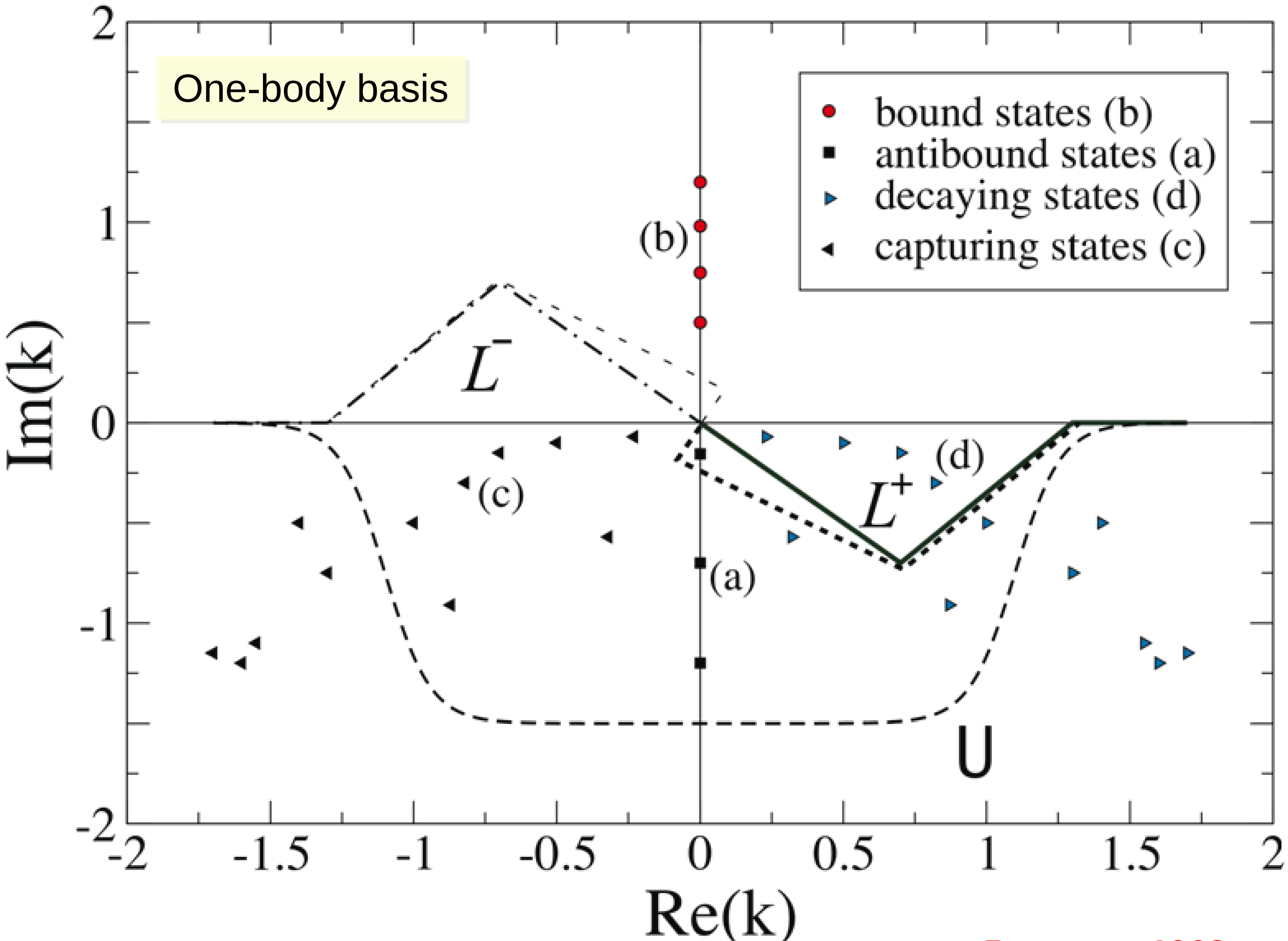


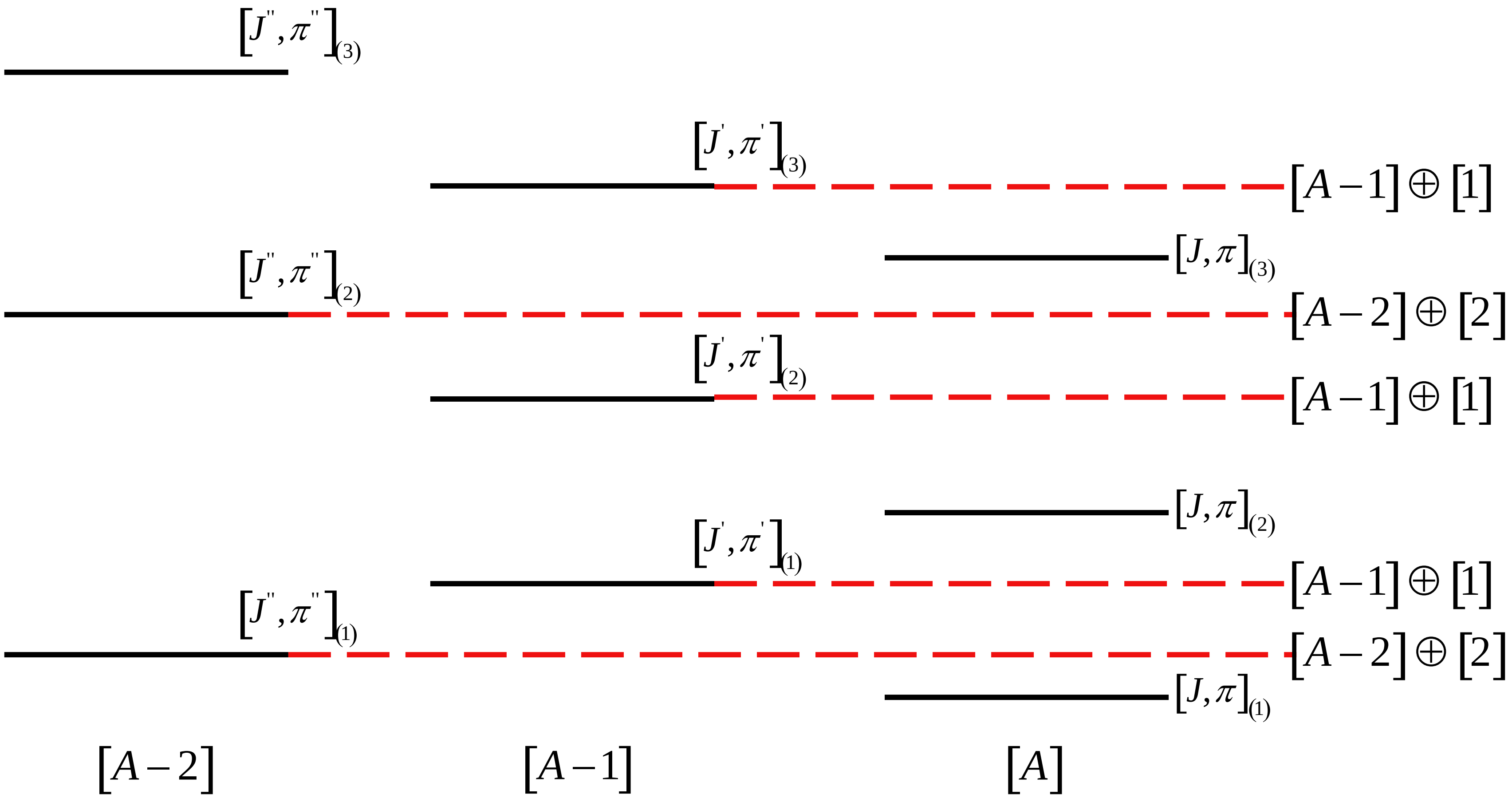
Berggren ensemble for a given *jl* channel:

