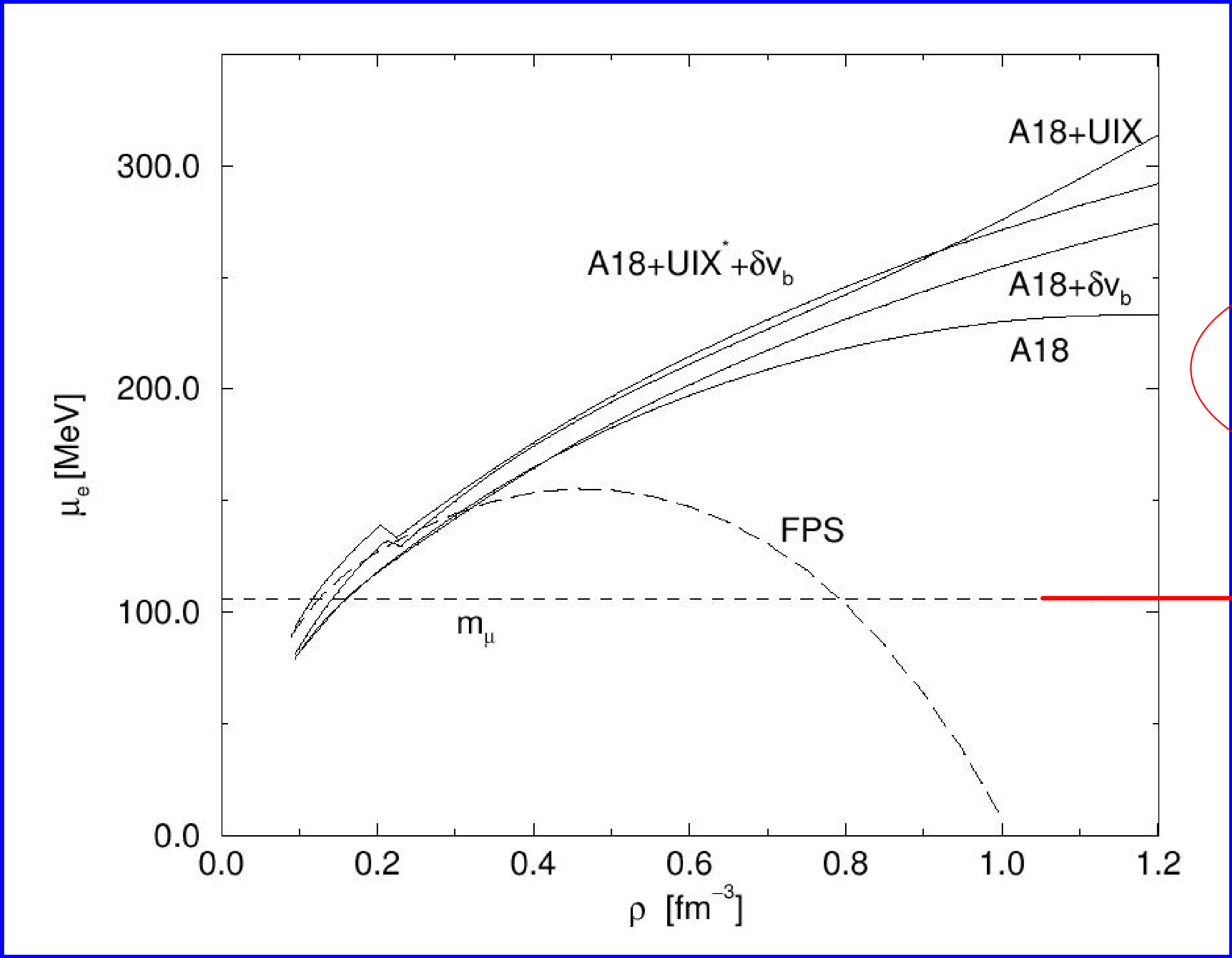
Proton fraction $x_p = n_p/(n_p+n_n)$ in matter in beta equilibrium



direct URCA cooling

Electron chemical potential vs. baryon density

$e^- \Rightarrow \mu^-$ when $\mu_e > m_\mu$



μ mesons present

Neutron Star Models

Equation of state: E = energy density = ρc^2

n_b = baryon density

$P(\rho)$ = pressure = $n_b^2\, \partial(E/n_b)/\partial n_b$

Tolman-Oppenheimer-Volkoff equation of hydrostatic balance:

