CMB: Overview and Current Status

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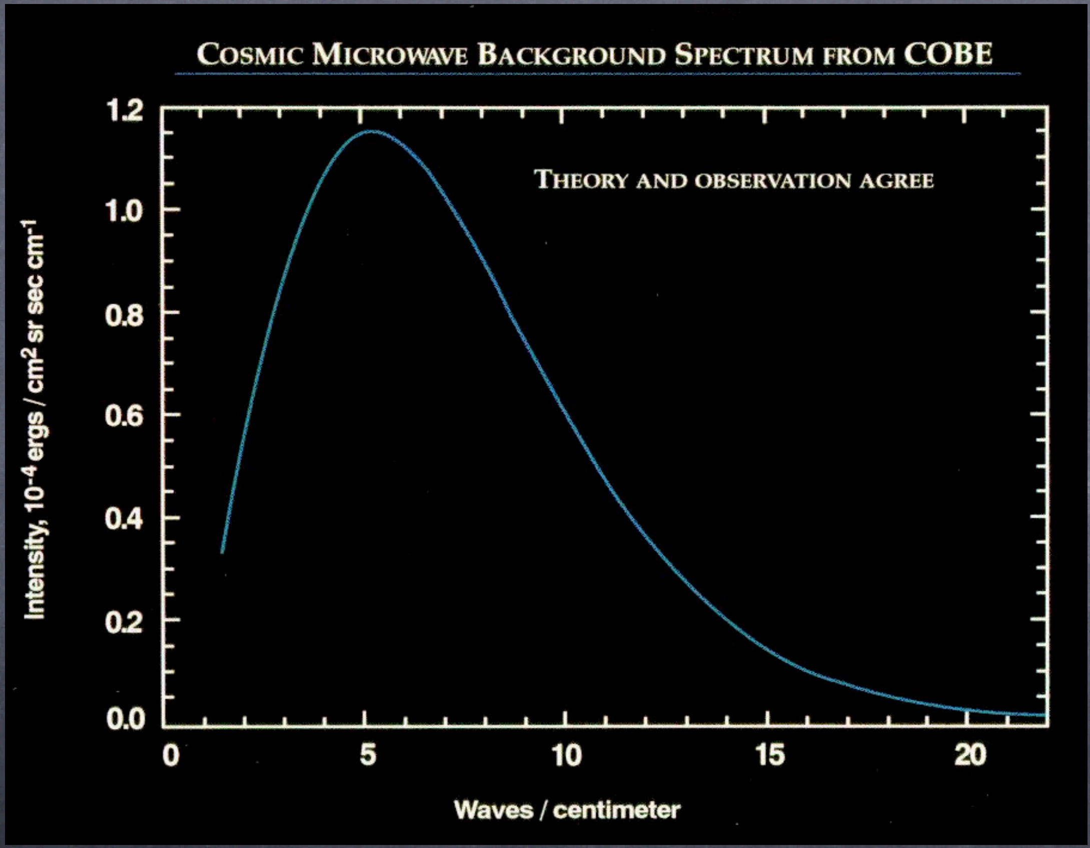
We now have a standard cosmological model

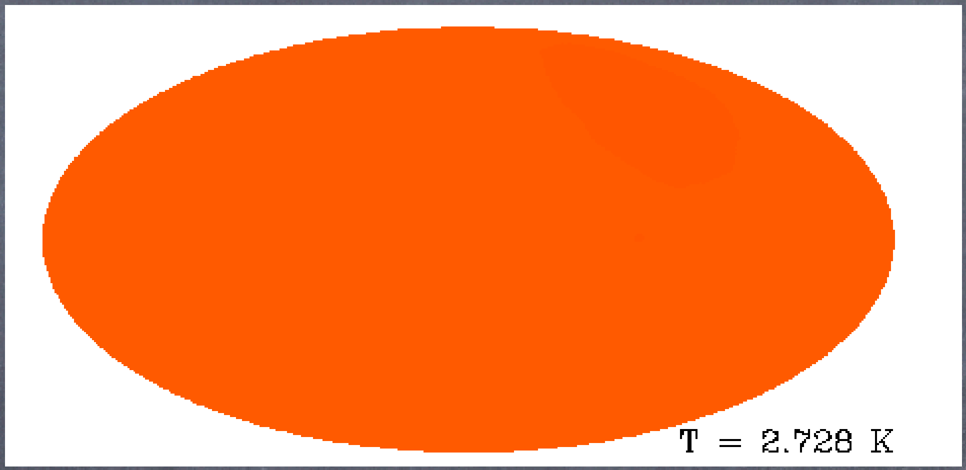
- General Relativity + Uniform Universe ⇒ Big Bang
  - Density of universe determines its fate + shape
- Universe is flat (total density = critical density)
  - Atoms 4%
  - Dark Matter 23%
  - Dark Energy (cosmological constant?) 72%
- Universe has tiny ripples

THIS MODEL FITS ALL OF THE COSMOLOGICAL DATA!

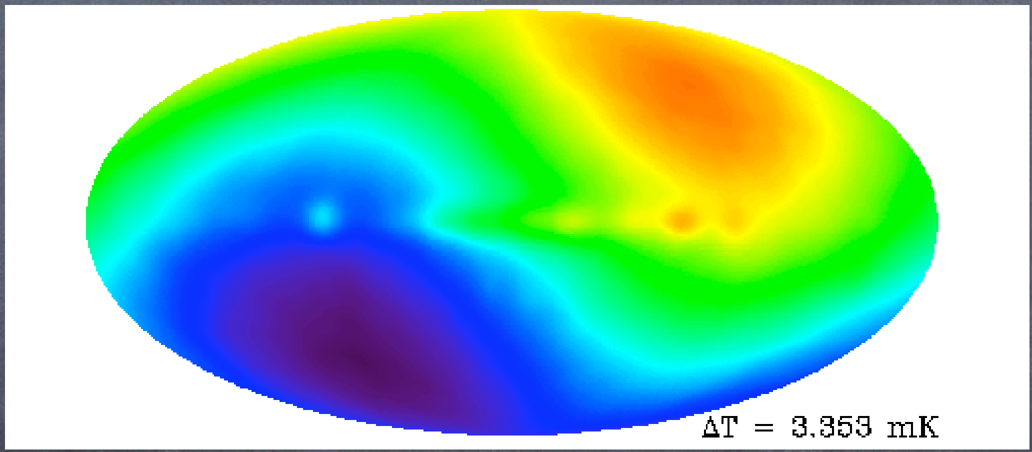
Observing the CMB



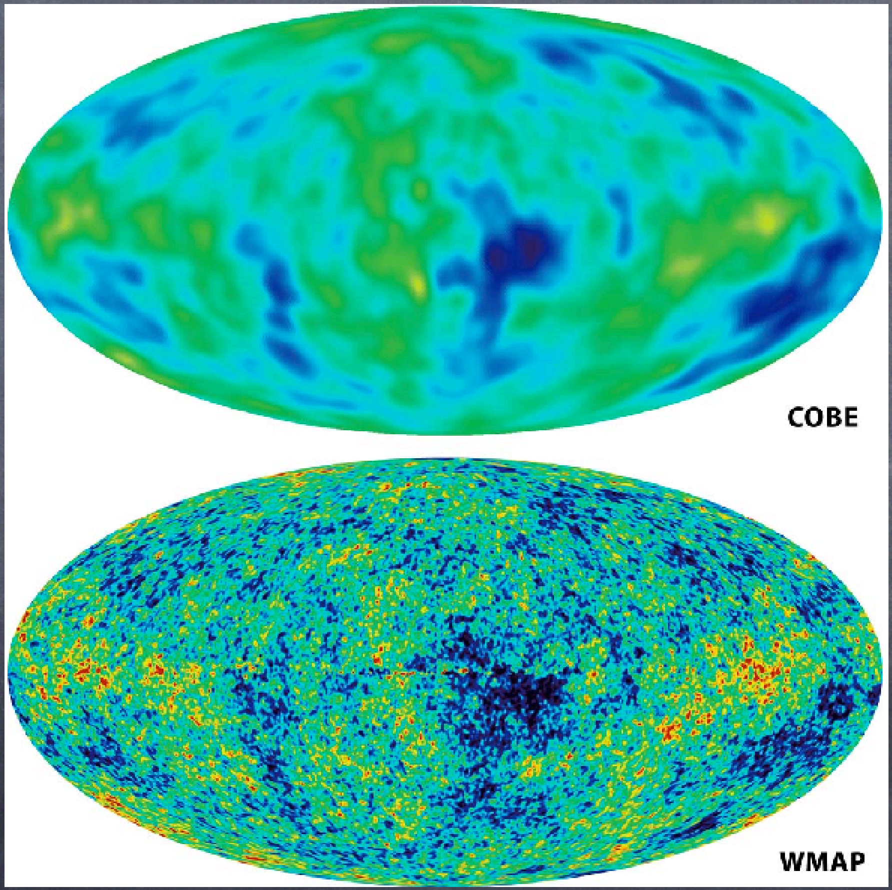
Looking for CMB Fluctuations



CMB Dipole



CMB Fluctuations



THE ASTROPHYSICAL JOURNAL, 162:815–836, December 1970
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PRIMEVAL ADIABATIC PERTURBATION IN AN EXPANDING UNIVERSE*

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ABSTRACT

The general qualitative behavior of linear, first-order density perturbations in a Friedmann-Lemaître cosmological model with radiation and matter has been known for some time in the various limiting situations. An exact quantitative calculation which traces the entire history of the density fluctuations is lacking because the usual approximations of a very short photon mean free path before plasma recombination, and a very long mean free path after, are inadequate. We present here results of the direct integration of the collision equation of the photon distribution function, which enable us to treat in detail the complicated regime of plasma recombination. Starting from an assumed initial power spectrum well before recombination, we obtain a final spectrum of density perturbations after recombination. The calculations are carried out for several general-relativity models and one scalar-tensor model. One can identify two characteristic masses in the final power spectrum: one is the mass within the Hubble radius ct at recombination, and the other results from the linear dissipation of the perturbations prior to recombination. Conceivably the first of these numbers is associated with the great rich clusters of galaxies, the second with the large galaxies. We compute also the expected residual irregularity in the radiation from the primeval fireball. If we assume that (1) the rich clusters formed from an initially adiabatic perturbation and (2) the fireball radiation has not been seriously perturbed after the epoch of recombination of the primeval plasma, then with an angular resolution of 1 minute of arc the rms fluctuation in antenna temperature should be at least $\delta T/T = 0.00015$.