Berggren 1968

$$\sum_{n\in(b,d)} |u_n\rangle\langle u_n| + \int_{L^+} |u(k)\rangle\langle u(k)| dk = 1.$$

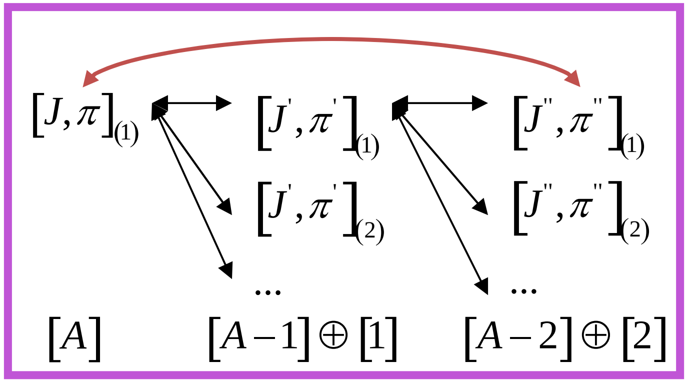
Real Energy Treatment

Coupling scheme for A-particle configurations



Closed space: SM space

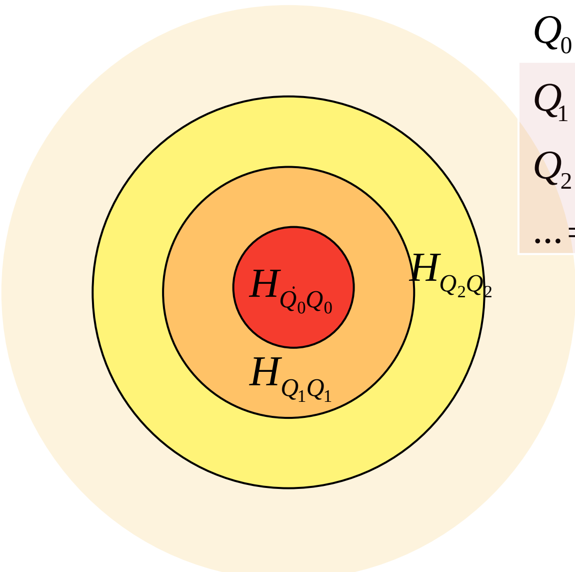
Open space involves particles in the continuum



From M. Ploszajczak

Hilbert space formulation : Shell Model Embedded in the Continuum (1999)

Closed space: SM space



$Q_0 = [A]$

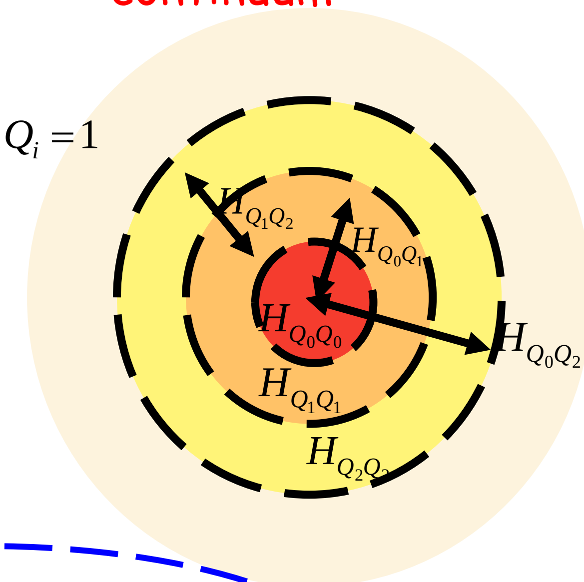
$Q_1 = [A-1] \oplus [1]$

$Q_2 = [A-2] \oplus [2]$

$\ldots = \ldots$

$$\sum_{i=0}^{A} Q_i = 1$$

Open space involves particles in the scattering continuum



$$H_{Q_0Q_0} \rightarrow H_{Q_0Q_0}^{eff}(E)$$

$$\langle \Phi_i^{SM} | H_{Q_0Q_0}^{eff}(E) | \Phi_j^{SM} \rangle = \langle \Phi_i^{SM} | H_{Q_0Q_0} | \Phi_j^{SM} \rangle + \sum_{c=1}^{\Lambda} P\left(\int_{\varepsilon_c}^{\infty} dE' \frac{\gamma_i^c \gamma_j^c}{E - E'} \right) - \frac{i}{2} \sum_{c=1}^{\Lambda} \gamma_i^c \gamma_j^c$$