$$\frac{\partial P(r)}{\partial r} = -\frac{G}{r^2}\frac{\left(\rho(r) + P(r)/c^2\right)}{1 - 2m(r)G/rc^2}\left(m(r) + 4\pi P(r) r^3/c^2\right)$$

general relativistic corrections

$$m(r) = \int_0^r \rho(r') 4\pi r'^2 dr'$$ = mass within radius r

1) Choose central density: $\rho(r=0) = \rho_c$

2) Integrate outwards until P=0 (at radius R)

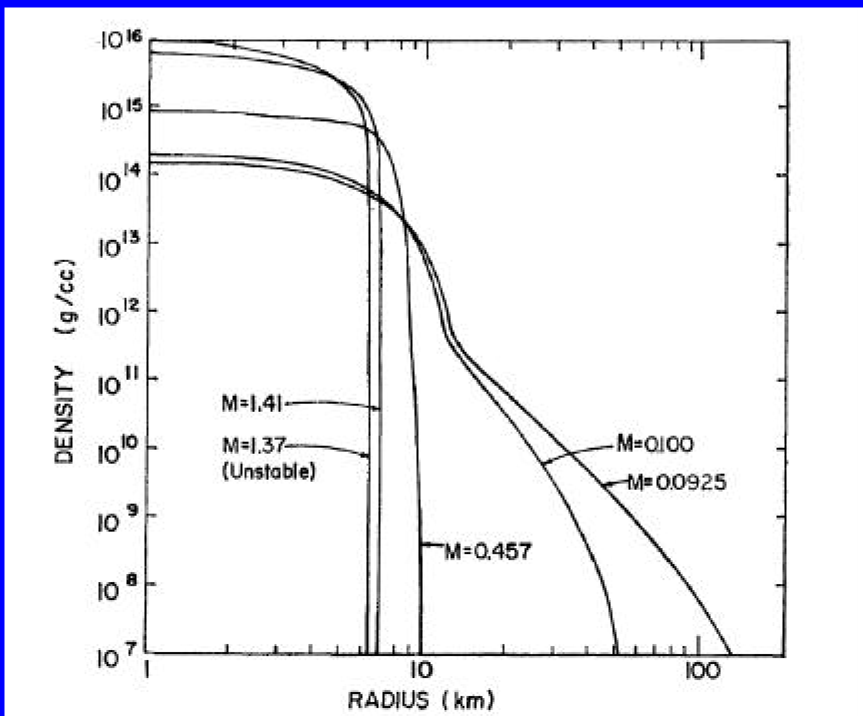
3) Mass of star

$$M = \int_0^R \rho(r) 4\pi r^2 dr$$

Problem: Solve the TOV equation analytically for constant mass density, ρ

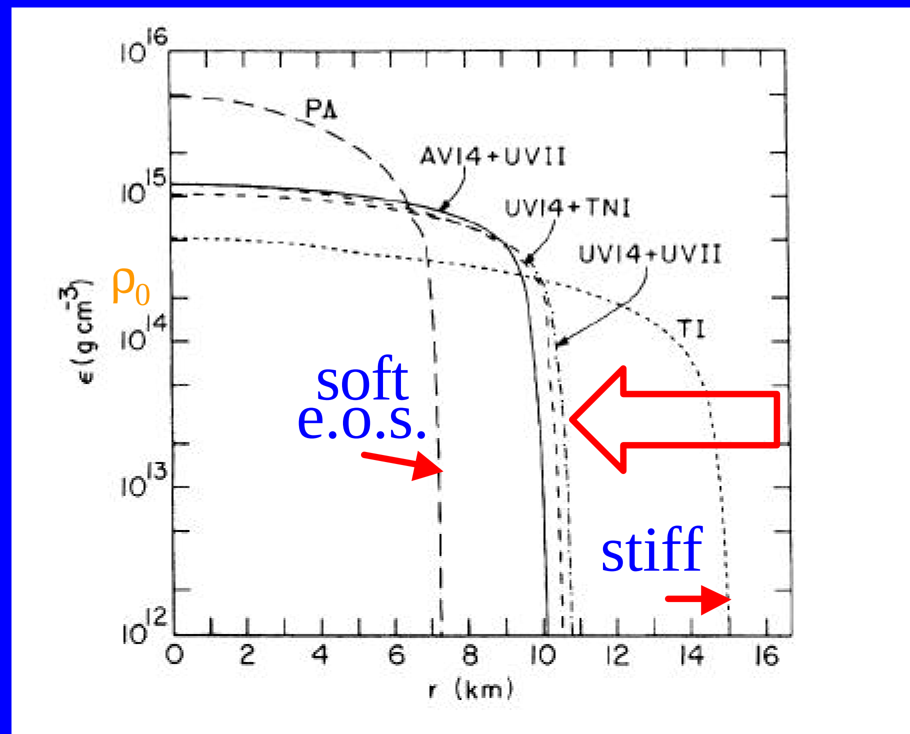
Mass density vs. radius inside neutron star for various equations of state