I. INTRODUCTION

1970Ap&SS...7....3S

SMALL-SCALE FLUCTUATIONS OF RELIC RADIATION*

R. A. SUNYAEV and YA. B. ZELDOVICH

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(Received 11 September, 1969)

Abstract. Perturbations of the matter density in a homogeneous and isotropic cosmological model which leads to the formation of galaxies should, at later stages of evolution, cause spatial fluctuations of relic radiation. Silk assumed that an adiabatic connection existed between the density perturbations at the moment of recombination of the initial plasma and fluctuations of the observed temperature of radiation $\delta T/T = \delta \varrho_m / 3\varrho_m$. It is shown in this article that such a simple connection is not applicable due to:

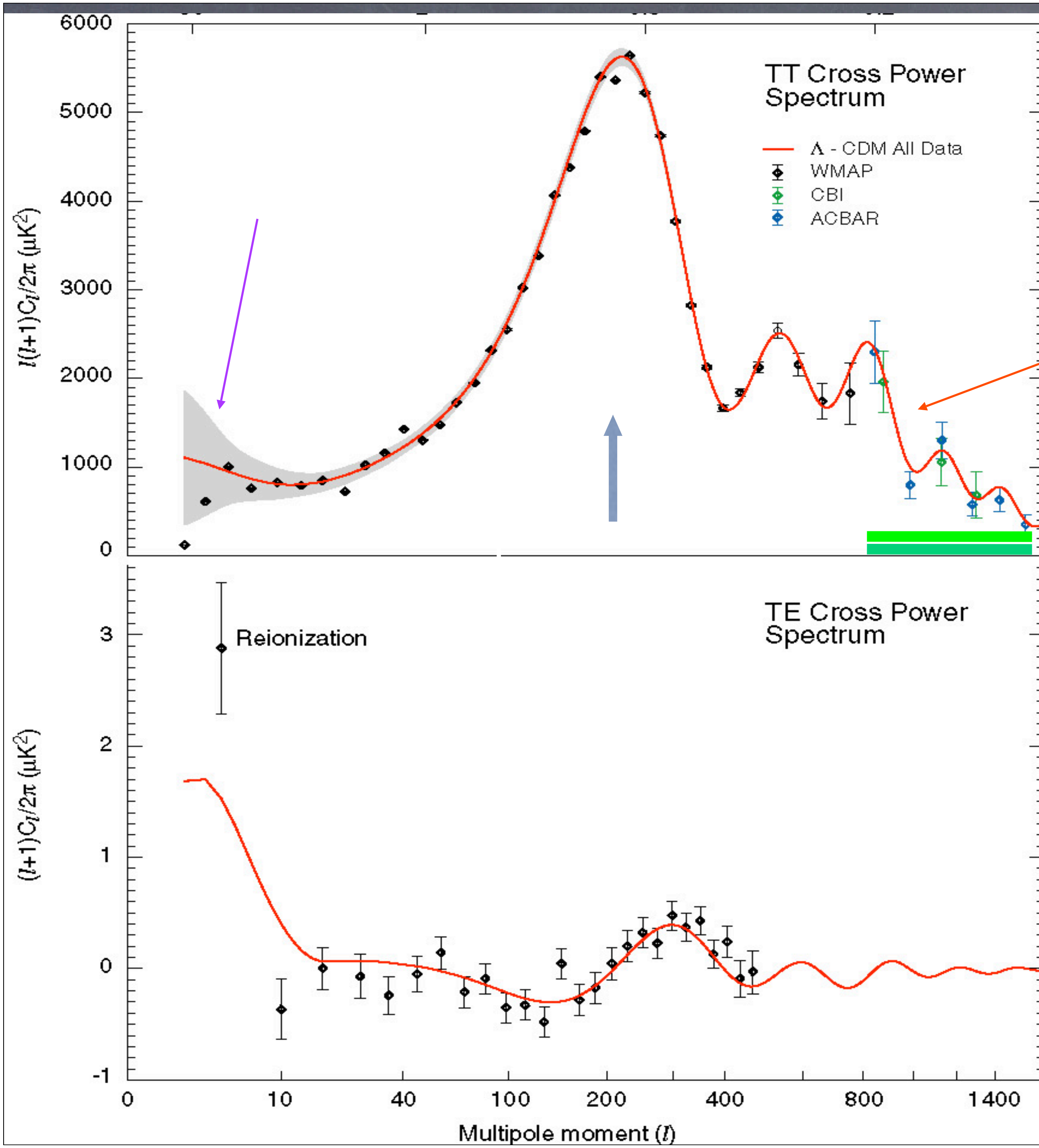
(1) The long time of recombination;

(2) The fact that when regions with $M < 10^{15} M_{\odot}$ become transparent for radiation, the optical depth to the observer is still large due to Thompson scattering;

(3) The spasmodic increase of $\delta \varrho_m / \varrho_m$ in recombination.

As a result the expected temperature fluctuations of relic radiation should be smaller than adiabatic fluctuations. In this article the value of $\delta T/T$ arising from scattering of radiation on moving electrons is calculated; the velocity field is generated by adiabatic or entropy density perturbations. Fluctuations of the relic radiation due to secondary heating of the intergalactic gas are also estimated. A detailed investigation of the spectrum of fluctuations may, in principle, lead to an understanding of the nature of initial density perturbations since a distinct periodic dependence of the spectral density of perturbations on wavelength (mass) is peculiar to adiabatic perturbations. Practical observations are quite difficult due to the smallness of the effects and the presence of fluctuations connected with discrete sources of radio emission.