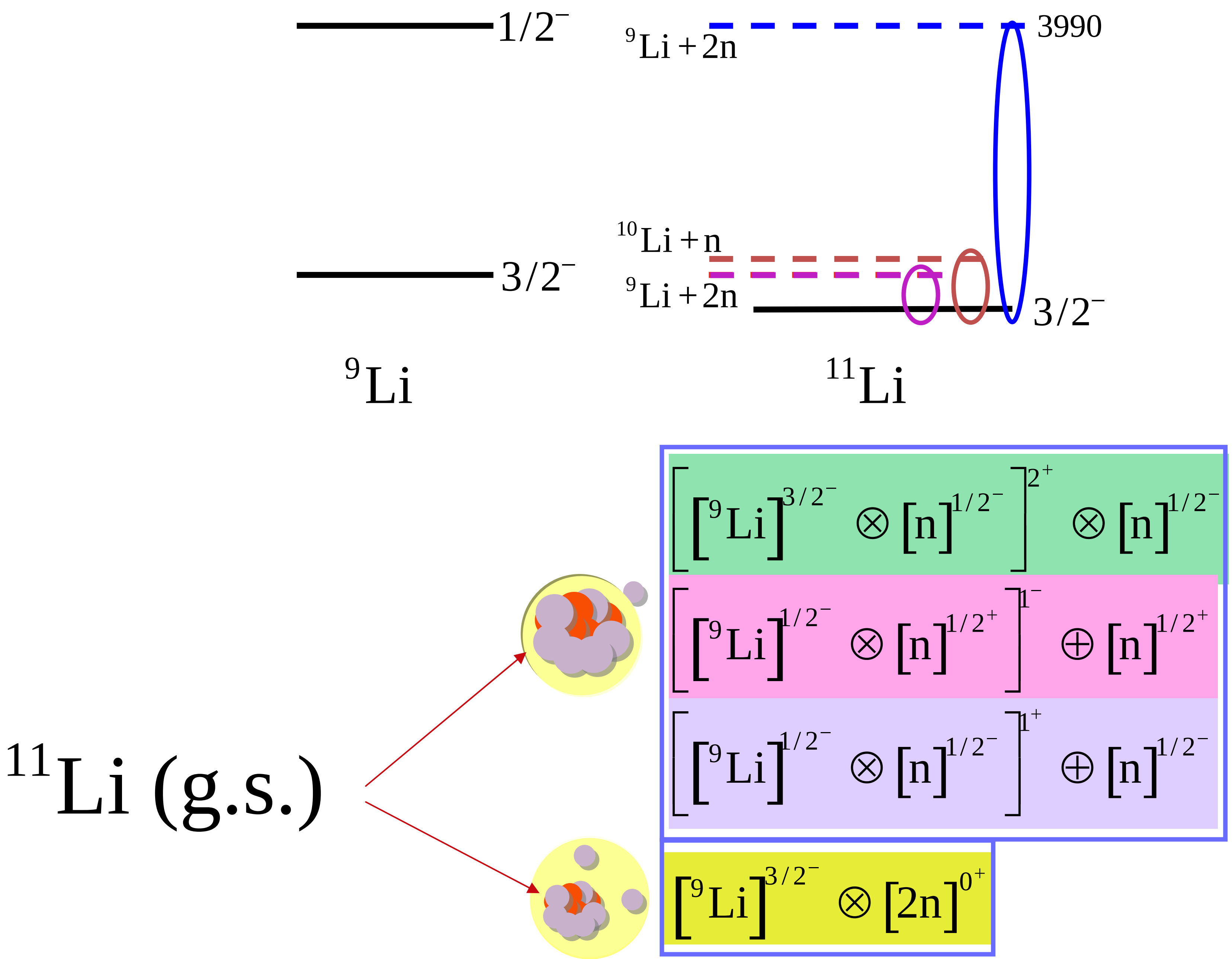
real — imaginary

$$\gamma_i^c(E, E') = \langle i | H - E | c; E' \rangle$$

$$\gamma_i^c(E) = \gamma_i^c(E, E)$$

coupling between closed and open space. It vanishes if $E<E_c$

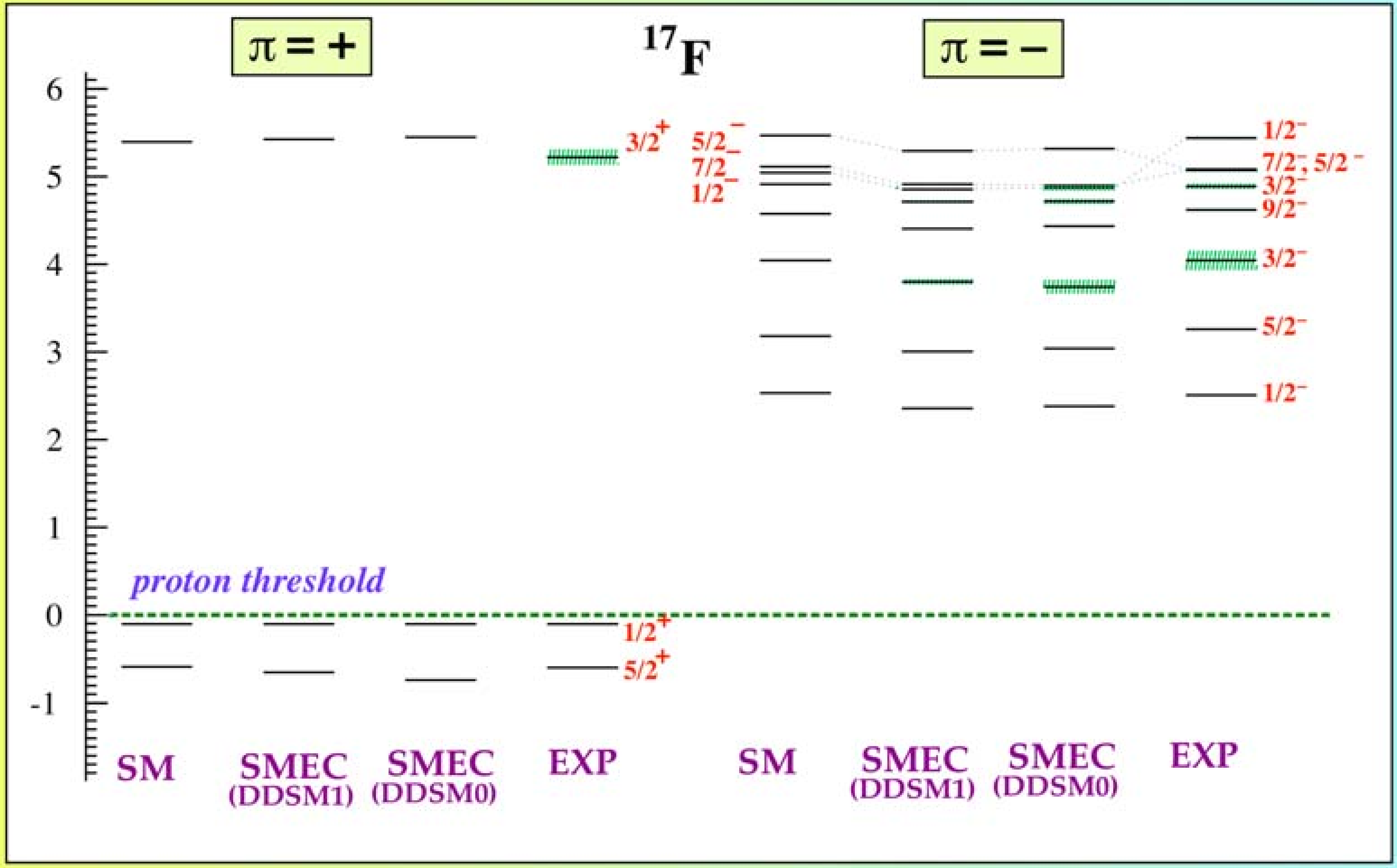
From M. Ploszajczak


$1/2^-$
$^9Li + 2n$
3990
$^{10}Li + n$
$3/2^-$
$^9Li + 2n$
$3/2^-$
^9Li
^{11}Li
$\left[\left[^9Li \right]^{3/2^-} \otimes [n]^{1/2^-} \right]^{2^+} \otimes [n]^{1/2^-}$
