SUMMARY OF PRIMORDIAL DENSITY PERTURBATIONS GENERATED AT INFLATION

* ORIGIN: VACUUM FLUCTUATIONS OF INFLATON FIELD, FROZEN IN WHEN CROSSING OUT HORIZON

$$\langle 0 | \phi^2(\vec{x}) | 0 \rangle = \int \left[\frac{H^2}{2\pi} \right] \frac{dk}{k}$$

EVALUATED AT HORIZON CROSSING,

$$\frac{k}{a(t)} = H$$

$$\boxed{\delta\phi = \frac{H}{\sqrt{2\pi}}}$$ IN LOGARITHMIC INTERVAL OF k

* Density perturbations

$$\frac{\delta\rho}{\rho} = \frac{H}{\dot{\varphi}} \delta\varphi = \frac{H^2}{\sqrt{2\pi}} \cdot \frac{1}{\dot{\varphi}}$$

$$\Downarrow$$

$$\Delta_s \equiv \frac{\delta\rho}{\rho} = \frac{H^2}{2\pi\dot{\varphi}} = 3\,\frac{H^3}{V'}$$

$$\boxed{\frac{\delta\rho}{\rho} = 3 \cdot \left(\frac{8\pi}{3}\right)^{3/2} \frac{V^{3/2}}{M_{Pl}^3 V'}} \sim 10^{-5}$$

$\Downarrow$

FLAT INFLATON POTENTIAL $V(\varphi)$

Need $\Delta(k) \sim 10^{-5}$

$$\Downarrow$$

$$\frac{1}{M_{Pl}^3}\left[\frac{V^{3/2}}{V'}\right]_{\varphi \sim M_{Pl}} \sim 10^{-7}$$

$\boxed{\text{PROBLEM}}$

NOT QUITE CORRECT: INTERESTING MODES EXIT HORIZON BEFORE $\varphi \sim M_{Pl}$

WANT VERY FLAT POTENTIAL.

E.G. $V = \lambda\varphi^4 \Rightarrow \dfrac{V^{3/2}}{V'} = \dfrac{\sqrt{\lambda}}{4}\varphi^3$

$$\Rightarrow \sqrt{\lambda} \sim 10^{-6} \Rightarrow \lambda \sim 10^{-12} \quad (!).$$

HOMEWORK # 2
FOR BETTER ESTIMATE

TILT IN SPECTRUM.

$$\Delta_S(k) = \frac{H^2}{2\pi\dot{\varphi}}$$ AT TIME WHEN $\dfrac{k}{a(t_k)} = H(t_k)$

Let k_* BE SOME CENTRAL VALUE,
SAY, $\dfrac{k_*}{a_0} \sim \dfrac{1}{1\,\text{Mpc}}$ TODAY

THEN $\dfrac{k_*}{a(k_*)} = H$

FOR DIFFERENT k WE ALSO HAVE $\dfrac{k}{a(k)} = H$

varies fast

Varies slowly

$$\Downarrow$$

$$\frac{a(k)}{a(k_*)} = \frac{k}{k_*}$$

PROBLEM FOR TODAY

(PREPARATION FOR HOMEWORKS # 2, 3)

CONSIDER INFLATON WITH POTENTIAL

$$V(\varphi) = \lambda \varphi^4$$

Estimate

AT WHAT VALUE OF φ A MODE OF PRESENT WAVELENTH 1 Mpc EXITS THE HORIZON AT INFLATION.

ASSUME THAT THE UNIVERSE REHEATS INSTANTANEOUSLY AFTER INFLATION.