Sunyaev & Zeldovich



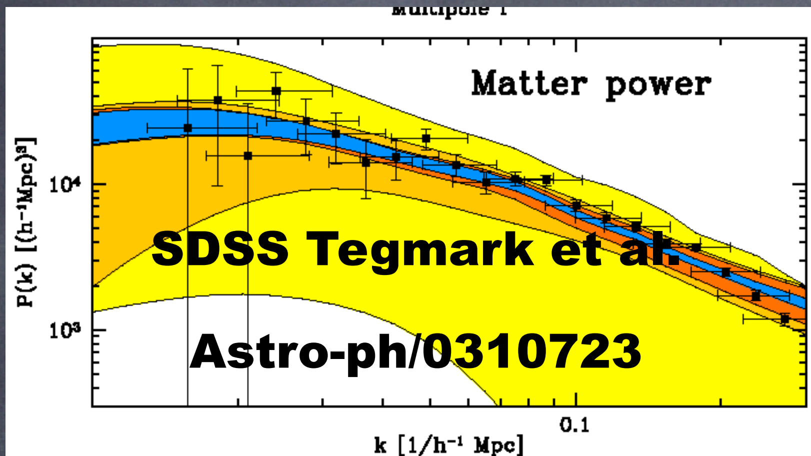
Temperature

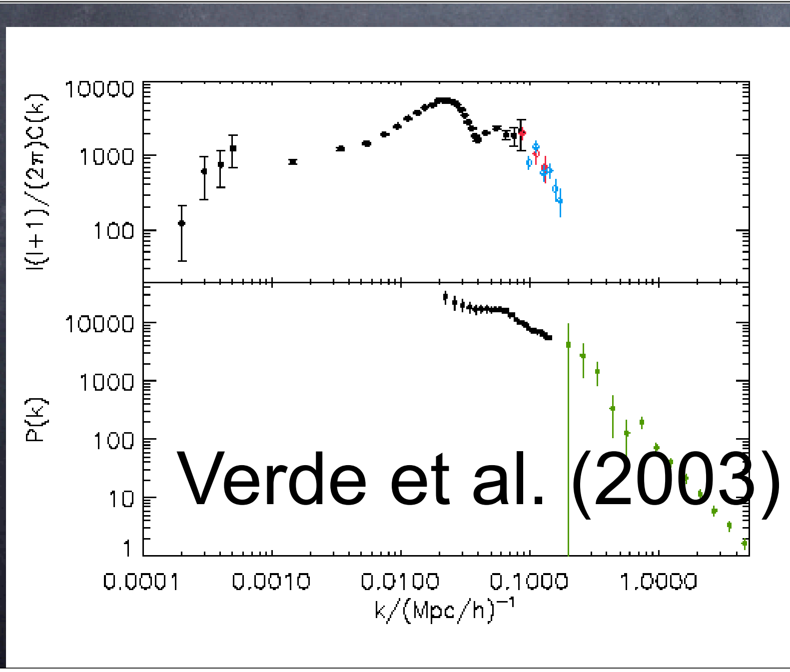
85% of sky

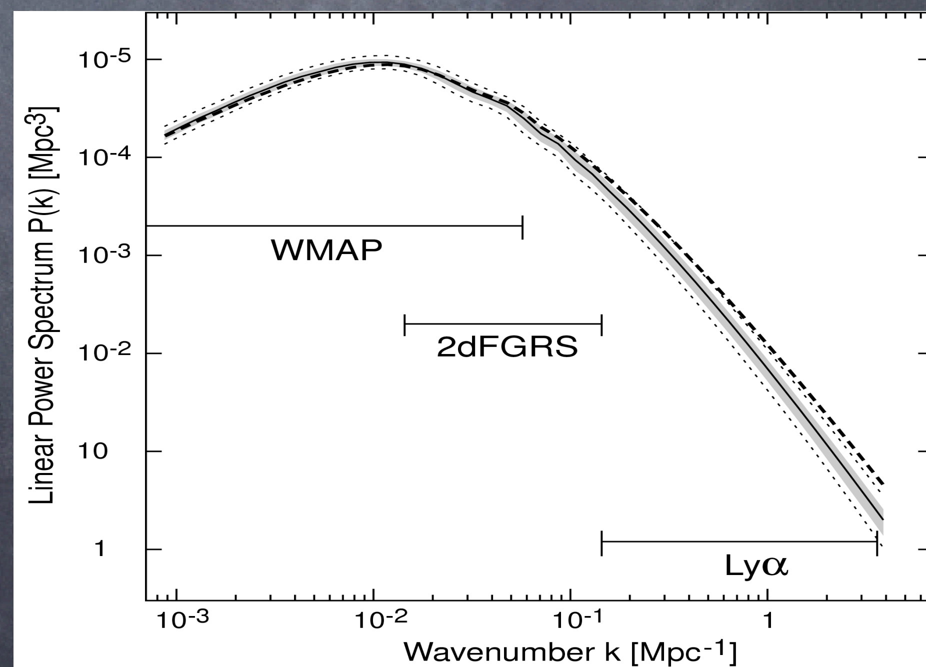
Best fit model

	WMAP
A	0.9 ± 0.1
n_s	0.99 ± 0.04
τ	$0.166^{+0.076}_{-0.071}$
h	0.72 ± 0.05
$\Omega_m h^2$	0.14 ± 0.02
$\Omega_b h^2$	0.024 ± 0.001
χ^2_{eff}/ν	1431/1342

Model Predicts Today's Universe







Consistent Parameters

	WMAP+CBI +ACBAR	All CMB(Bond)	CMB+ 2dFGRS	CMB+SDSS (Tegmark)
$\Omega_b h^2$	.023 ± .001	.0230 ± .0011	.023 ± .001	.0232 ± .0010
$\Omega_x h^2$	.117 ± .011	.117 ± .010	.121 ± .009	.122 ± .009
h	.73 ± .05	.72 ± .05	.73 ± .03	.70 ± .03
n_s	.97 ± .03	.967 ± .029	.97 ± .03	.977 ± .03
σ_8	.83 ± .08	.85 ± .06	.84 ± .06	.92 ± .08

Standard Cosmology

- We know have a standard cosmological model that answers most of the “old” cosmological questions:
  - What is the shape and size of the universe?
  - How fast is the universe expanding?
  - How old is the universe?
  - What is the composition of the universe?
  - How do galaxies form?
  - What is the origin of the primordial fluctuations?

New Questions

- Physics that we don't know (String theory, quantum cosmology,…
  - How did the universe begin?
  - What is the dark energy?
  - What is the dark matter?
- Physics that we don't know how to calculate (Non-linear hydro, star formation…
  - First stars
  - Galaxy formation

CMB Observations

- Large angle observations (> 5 l < 2000) probes physics at the surface of last scatter (z ~1100)
- Small angle observations (< 5′ l > 2000) probes local (z~0.5) physics

Lecture Outline

- Lecture 1
  - Overview
  - Gaussian Random Variables
  - CMB and Mass Fluctuations
- Lecture 2: Evolution of Fluctuations
- Lecture 3: Polarization
- Lecture 4: Statistics of CMB Fluctuations
- Lecture 5: Clusters: SZ Effect and Probing the Growth of Dark Energy

x is a Gaussian Variable

$$p(x) = \frac{1}{\sqrt{\pi}\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

$$\int p(x)dx = 1$$

$$< x >= \int p(x)xdx = 0$$

$$< x^2 >= \int p(x)x^2dx = \sigma^2$$

$$< x^{2N+1} >= 0$$

$$< x^{2N} >= \frac{(2N)!}{2^N}\sigma^{2N}$$

$$< x_i x_j >= \sigma_i^2 \delta_{ij}$$

$$y_k = \sum_i \alpha_{ki} x_i$$

$$\begin{aligned} < y_k y_l > &= \sum_i \sum_j \alpha_{ki} \alpha_{lj} < x_i x_j > \\ &= \sum_i \alpha_{ki} \alpha_{li} \sigma_i^2 \equiv \mathbf{C_{kl}} \end{aligned}$$

$$< y_k^{2N+1} >= 0$$

$$< y_k^{2N} >= \frac{(2N)!}{2^N} [C_{kk}]^{(2N)}$$

The sum of random variables is also a Gaussian variable

Density Fluctuations

$$\rho(\vec{x}) = \rho_0[1 + \int d^3k A(\vec{k}) \exp(i\vec{k} \cdot \vec{x})]$$

$$< A(\vec{k})A(\vec{k}') >= P(k)$$

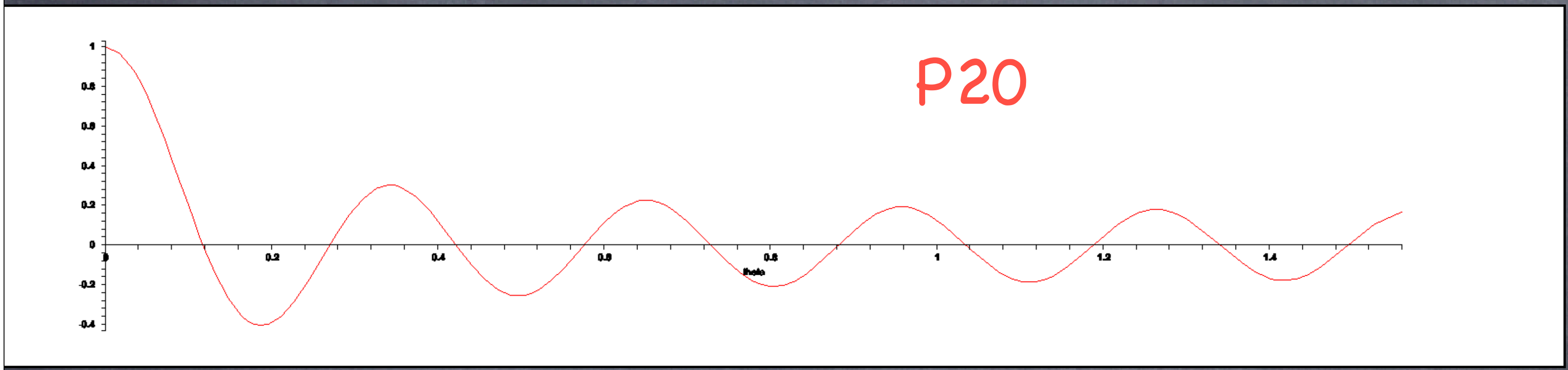
$$\rho_s(\vec{x}) = \int d^3 \rho(\vec{x}) W(|\vec{x} - \vec{x}')$$

$$W(r) = \exp[-r^2/(2r_0^2)]$$

$$\rho_s(\vec{x}) = \int d^3k A(\vec{k}) W(k) \exp(i\vec{k} \cdot \vec{x})]$$

$$W(k) = exp(-k^2 r_0^2/2)$$

Legendre Polynomials



First null near
2/l radians

$$\sum_m Y_{lm}(\hat{n}) Y_{lm}(\hat{n}') = \frac{2l+1}{4\pi} P_l(\hat{n}\cdot\hat{n}')$$

$$\theta \sim \frac{180^o}{l}$$

Temperature Fluctuations

$$T(\hat{n}) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

$$< a_{lm} a_{l'm'} >= c_l$$

$$T_s(\hat{n}) = \int d^2n' T(\hat{n}) W(|\hat{n} - \hat{n}'|$$

$$T_s(\hat{n}) = \sum_{lm} a_{lm} w_l Y_{lm}(\hat{n})$$

$$\begin{aligned}
C(\cos\theta) &= \int \frac{d^2n}{4\pi}\frac{d^2n'}{4\pi} T(\hat{n})T(\hat{n}')\delta(\hat{n}\cdot\hat{n}' - \cos\theta) \\
&= \int \frac{d^2n}{4\pi}\frac{d^2n'}{4\pi} \sum_{lm}\sum_{l'm'} a_{lm}a_{l'm'}Y_{lm}(\hat{n})Y_{l'm'}(\hat{n}')\delta(\hat{n}\cdot\hat{n}' - \cos\theta) \\
&= \int \frac{d^2n}{4\pi}\frac{d^2n'}{4\pi} \sum_{l} c_l \frac{2l+1}{4\pi} P_l(\hat{n}\cdot n')\delta(\hat{n}\cdot\hat{n}' - \cos\theta) \\
&= \sum_l c_l \frac{2l+1}{4\pi} P_l(\cos\gamma)
\end{aligned}$$

The statistical properties of temperature and density fluctuations are characterized ENTIRELY by the amplitude of the power spectrum