$\left[\left[^9Li \right]^{1/2^-} \otimes [n]^{1/2^+} \right]^{1^-} \oplus [n]^{1/2^+}$
$\left[\left[^9Li \right]^{1/2^-} \otimes [n]^{1/2^-} \right]^{1^+} \oplus [n]^{1/2^-}$
^{11}Li (g.s.)
$\left[^9Li \right]^{3/2^-} \otimes [2n]^{0^+}$

Continuum Shell Model

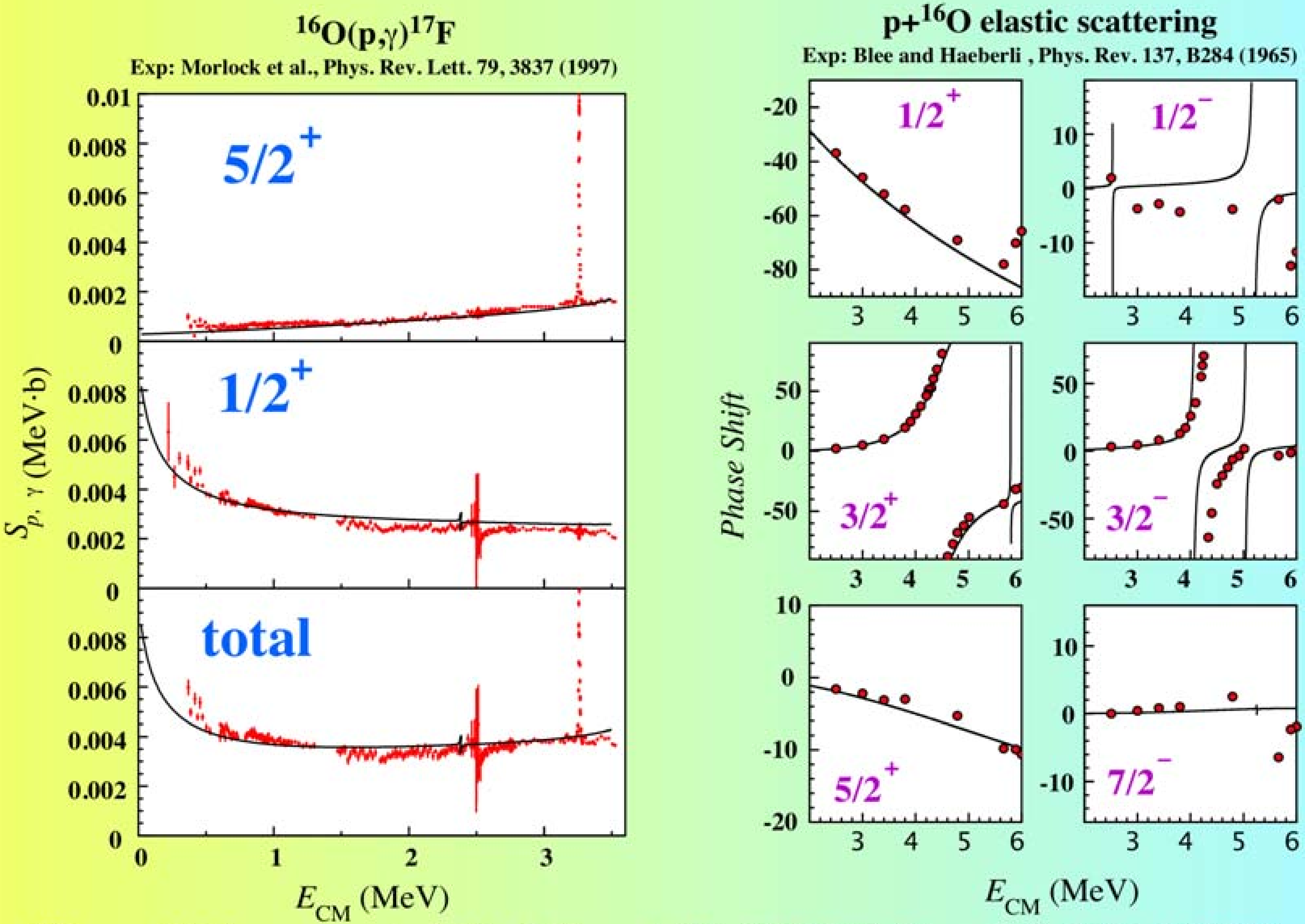
Q - quasi-bound states P - scattering states

H_{QQ} - shell model Hamiltonian; H_{PQ} - density-dependent



Theory: K. Bennaceur et al., Nucl. Phys. A651, 289 (1999); Phys. Lett. B488, 75 (2000)