M= 0.009 - 1.4M–



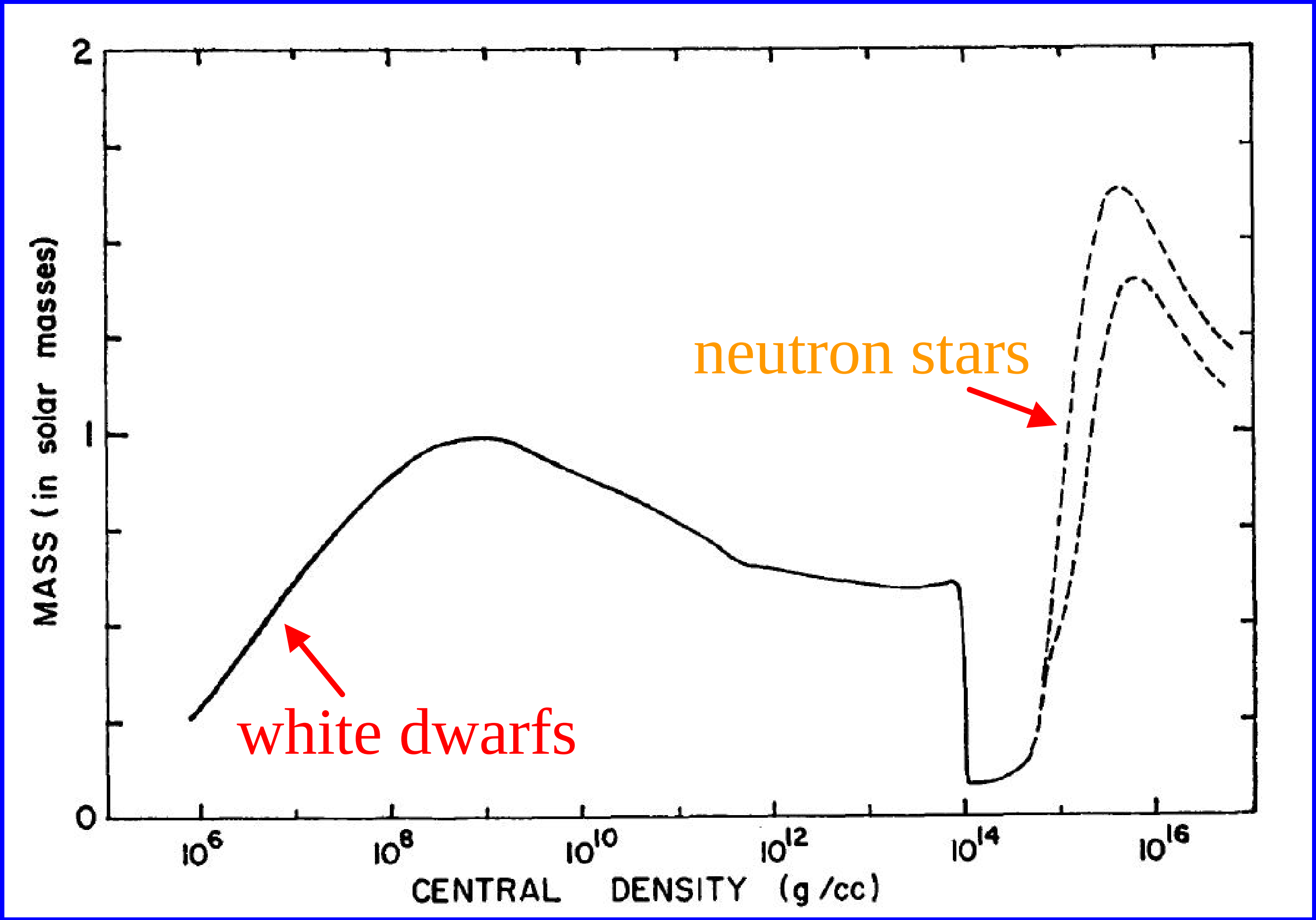
Baym, Pethick & Sutherland, Ap.J 170, 299 (1971)