Fluctuations from SLS

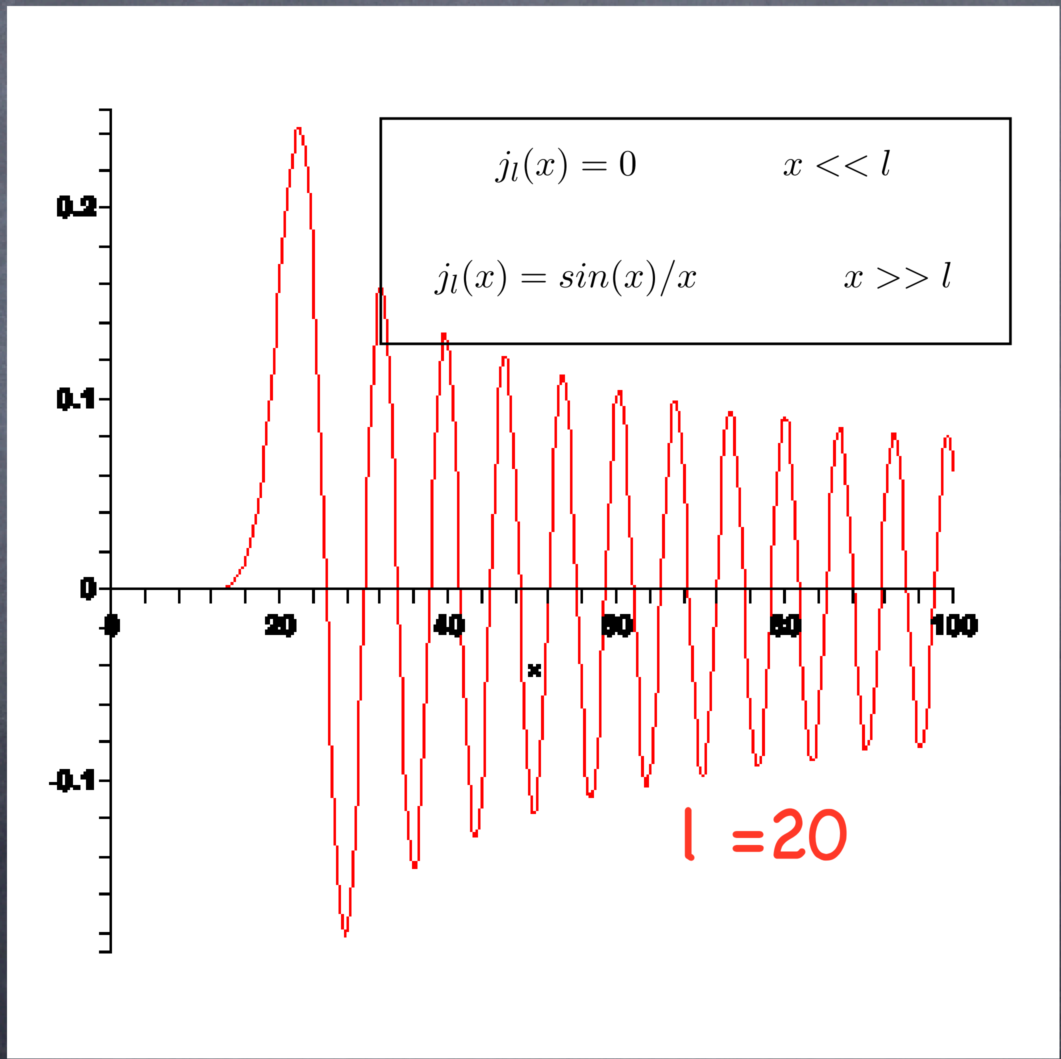
$$\begin{aligned} T(\hat{n}(\eta_0 - \eta_{LS})) &= \int d^3k T(\vec{k}, \eta_{LS}) \exp(i\vec{k} \cdot \hat{n}(\eta_0 - \eta_{LS})) \\ &= \int d^3k T(\vec{k}, \eta_{LS}) \sum_{lm} j_l[k(\eta_0 - \eta_{LS})] Y_{lm}(\hat{n}) Y^*_{lm}(\hat{k}) \end{aligned}$$

$$T(\hat{n}(\eta_0 - \eta_{LS})) = \sum_{lm} a_{lm} Y_{lm}(\hat{n})$$

$$a_{lm} = 4\pi \int k^2 dk T_{lm}(k) j_l[k(\eta_0 - \eta_{LS}))]$$

$$\eta = \int_0^t \frac{\mathrm{d}t'}{a(t')}$$

Bessel Function



Problem

In the next lecture, we will show that for adiabatic fluctuations in a matter dominated universe,

$$T(r, \eta_{LS}) = -\frac{\Phi(r)}{3} \tag{21}$$

Assume a scale invariant spectrum for Φ:

$$< \Phi_{lm}(k)\Phi^*_{lm}(k') >= Ak^{-3}\delta_{lm}^{l'm'}\delta(k-k') \tag{22}$$

and compute the angular power spectrum.

Solution

$$a_{lm} = \frac{1}{3}\int k^2 dk \Phi_{lm}(k) j_l(k(\eta_0 - \eta_{LS})$$

$$\begin{aligned}
c_l &= < a_{lm} a_{lm} > \\
&= \frac{1}{9}\int k^2 dk (k')^2 dk' < \Phi_{lm}(k)\Phi_{lm}(k') > j_l(k(\eta_0 - \eta_{LS}) j_l(k'(\eta_0 - \eta_{LS}) \\
&= \frac{A}{9}\int \frac{dk}{k} j_l^2(k\eta_0 - \eta_{LS}) \\
&\propto \frac{A}{l(l+1)}
\end{aligned}$$

Lecture 2 Evolution of Fluctuations

$$c_l = \int k^2 dk P(k) \Theta_l^2(k)$$

Initial Conditions → $P(k)$

Evolution → $\Theta_l^2(k)$

How do we compute CMB angular power spectrum?

- Download CMBFAST or CAMB from the web. These fast computer programs evaluate the CMB angular power spectrum and the matter power spectrum for different sets of initial conditions
- Solve coupled linear gravity-Boltzmann equation numerically (see Dodelson's textbook)
- Use tight coupling approximation to estimate the angular power spectrum (Hu & Sugiyama)

Initial Conditions

$$\Phi(\vec{x}, t_{initial}) = \int d^3k \exp(i\vec{k}\cdot\vec{x})\Phi(\vec{k}, t_{initial})$$

$$\nabla^2\Phi = 4\pi G\rho a^2\Delta$$

$$\Delta(\vec{k}) = \frac{k^2 a}{3/2\Omega_m H_0^2}\Phi(\vec{k})$$

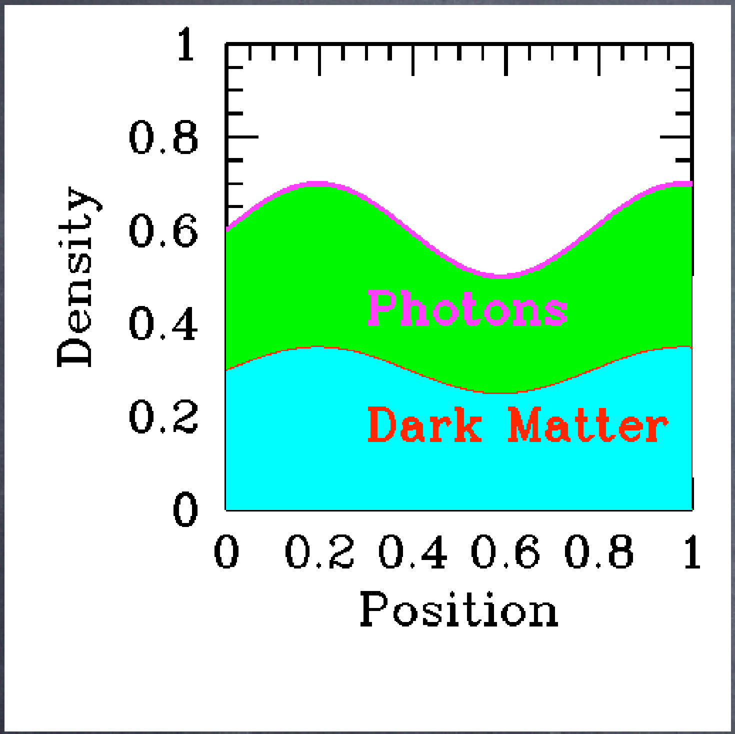
$$\Delta_{tot} = \Delta_\gamma + \Delta_{baryon} + \Delta_\nu + \Delta_{DM}$$