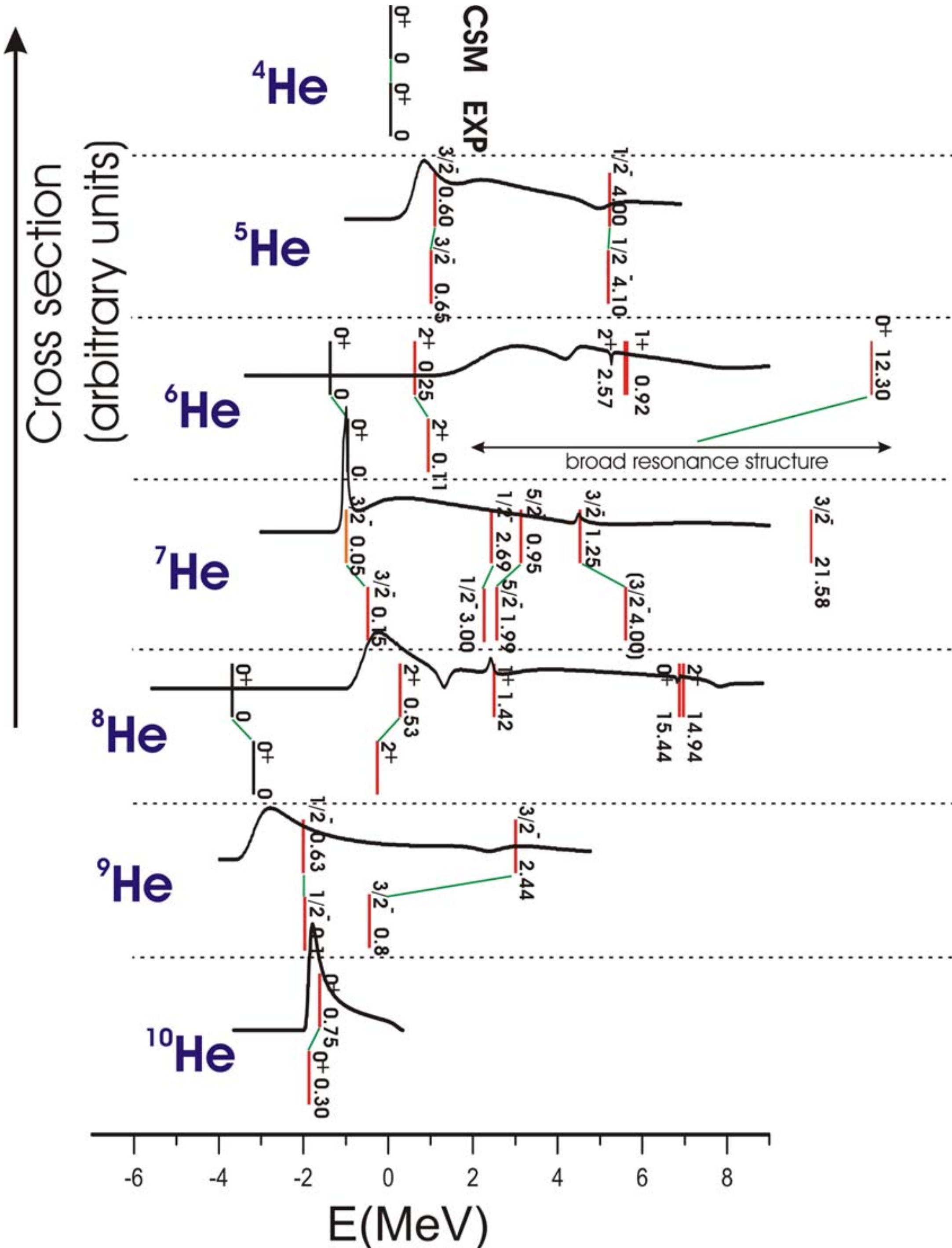
Continuum Shell Model



Theory: K. Bennaceur et al., Nucl. Phys. A651, 289 (1999); Phys. Lett. B488, 75 (2000)

Continuum Shell Model
He isotopes

- Cross section and structure within the same formalism
- Reaction l=1 polarized elastic channel

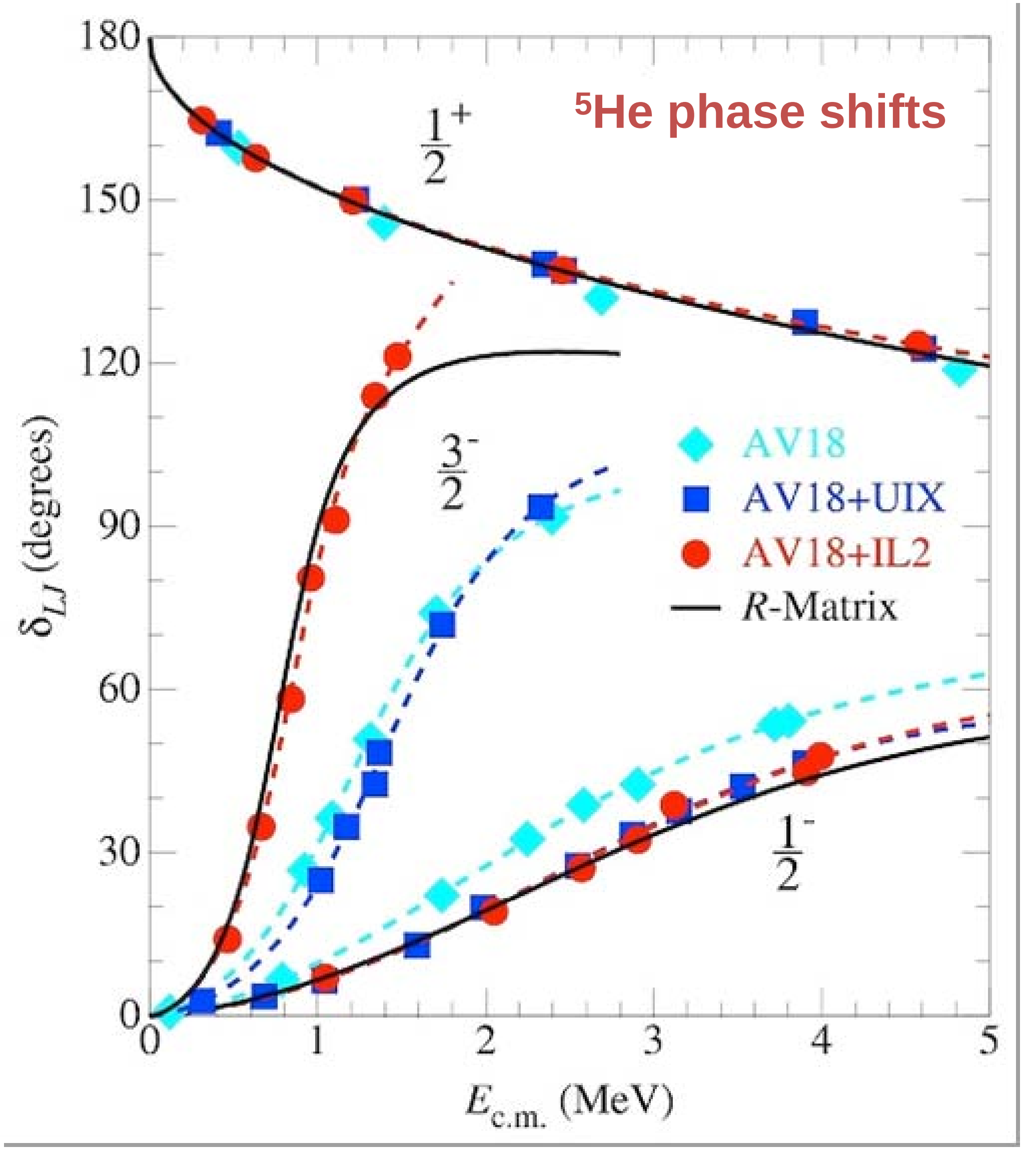


References

[1] A. Volya and V. Zelevinsky, Phys. Rev. Lett 94 (2005) 052501.
[2] A. Volya and V. Zelevinsky, Phys. Rev. C 67 (2003) 54322