$$t_k = t_{k_*} + \Delta t_k$$

times of HORIZON CROSSING

$\Downarrow$

$$\frac{k}{k_*} = \frac{a(k)}{a(k_*)} = e^{H \Delta t_k} \Rightarrow \boxed{\Delta t_k = \frac{1}{H} \ln \frac{k}{k_*}}$$

$$\Delta(k) = \frac{H^2}{2\pi \dot{\varphi}}(t_k)$$

$$= \frac{H^2}{2\pi\dot{\varphi}}(t_*) \left[1 + \Delta t_k \cdot \frac{\partial \ln \frac{H^2}{2\pi\dot{\varphi}}}{\partial t} \right]$$

$$\Delta(k) = \Delta(k_*) \left[1 + \left(\frac{2\dot{H}}{H} - \frac{\ddot{\varphi}}{\dot{\varphi}} \right) \frac{1}{H} \ln \frac{k}{k_*} \right]$$

$$\Delta_s^2(k) = \Delta_s^2(k_*) \left(\frac{k}{k_*} \right)^{n_s - 1}$$

↑ def. of spectral index

$$\text{TILT} \equiv n_s - 1 = 2 \left(2 \frac{\dot{H}}{H^2} - \frac{\ddot{\varphi}}{H \dot{\varphi}} \right)$$

⇐ BOTH NEGATIVE FOR POWER-LAW $V(\varphi)$

$2 \times (-\epsilon)$ $\quad$ $-(\epsilon - \eta)$

$$n_s - 1 = -6\epsilon + 2\eta \quad < 0$$

FOR POWER LAW $V(\varphi)$

SMALL

Homework #2 for numerics

GRAVITATIONAL WAVES:

$$h = \frac{\sqrt{32\pi}}{M_{Pl}} \tilde{h}.$$

$\tilde{h}$ BEHAVES EXACTLY LIKE MASSLESS SCALAR FIELD, FOR EVERY POLARIZATION

$\Downarrow$

INFLATION GENERATES STOCHASTIC PRIMORDIAL GRAVITY WAVES WITH (ALMOST) FLAT SPECTRUM AND AMPLITUDE

$$\Delta_h = \sqrt{2}\,\frac{\sqrt{32\pi}}{M_{Pl}}\,\Delta_\phi$$

$\Delta_\phi \leftarrow \frac{H}{2\pi}$

two polarizations

$$\Delta_T \equiv \boxed{\Delta_h = \frac{4}{\sqrt{\pi}}\,\frac{H}{M_{Pl}}}$$

MAY BE QUITE LARGE, SAY, 10^{-5}!

tensor modes

RECALL $\Delta_S = \frac{H^2}{2\pi\dot{\varphi}}$; $\dot{\varphi} = -\frac{V'}{3H}$

$\Downarrow$

$$\frac{\Delta_T}{\Delta_S} = 8\sqrt{\pi}\,\frac{V'}{3M_{Pl}H^2} = \frac{1}{\sqrt{\pi}}\,\frac{M_{Pl}\cdot V'}{V} = 4\sqrt{\epsilon}$$

$H^2 \leftarrow \frac{8\pi}{3}V$

one-field inflation: SMALL, BUT NOT TINY

$$\boxed{\frac{\Delta_T^2}{\Delta_S^2} = 16\epsilon}$$

$\Delta_T \sim 10^{-5} \div 10^{-6}$

- Gravity waves produce CMB anisotropy AND POLARIZATION.

EFFECTS HAVE <u>NOT</u> BEEN OBSERVED (YET)

$$\Downarrow \text{ OBS.}$$

$$\Delta_T \lesssim 10^{-5}$$

$$\Downarrow$$

$$\frac{H_{infl}}{M_{Pl}} < 10^{-5}$$

INFLATION HAPPENED WELL BELOW PLANCK SCALE : EXPANSION RATE <u>small</u> compared to $\frac{1}{t_{Pl}}$

$$H^2 = \frac{8\pi}{3} G\rho \Rightarrow \rho < 10^{-11} M_{Pl}^4$$

ENERGY DENSITY AT INFLATION <u>SMALL</u> compared to Planck density

NO PROBLEM WITH QUANTUM GRAVITY

THUS, INFLATION PRODUCES

- ALMOST FLAT SPECTRUM OF DENSITY PERTURBATIONS

NB: NOTHING PARTICULARLY EXCITING, IF EXACTLY FLAT $\Leftrightarrow$ SCALE - INVARIANCE

HARRISON; ZELDOVICH LONG BEFORE INFLATION INVENTED

- GAUSSIAN PERTURBATIONS

NB: RATHER GENERIC EITHER: SUPERHORIZON MODES DO NOT TALK TO EACH OTHER

BUT: OBSERVATION OF NON-GAUSSIANITY WOULD DISFAVOR INFLATION.