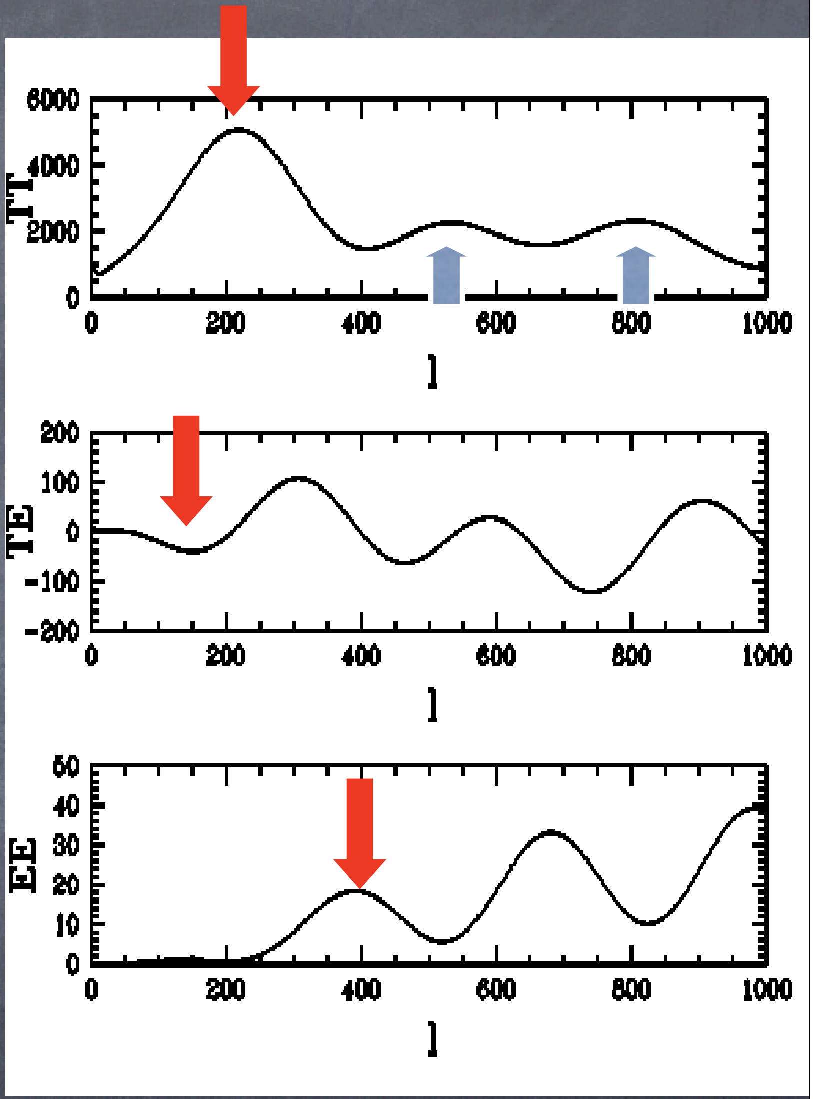
$$S_i = \frac{n_i}{n_\gamma}$$

Density and Potential Fluctuations are Gaussian Random Variables

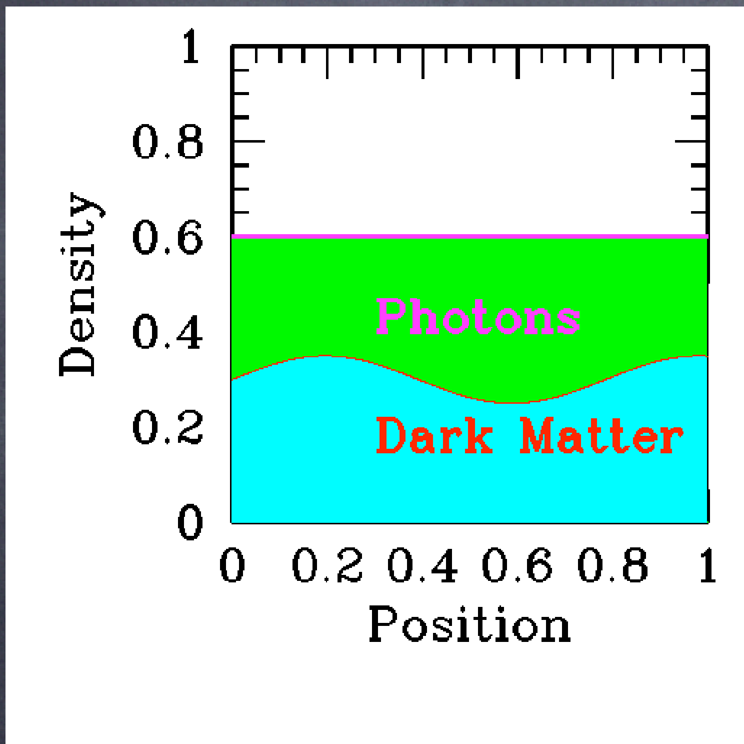
Adiabatic Initial Conditions: S(x,t) = S

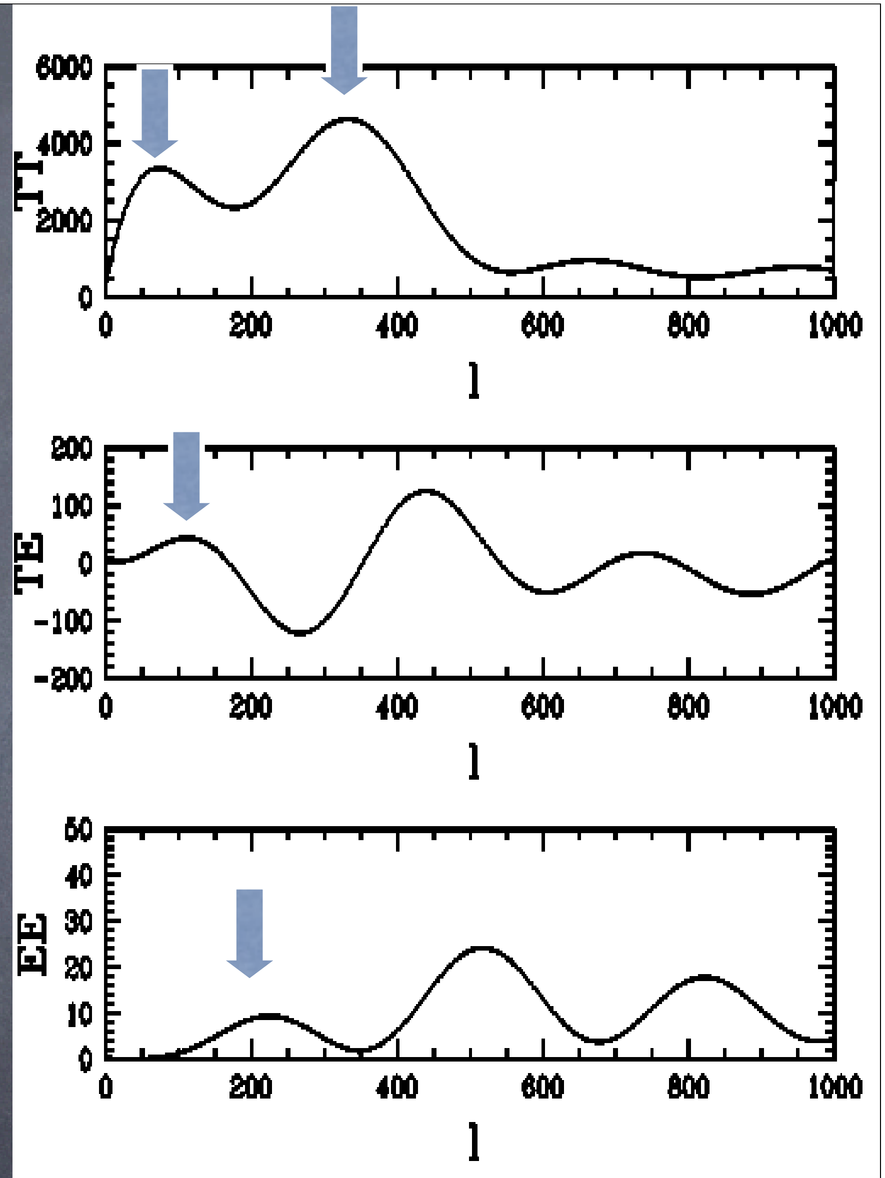
Isocurvature Initial Conditions $\Phi(\vec{x}, t_{initial})$ = 0





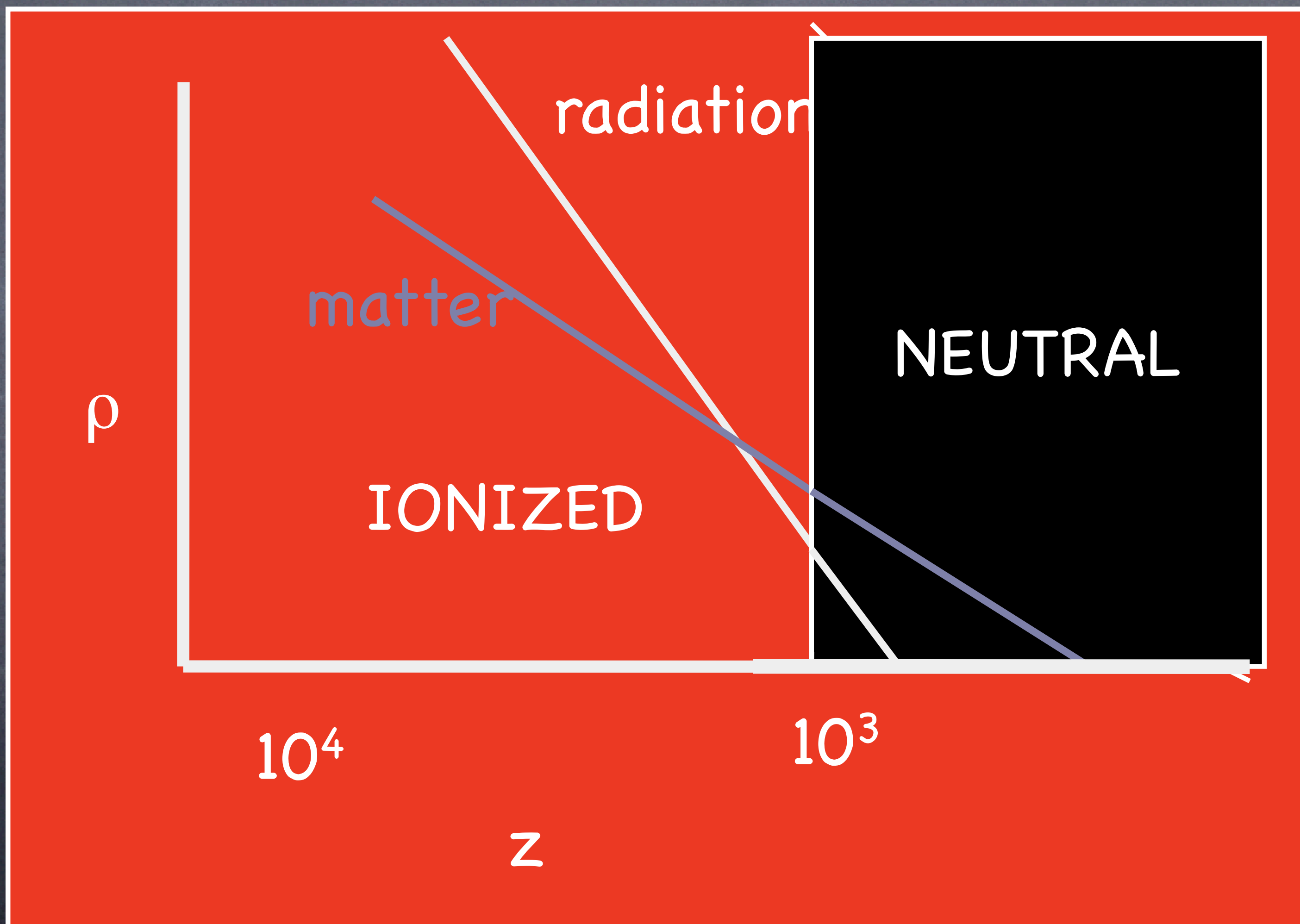
ADIABATIC DENSITY FLUCTUATIONS





ISOCURVATURE ENTROPY FLUCTUATIONS

Thermal History of Universe



Decoupling Happens Fast

$$\gamma + H < - > e^- + p$$

$$n_\gamma(E > 13.6eV)/n_e^- \sim 10^{10} \exp(-13.6eV/T_\gamma)$$

The ionized fraction rapidly drops from 1 to 0.001 between z=1200 and z = 1000

At z > 1200, photons are tightly coupled to electrons. At z < 1000, photons stream freely with minimal interactions with matter