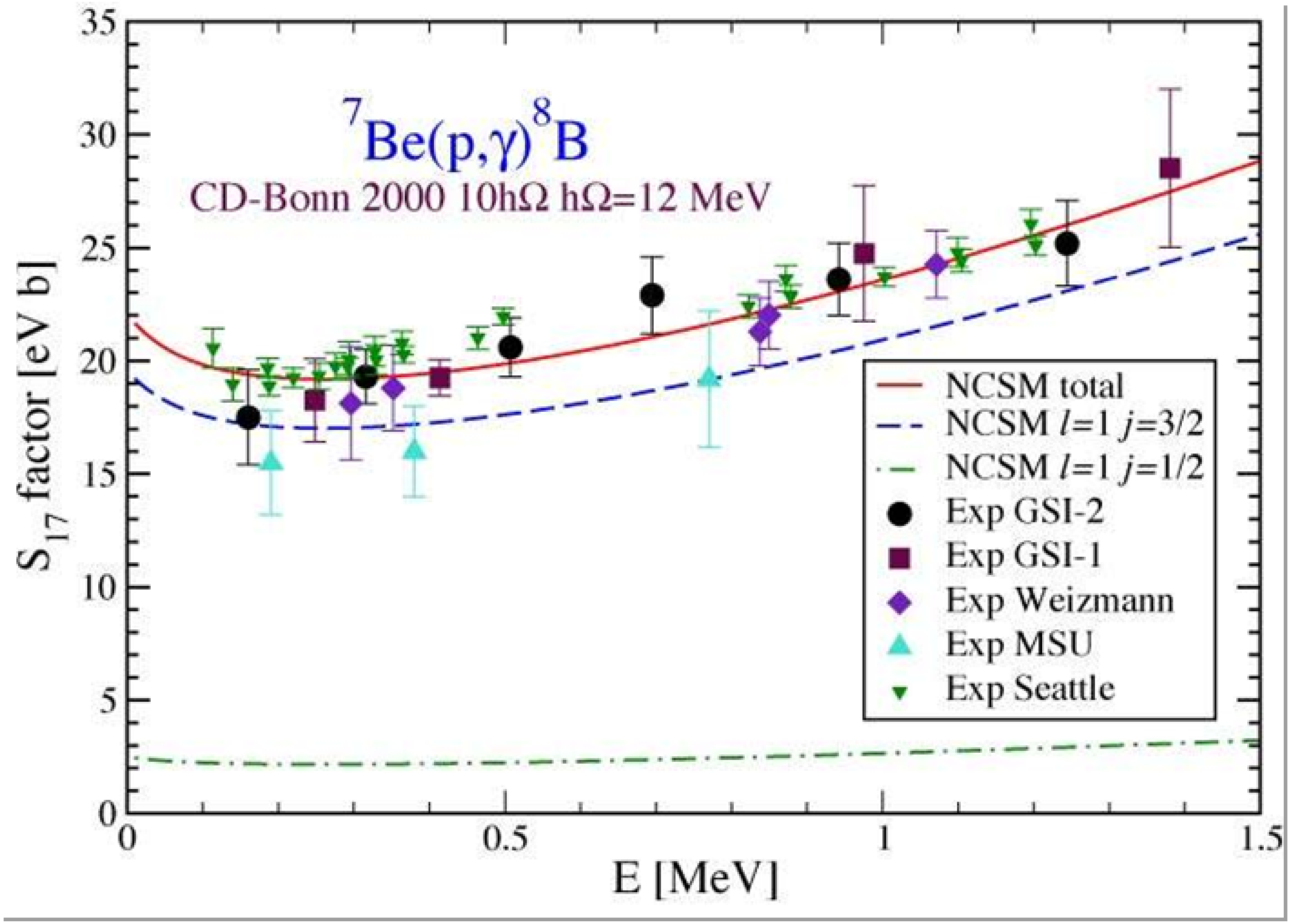
Neutron Scattering with Quantum Monte Carlo