- SMALL TILT IN SPECTRUM

RED IN SINGLE-FIELD MODELS, BUT SEE HOMEWORK #3

- APPRECIABLE STOCHASTIC GRAVITY WAVES; PRIMORDIAL SPECTRUM ALMOST, BUT NOT EXACTLY, FLAT

OBSERVATION OF ANY OF THESE $\Downarrow$ STRONG SUPPORT OF INFLATION

NOT GENERIC PREDICTION, SEE HOMEWORK # 3.

- ADIABATIC MODE OF $\frac{\delta\rho}{\rho}$

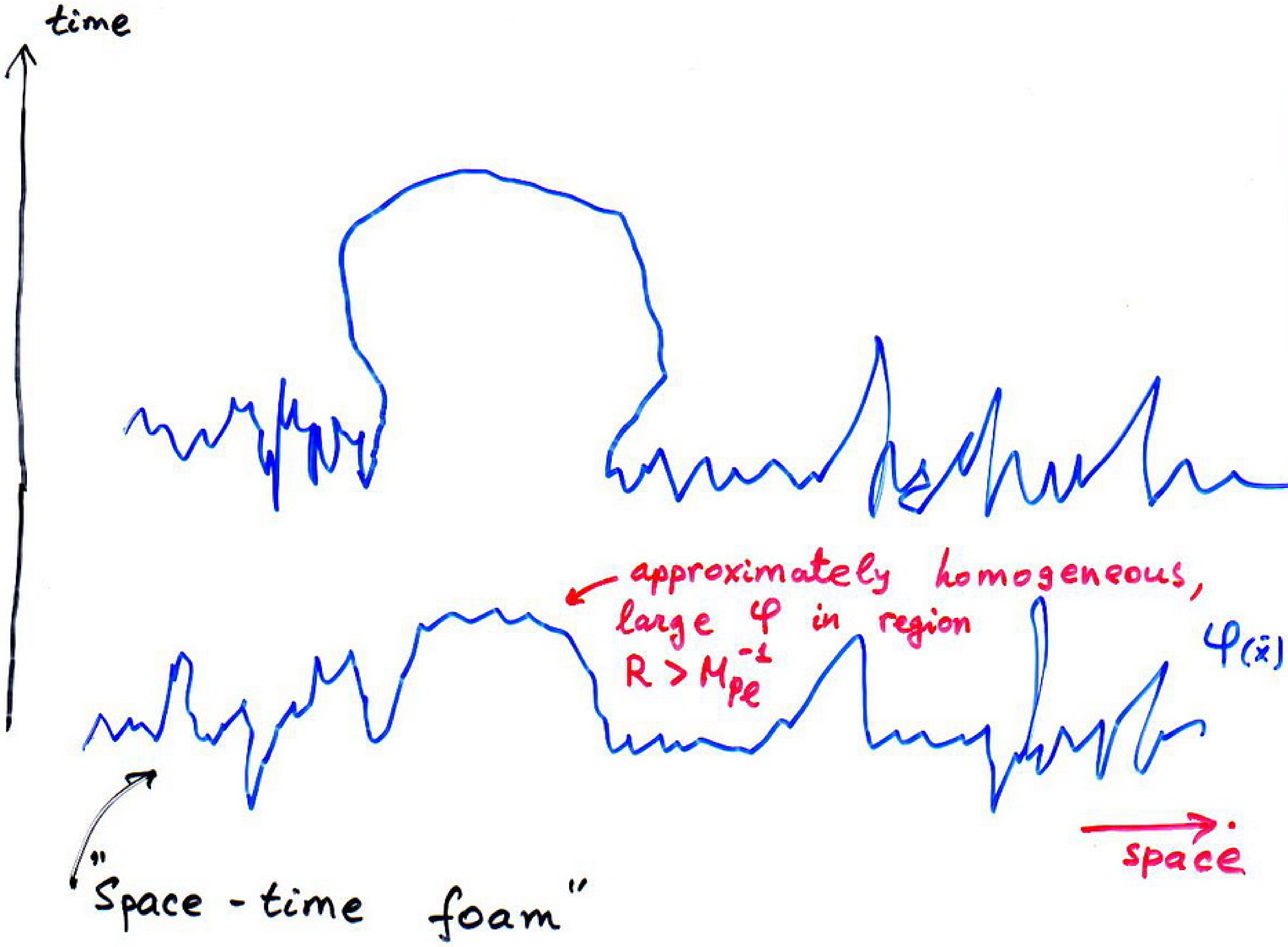
ALSO NOTHING VERY EXCITING.