Evolution of Photon Fluctuations

- Tight coupling regime: fluctuations oscillate as acoustic waves
- Silk damping regime ($1200 < z < 1000$): photons diffuse out of fluctuations
- Free streaming

Tight Coupling

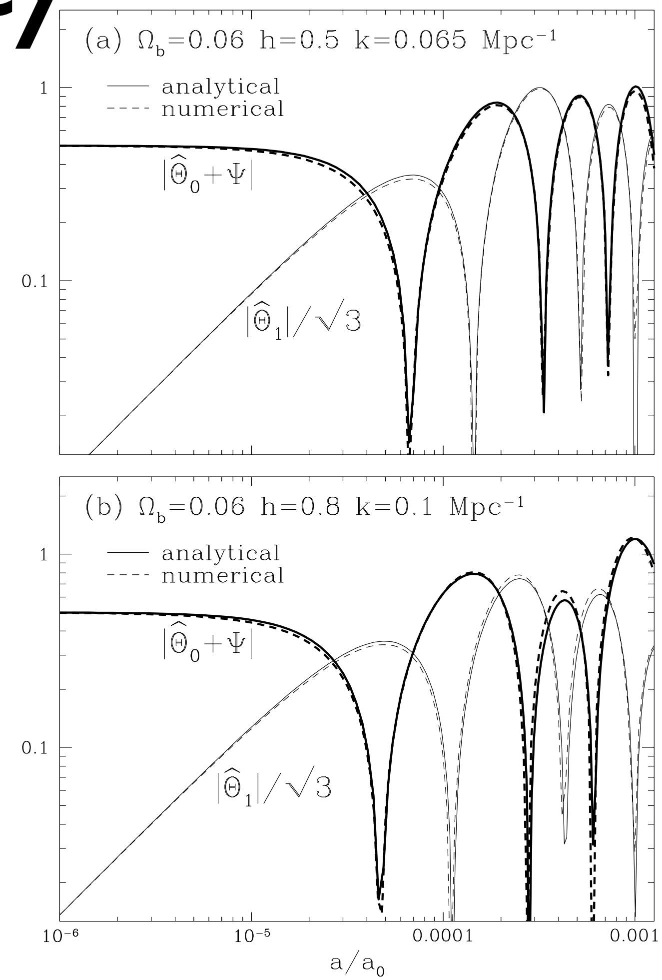
$$\ddot{\Theta} + c_s^2 k^2 \Theta = F(\Phi)$$

$$\Theta = A_0 \cos(kr_s) + B_0 \sin(kr_s)$$

$$r_s = \int c_s(\eta) d\eta$$

$$c_s^2 = \frac{p}{\rho} = \frac{1/3\rho_\gamma}{\rho_\gamma + (4/3)\rho_{baryon}}$$

Velocity



$$\dot{\Theta}_0 + k\Theta_1 = 0$$

$$\Theta_1 = c_s A \sin(kr_s)$$

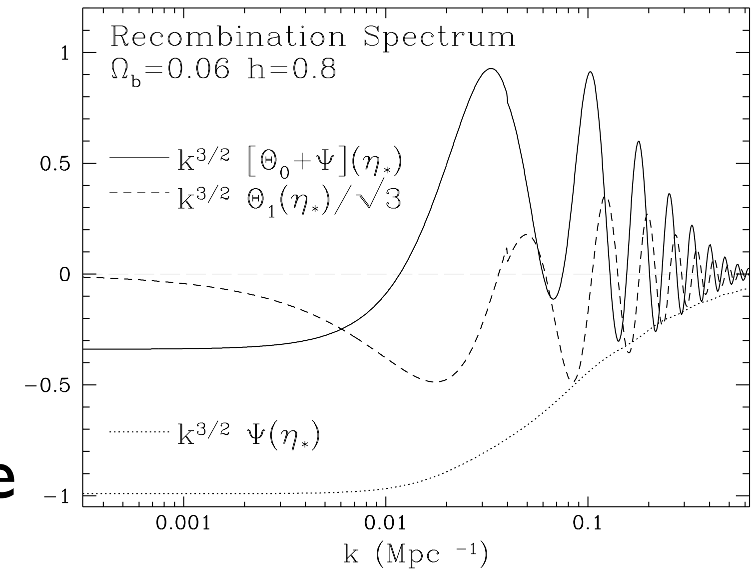
$$\dot{v}_b = n_e \sigma_T R (v_b - \Theta_1)$$

Silk Damping

$$\Theta_0, \Theta_1 \sim \exp(-k^2/k_D^2)$$

$$k_D^{-2} \simeq \int \frac{d\eta}{n_e \sigma_T a}$$

Photons diffusing out of density fluctuations damp the fluctuations



Propagation

$$T(k,\eta_D) = \left[-\frac{4}{3} + \cos(kr_s^*)\right]\exp(-k^2/k_D^2)\Phi(k) + ic_s\Phi(k)\sin(kr_s^*)\exp(-k^2/k_D^2)$$

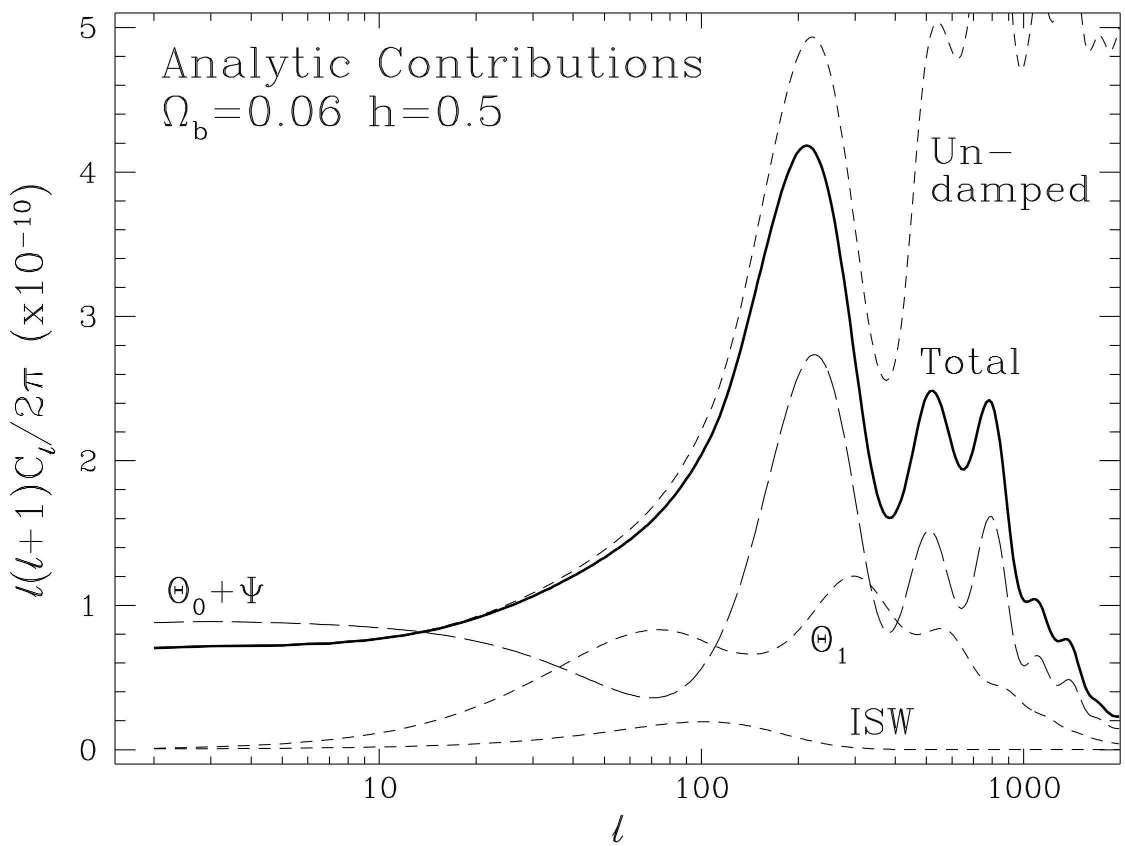
$$\begin{aligned} a_{lm} &= \left\{\Phi_{lm}(k)\left[-\frac{4}{3} + \cos(kr_s^*)\right]\exp(-k^2/k_D^2) \right. \\ &\quad \left. + ic_s\Phi_{lm}i(k)\sin(kr_s^*)\exp(-k^2/k_D^2)\right\} j_l(k(\eta_0 - \eta_{LS})) \end{aligned}$$

$$c_l = \int P(k)\left\{\left[-\frac{4}{3} + cos(kr_s^*)\right]^2 \exp(-2k^2/k_D^2) + c_s^2\sin^2(kr_s^*)\right\} j_l^2(k(\eta_0 - \eta_{LS}))$$

$$k_{peak}r_s = n\pi$$

$$l_{peak} = k_{peak}\eta_0 = \pi\eta_0/r_s \simeq 200$$

Approximate Angular Power Spectrum



ISW Term

- In a flat matter dominated universe, the gravitational potential is constant

$$\delta T = 2 \int \dot{\Phi} dt$$

- Early ISW: matter radiation equality is at z~2000. At z ~500-1000, radiation still contributes 12-25% of the total energy density. Radiation doesn't cluster so it suppresses the growth of fluctuations