K. Nollett et al., Phys. Rev. Lett. 99, 022502 (2007)

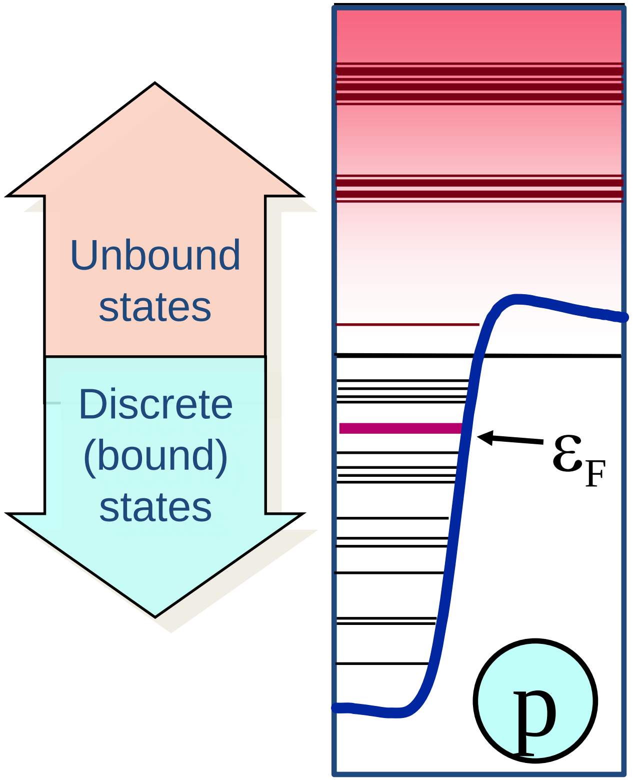


No Core Shell Model + Resonating Group Method

(S. Quaglioni and P. Navratil, arXiv 0901.0950v1

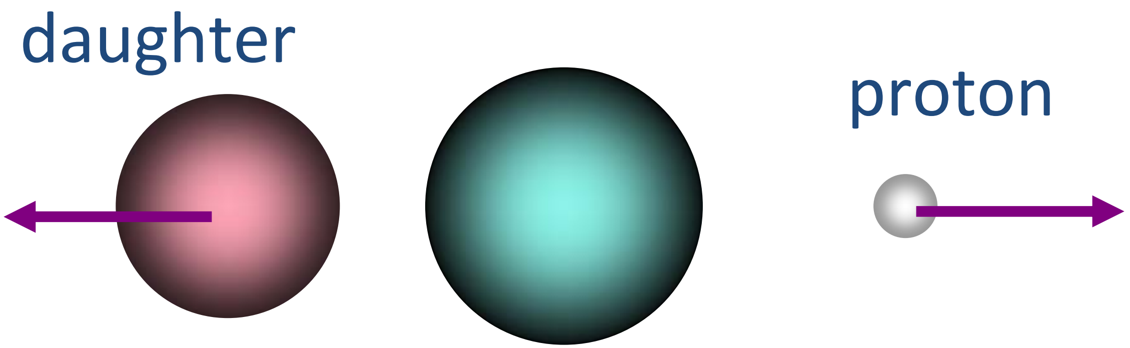


Description of Proton Emitters

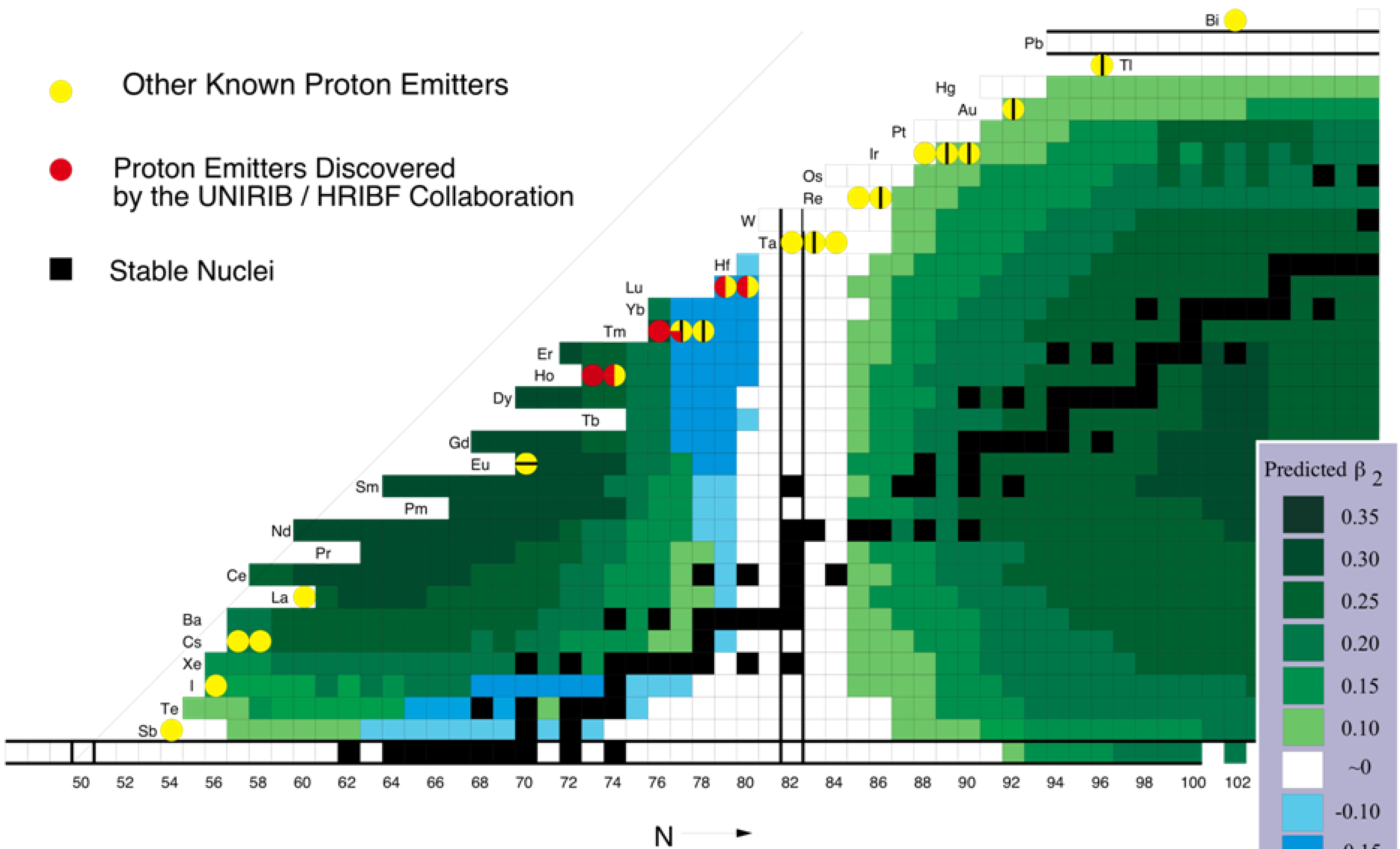


$$\ln 2 / T_{1/2} \propto \left(Ze^2 / R_c - Q_{p,nucl}\right)^{-1/2} \exp\left(-2G_{jl}\right)$$