ADMIXTURE OF NON-ADIABATIC MODE WOULD BE POSSIBLE TO INCORPORATE INTO INFLATIONARY MODELS.

MORE ABOUT INFLATION

- HOW DOES INFLATION BEGIN?

ROUGH IDEAS ONLY...



SHALL WE EVER KNOW? HARDLY...

- It is likely that there are many regions in UNIVERSE THAT INFLATED, OR STILL INFLATE

Many of these, including ours, are MUCH larger than VISIBLE part of the UNIVERSE.

SHALL WE EVER KNOW?

MORE "PRACTICAL" QUESTIONS:

Q1. • WHAT ARE IMPLICIT ASSUMPTIONS ENTERING CALCULATIONS OF DENSITY PERTURBATIONS AND/OR GRAVITY WAVES?

• GENERAL RELATIVITY AT SHORT DISTANCES

$$\ell \sim H_{infl}^{-1} \quad (\text{say, } 10^{-28}\text{ cm})$$

• CONVENTIONAL RELATIVISTIC QUANTUM FIELD THEORY DOWN TO THESE DISTANCES.

QUITE STRONG!

• In fact, even STRONGER ASSUMPTION: CONVENTIONAL QUANTUM FIELD THEORY AT DISTANCES SHORTER THAN ℓ_{pl}.

"TRANS-PLANCKIAN PROBLEM"

FOR GIVEN CONFORMAL momentum k WAVELENGTH $\lambda = \frac{1}{p} = \frac{a}{k}$

is VERY SHORT AT EARLY STAGE OF INFLATION

$$\lambda(t_i) \sim \frac{a_i}{k} \sim \frac{a_{end}}{k} \cdot \underbrace{\frac{a_i}{a_{end}}}_{10^{-10^6}} \lll \ell_{pl}$$

YET ONE ASSUMES THE MODE IS IN VACUUM STATE!

NO PROBLEM FOR PERFECTLY LORENTZ-INVARIANT THEORY

LORENTZ BOOSTS

SHORT DISTANCES $\Longleftrightarrow$ LONG DISTANCES

WHAT IF LORENTZ-INVARIANCE DOES NOT HOLD AT SHORT DISTANCES,

$$\ell \lesssim M_{Pl}^{-1} \equiv \ell_{Pl} = 10^{-33}\,\text{cm}$$

OR even

$$\ell \lesssim H^{-1} \qquad (\sim 10^{-28}\,\text{cm, say}) \;?$$

A1. SURPRISINGLY ENOUGH: NOTHING DRAMATIC in many cases.

Example: LORENTZ-VIOLATING DISPERSION RELATION FOR INFLATON

$$\omega = \omega(p) \neq |\vec{p}|$$

ASSUME $\omega(p)$ monotonic function

BEFORE HORIZON CROSSING (DROPPING 2's, π's)

$$\phi = \int \frac{d^3k}{\sqrt{\omega(p)}\;a^{3/2}(t)} \left[e^{i\vec{k}\vec{x} - i\int \omega dt} a_k + h.c. \right]$$

$$p = \frac{k}{a(t)}$$

HORIZON CROSSING

STANDARD

$$\omega\left(\frac{k}{a}\right) = H \qquad\qquad \frac{k}{a} = H$$

$$\Downarrow$$

$$a = k\, f(H) \qquad\qquad a = \frac{k}{H}$$

$$f(H) = \frac{1}{H}$$

$$\Downarrow$$

$$\sqrt{\omega}\; a^{3/2} = \sqrt{H} \cdot k^{3/2} f^{3/2}(H) \qquad\qquad \frac{1}{H} \cdot k^{3/2}$$