M=1.4M–



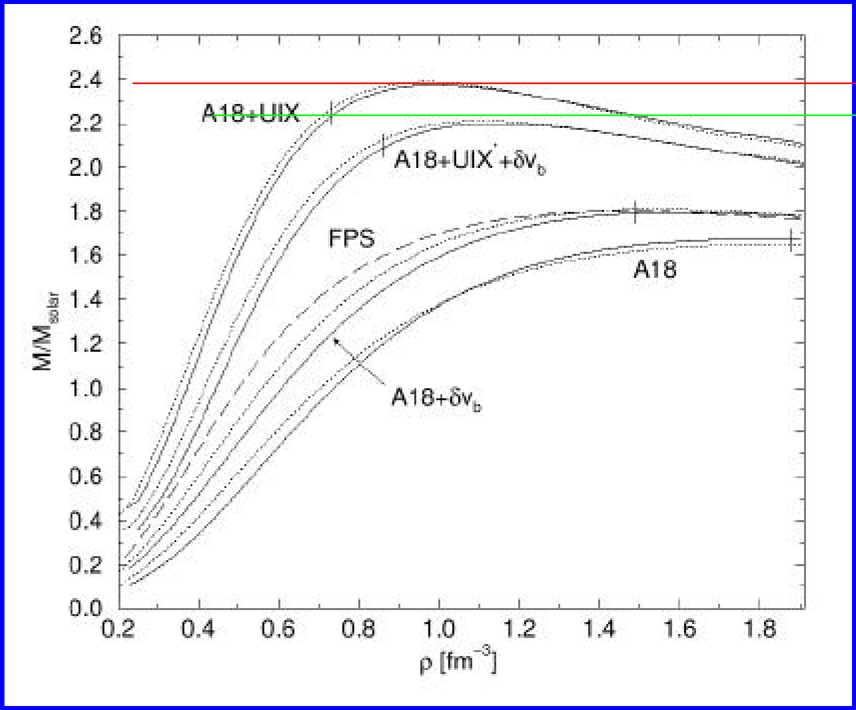
Wiringa, Faks, & Fabrocini, PR C38 (1988)