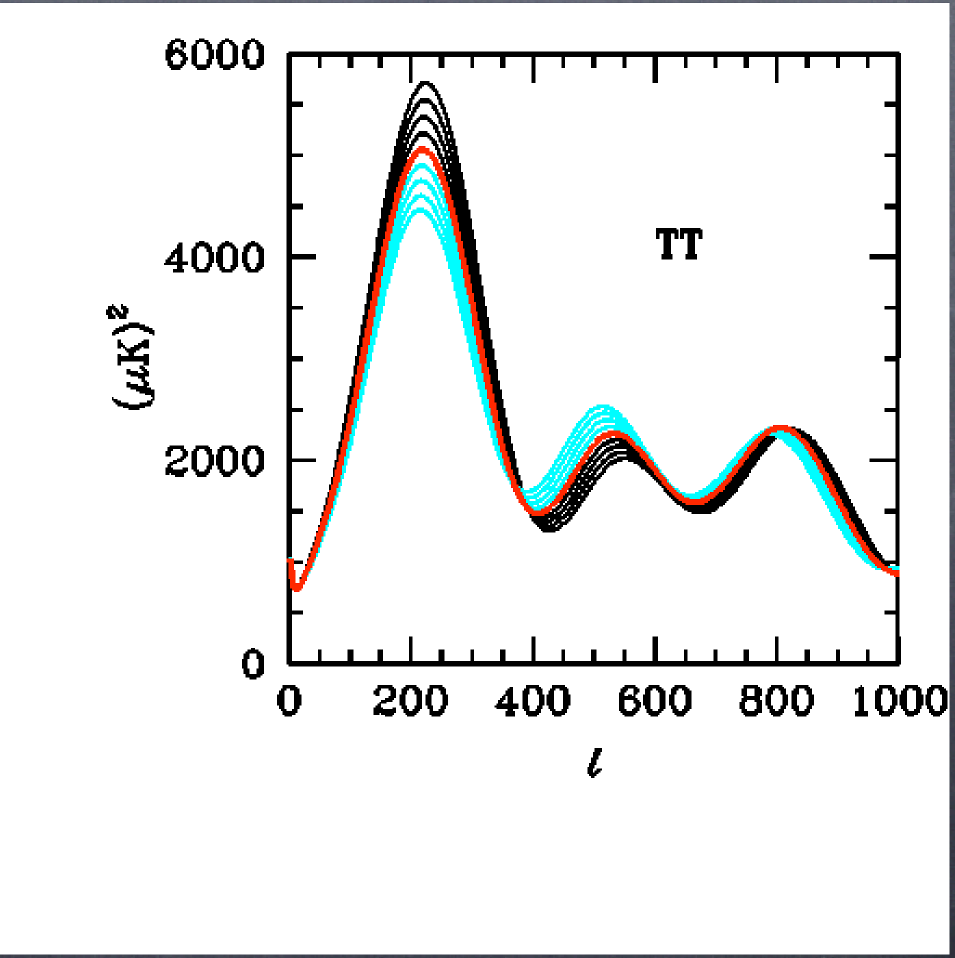
- Late ISW: once the universe becomes dark energy dominated, then the gravitational potential decays