AFTER CROSSING OUT HORIZON

$$\phi = \int \frac{d^3k}{k^{3/2}} \underbrace{\frac{1}{\sqrt{H}\, f^{3/2}(H)}}_{\text{STANDARD: } H} \left(a_k^+ + a_k\right)$$

STANDARD: H^2

$$\Downarrow$$

$$\langle 0|\phi^2|0\rangle = \int \frac{d^3k}{k^3} \frac{1}{H f^3(H)}$$

SLOWLY VARYING $H \Rightarrow$ ALMOST FLAT SPECTRUM AGAIN
ALBEIT WITH DIFFERENT
AMPLITUDE AND TILT

BUT THESE DEPEND ON $V(\varphi)$
in STANDARD THEORY.

STEEP $f(H)$:
DIFFERENT STORY

$$\Downarrow$$

YET CONCEIVABLE: STRONG DEVIATIONS FROM FLAT SPECTRUM IN LARGE INTERVAL OF WAVELENGTHS

GRAVITY WAVES !

Q2: WHAT PHYSICS IS BEHIND THE INFLATON FIELD?

- FLAT POTENTIAL AT LARGE FIELDS, $\varphi \sim M_{Pl}$

- UNLIKELY THAT φ IS A COMPOSITE FIELD ??

  Pseudo-Goldstone?!

- SUPERSYMMETRIC THEORIES OFTEN HAVE ALMOST FLAT DIRECTIONS ... BUT SUPERGRAVITY / QUANTUM GRAVITY EFFECTS TEND TO DESTROY FLATNESS AT LARGE φ

$$\Delta V_{SUGRA}(\varphi) \simeq m^4 \Big[\underbrace{\ldots \left(\frac{\varphi}{M_{Pl}}\right)^2 + \ldots}_{\text{ORDER 1 coefficients.}} \left(\frac{\varphi}{M_{Pl}}\right)^4 + \ldots$$

NOT TRACTABLE AT $\varphi \gtrsim M_{Pl}$ IN THE GENERAL CASE.

In most cases even worse: POTENTIAL NOT FLAT ENOUGH AT $\varphi \gtrsim M_{Pl}$

$\Downarrow$

TOO LARGE DENSITY PERTURBATIONS.

HOMEWORK # 2

FOR POWER-LAW POTENTIAL OF INFLATON,

$$V(\varphi) = \varkappa \varphi^\alpha$$

- CALCULATE THE AMPLITUDE OF DENSITY PERTURBATIONS AT COSMOLOGICALLY INTERESTING SCALES, SAY, $\lambda \sim 1$ Mpc. Estimate $\varkappa$.

Hint: take into account that perturbations at these scales cross out horizon quite some time before inflation ends.

ASSUME THAT UNIVERSE GETS INTO THERMAL EQUILIBRIUM IMMEDIATELY AFTER INFLATION ENDS. SHOW THAT RELAXING THIS ASSUMPTION DOES NOT DRAMATICALLY CHANGE RESULTS

- CALCULATE SCALAR SPECTRAL INDEX, $(n_s - 1)$.
- Give numerical RESULTS FOR $(n_s - 1)$ AND $\varkappa$ for