Families of cold condensed objects: mass vs. central density

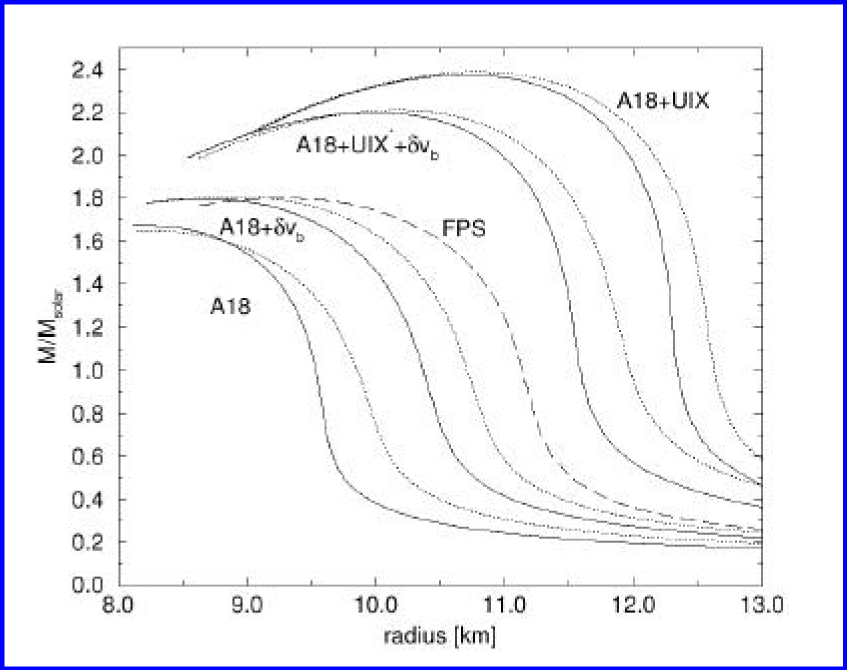


BPS, Ap.J 170, 299 (1971)