Determining Basic Parameters

Baryon Density

$\Omega_b h^2$ = 0.015,0.017..0.031

also measured through D/H

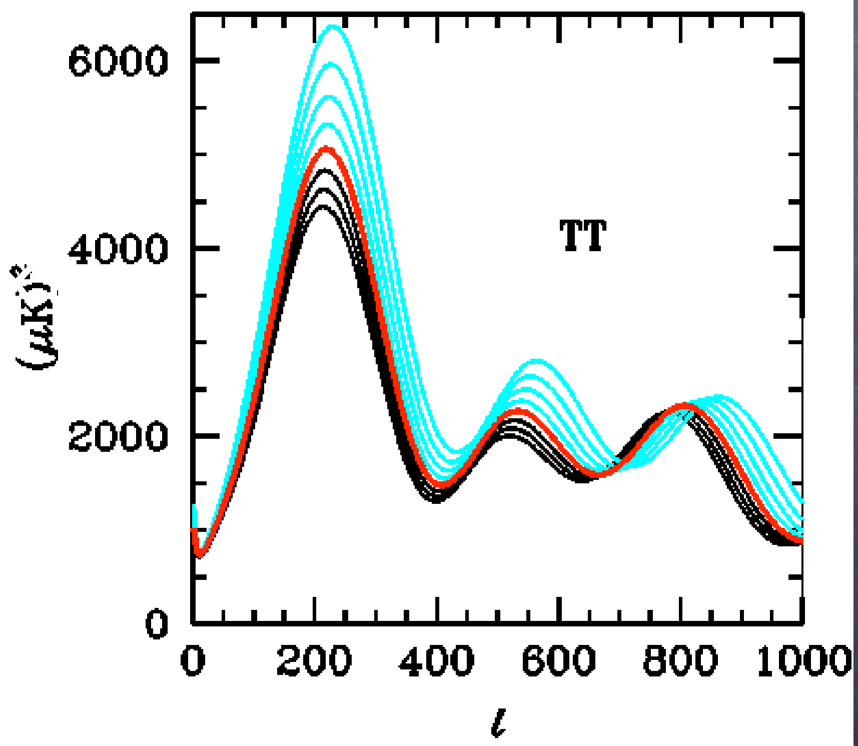


Determining Basic Parameters

Matter Density

$\Omega_m h^2 = 0.16,..,0.33$

Effect of forcing term in equation

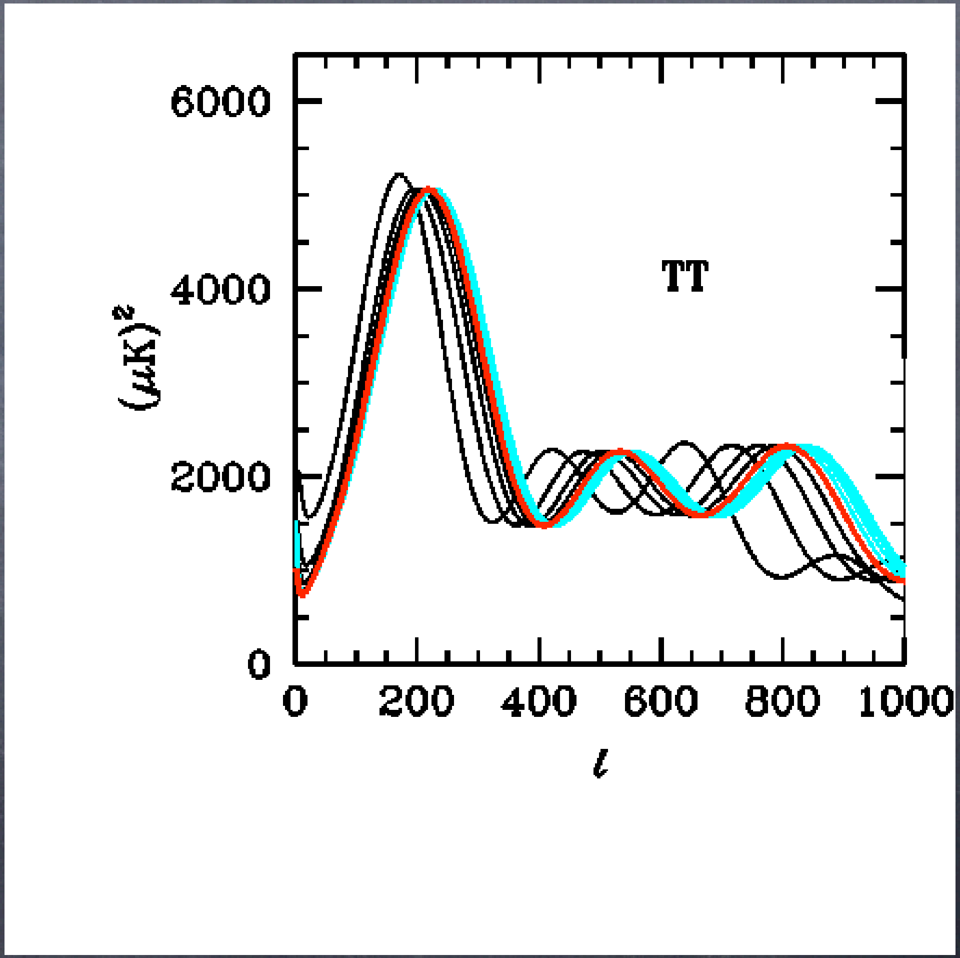


Determining Basic Parameters

Angular Diameter Distance

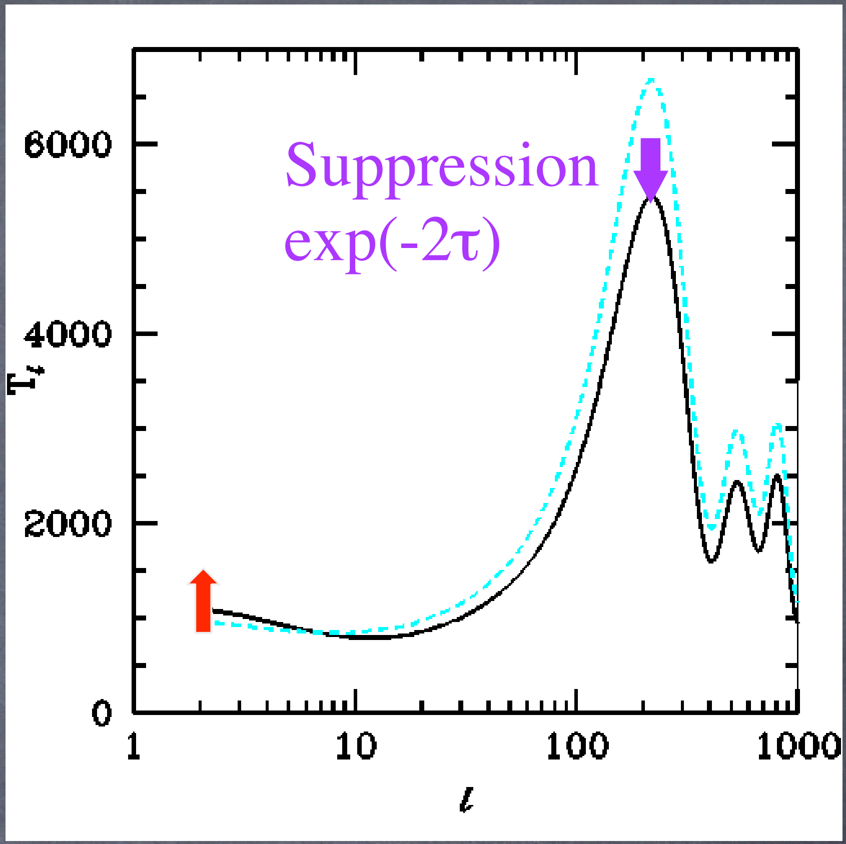
w = -1.8,..,-0.2

When combined with measurement of matter density constrains data to a line in Ω_m-w space



Reionization

- Suppression of small scale fluctuations
- Generation of additional large scale fluctuations
- Degenerate with spectral tilt (in temperature spectrum)



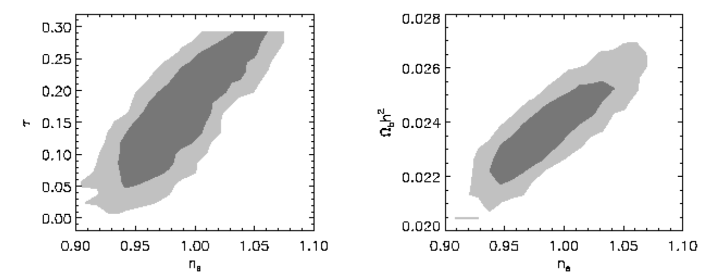
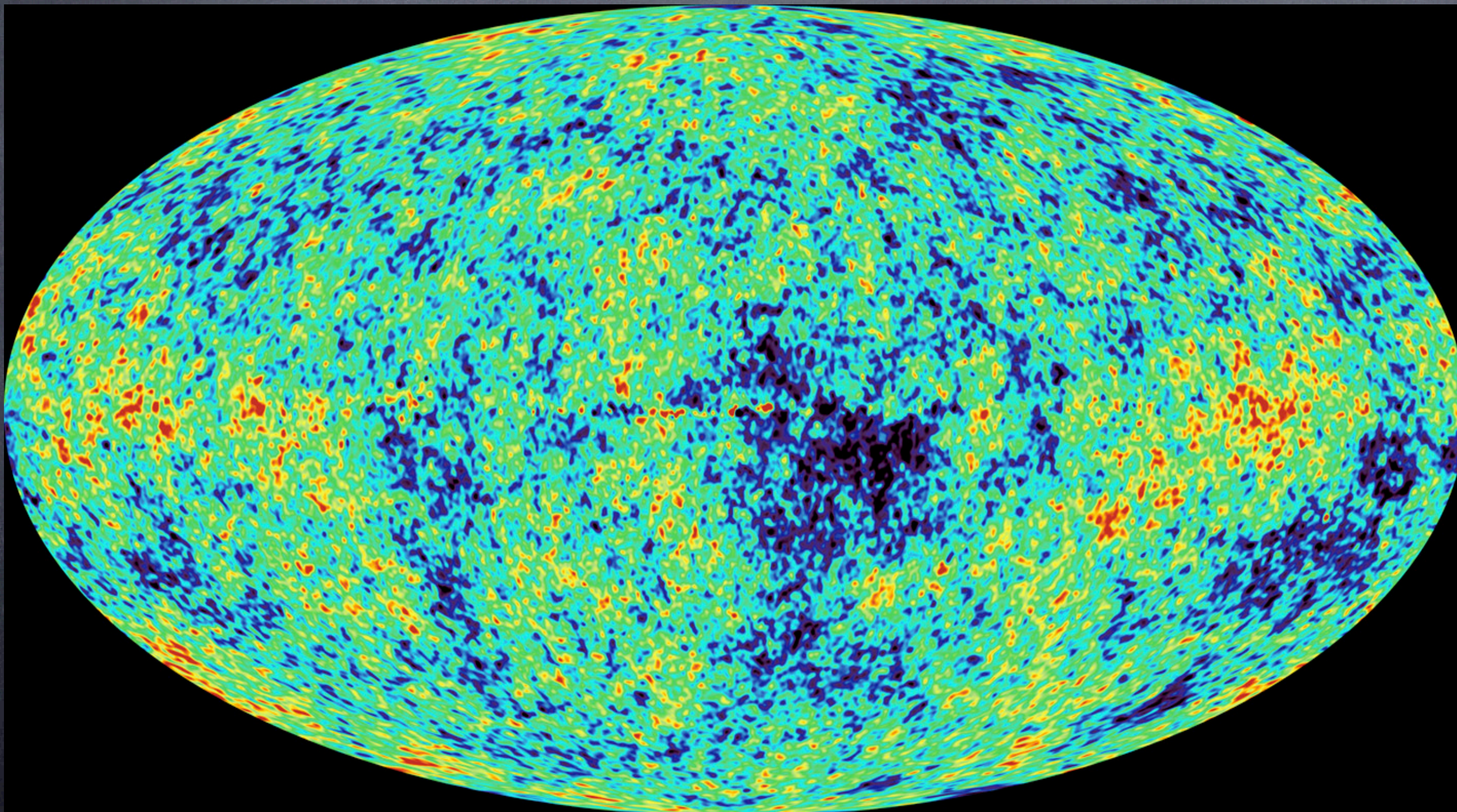


Fig. 5.— Spectral Index Constraints. Left panel: the $n_s - \tau$ degeneracy in the *WMAP* data for a power-law ΛCDM model. The TE observations constrain the value of τ and the shape of the C_l^{TT} spectrum constrain a combination of n_s and τ. Right panel: $n_s - \Omega_b h^2$ degeneracy. The shaded regions show the joint one and two sigma confidence regions.

FOREGROUND CORRECTED MAP



Current Status

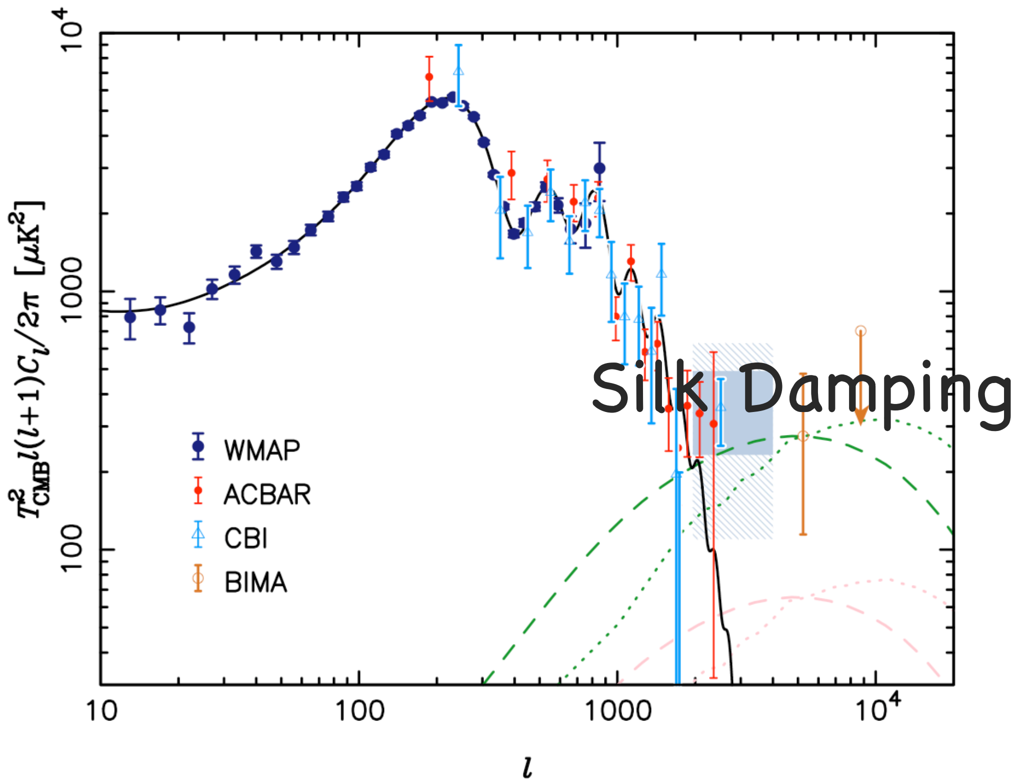


Fig. 14.— The CBI+*WMAP*+ACBAR Spectrum + high ℓ points from BIMA. The curves at high ℓ show the levels of SZ power expected in representative models using moving mesh hydrodynamics simulations (dotted) and smooth particle hydrodynamics (dashed) simulations (see text). The green and pink curves correspond to 30 GHz and 150 GHz, respectively. In these simulations $\sigma_8^{SZ} = 0.98$, which also fits well the WMAP and CBI observations at lower ℓ for the case of a running spectral index (see Table 5). The highest-ℓ ACBAR point has been displaced slightly to lower ℓ for clarity.

Problem

- Compute the location of the peaks in the angular power spectrum in a isocurvature model