$$\alpha = 2, \ 4 \text{ and } 6$$

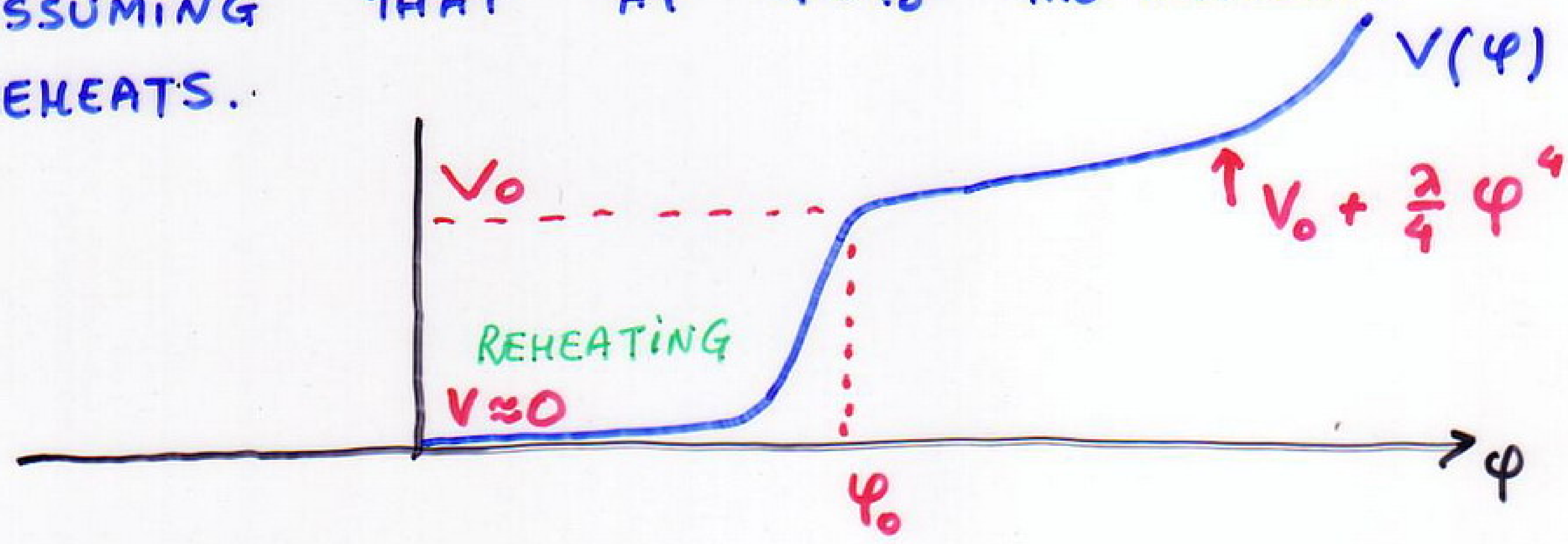
- CALCULATE THE AMPLITUDE OF GRAVITY WAVES. GIVE NUMERICAL RESULTS FOR SAME α.

HOMEWORK #3

CONSIDER INFLATION DRIVEN BY SCALAR FIELD WITH POTENTIAL

$$V(\varphi) = \begin{cases} \frac{\lambda}{4}\varphi^4 + V_0 & \varphi > \varphi_0 \\ \approx 0 & \varphi < \varphi_0 \end{cases}$$

ASSUMING THAT AT $\varphi < \varphi_0$ THE UNIVERSE REHEATS.



CHOOSE $\varphi_0 > M_{Pl}$ AND $V_0 \gg \lambda \varphi_0^4$

WHERE φ_0 AND V_0 ARE FREE PARAMETERS.

- SHOW THAT INFLATION TAKES PLACE ALL THE WAY DOWN TO $\varphi = \varphi_0$
- CALCULATE AMPLITUDES OF DENSITY PERTURBATIONS AND GRAVITY WAVES.
- CALCULATE SCALAR SPECTRAL INDEX $(n_s - 1)$
- CAN ONE ADJUST PARAMETERS IN SUCH A WAY THAT $\delta\rho/\rho \sim 10^{-5}$ AND $|n_s - 1| \lesssim 10^{-2}$?

HOMEWORK #3, cont'd

- WITH THIS CHOICE OF PARAMETERS, IS THE AMPLITUDE OF GRAVITY WAVES LARGER OR SMALLER THAN IN ONE-FIELD MODELS WITH POWER-LAW POTENTIALS?

- SHOW THAT THE SAME DYNAMICS EFFECTIVELY OCCURS IN TWO-FIELD MODEL: SCALAR FIELDS φ AND χ; SCALAR POTENTIAL

$$V(\varphi,\chi) = \frac{\lambda_1}{4}\left(\chi^2 - \chi_0^2\right)^2 + \frac{\lambda_2}{2}\varphi^2\chi^2 + \frac{\lambda_3}{4}\varphi^4$$

WITH APPROPRIATELY CHOSEN PARAMETERS χ_0, λ_1, λ_2, λ_3. WHAT CHOICE OF PARAMETERS IS "APPROPRIATE" IN THIS SENSE?