Maximum neutron star mass



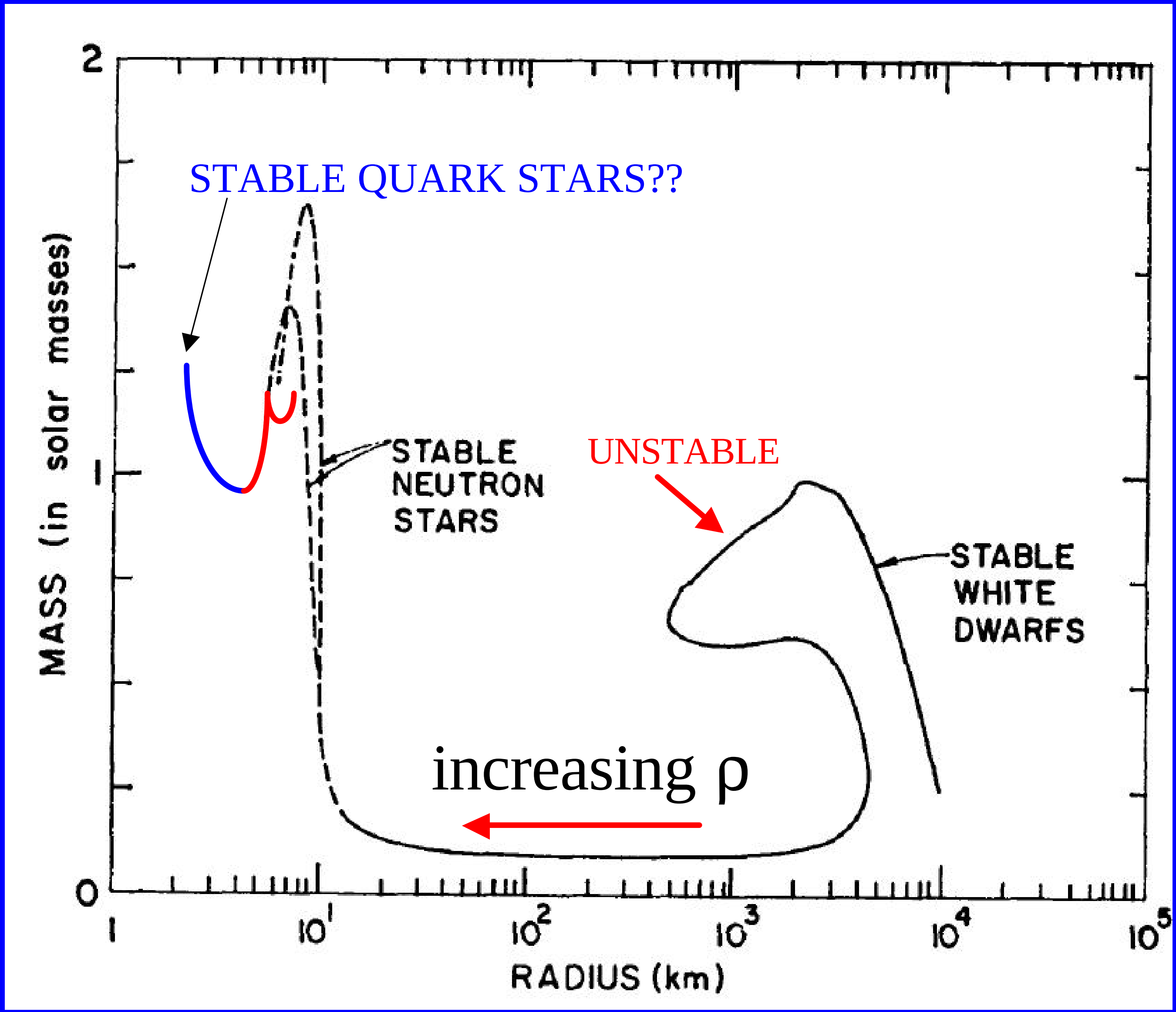
Mass vs. central density



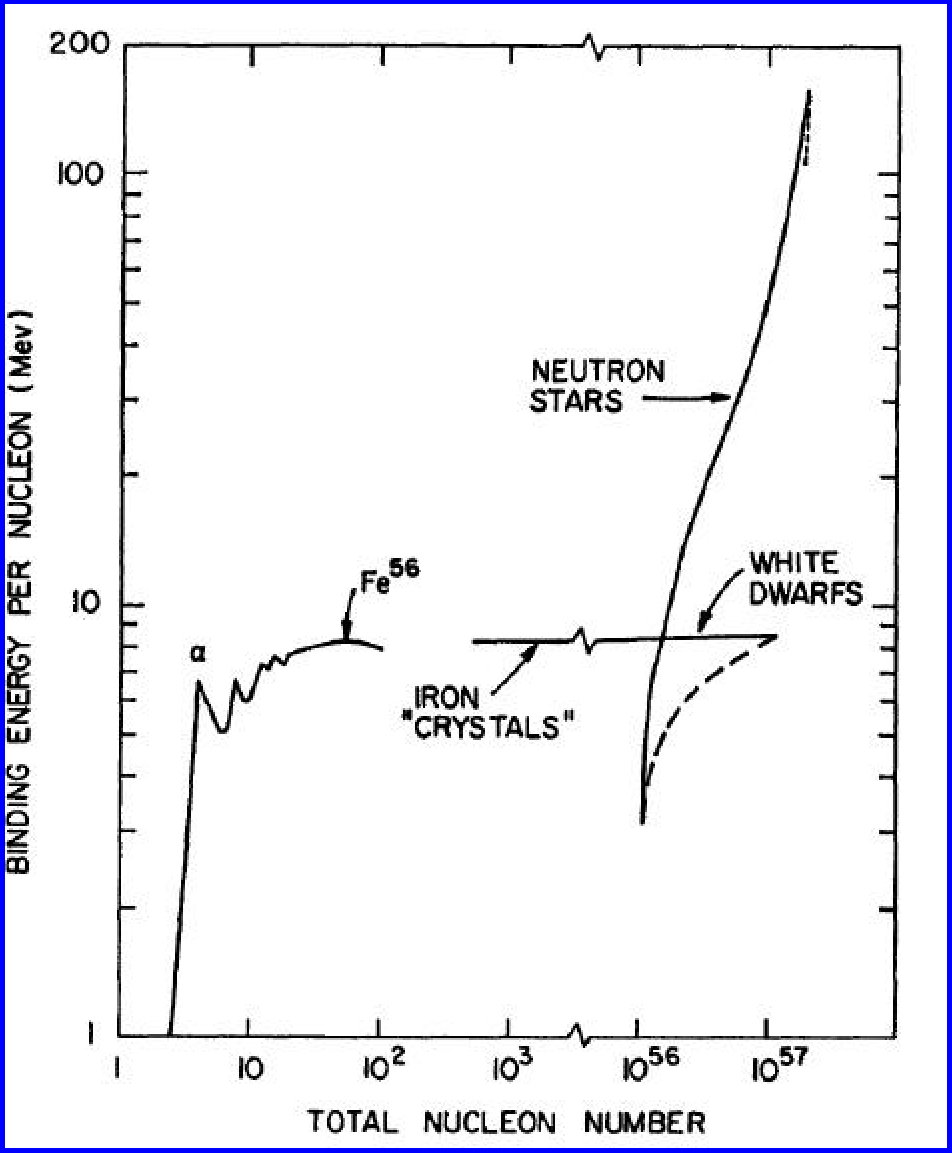
Mass vs. radius

Akmal, Pandharipande and Ravenhall, 1998

Mass vs. radius, and stability



Binding energy per nucleon (MeV) vs. nucleon number



BPS, Ap.J 170, 299 (1971)