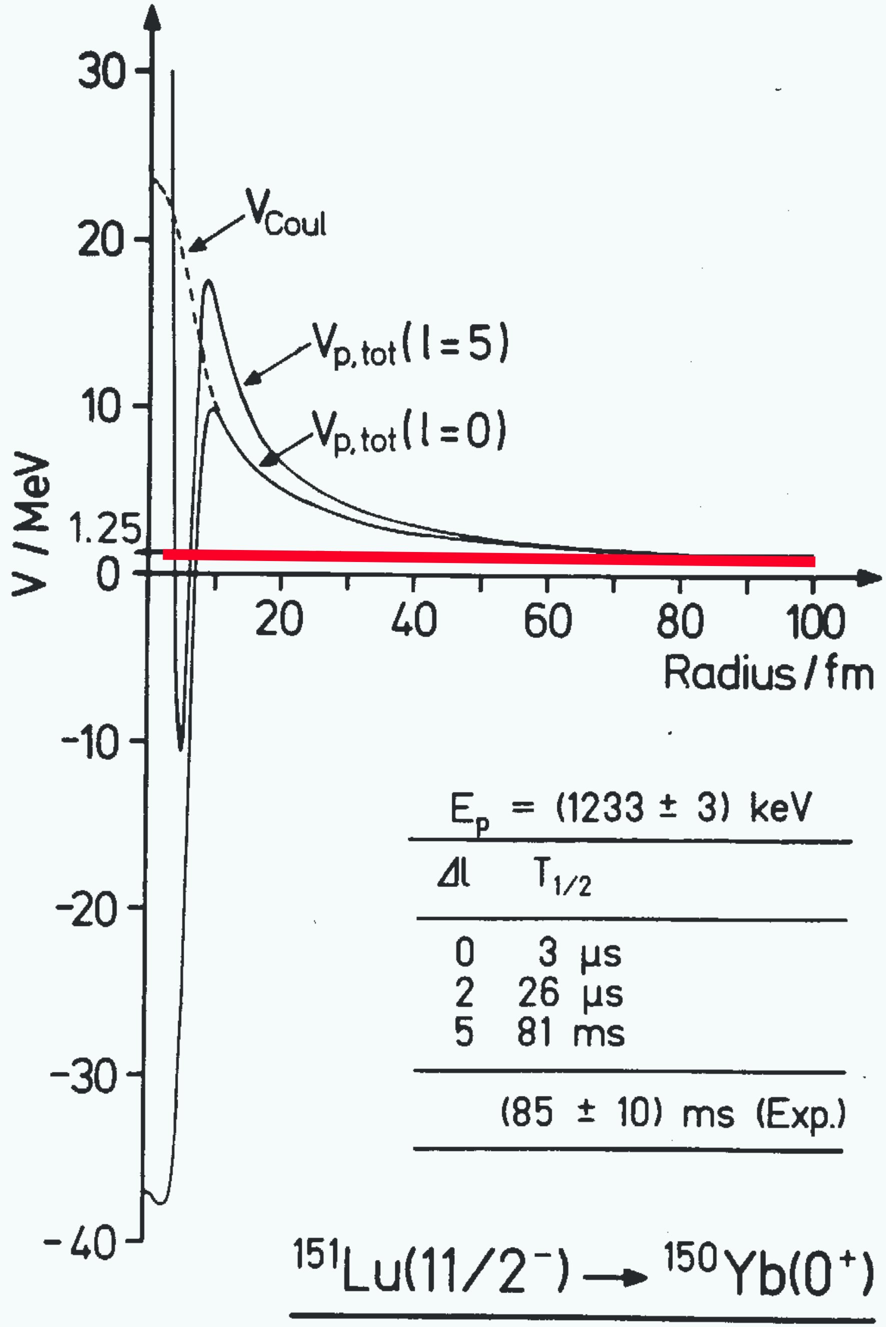
$$G_{jl} = \sqrt{2m/\hbar^2} \int_{R_{\text{in}}}^{R_{\text{out}}} dr \sqrt{V_{jl}(r) + V_{Coul}(r) + V_l(r) - Q_{p,\,nucl}}$$



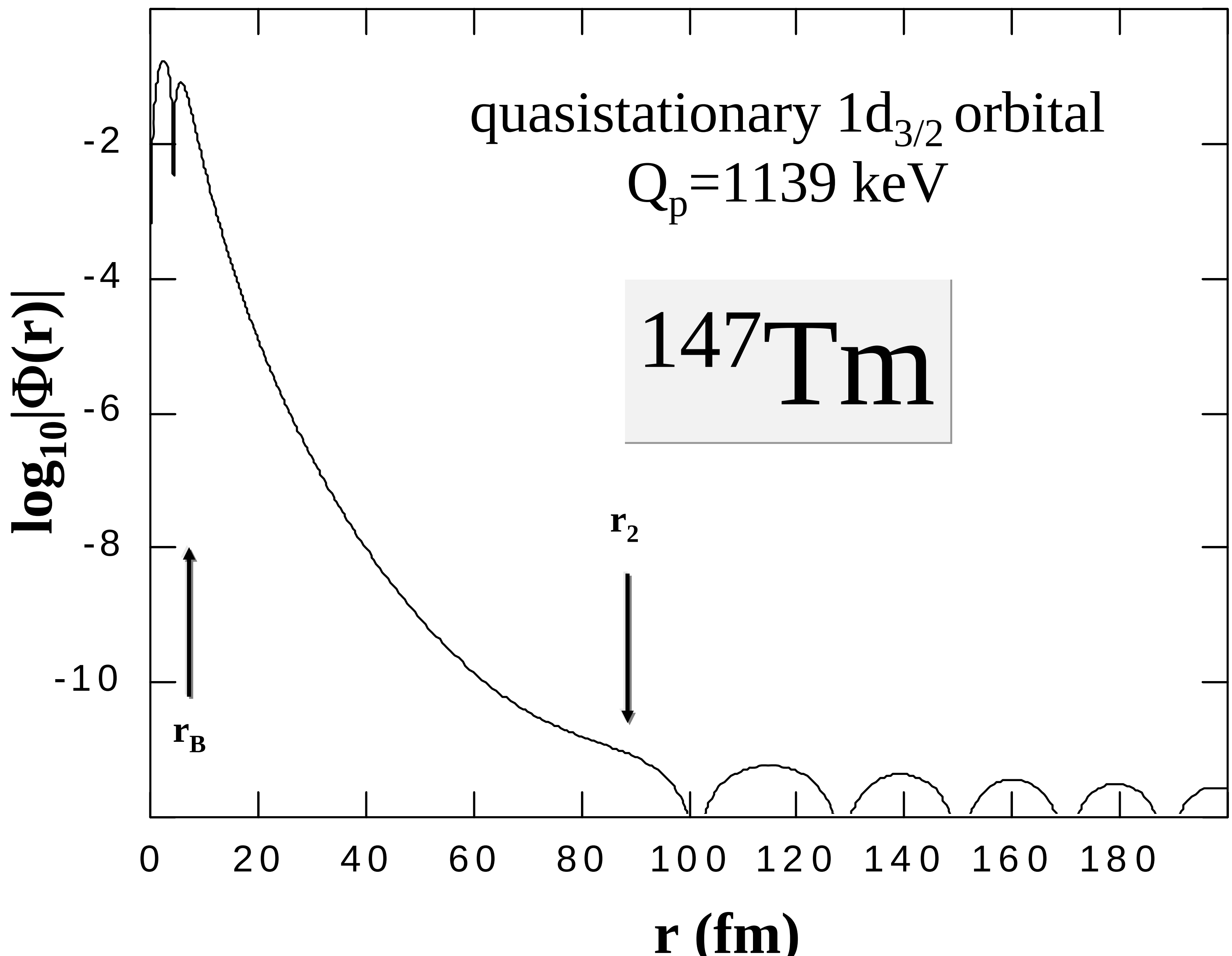
$$\Gamma = 2\pi T_{i\to f}^2$$



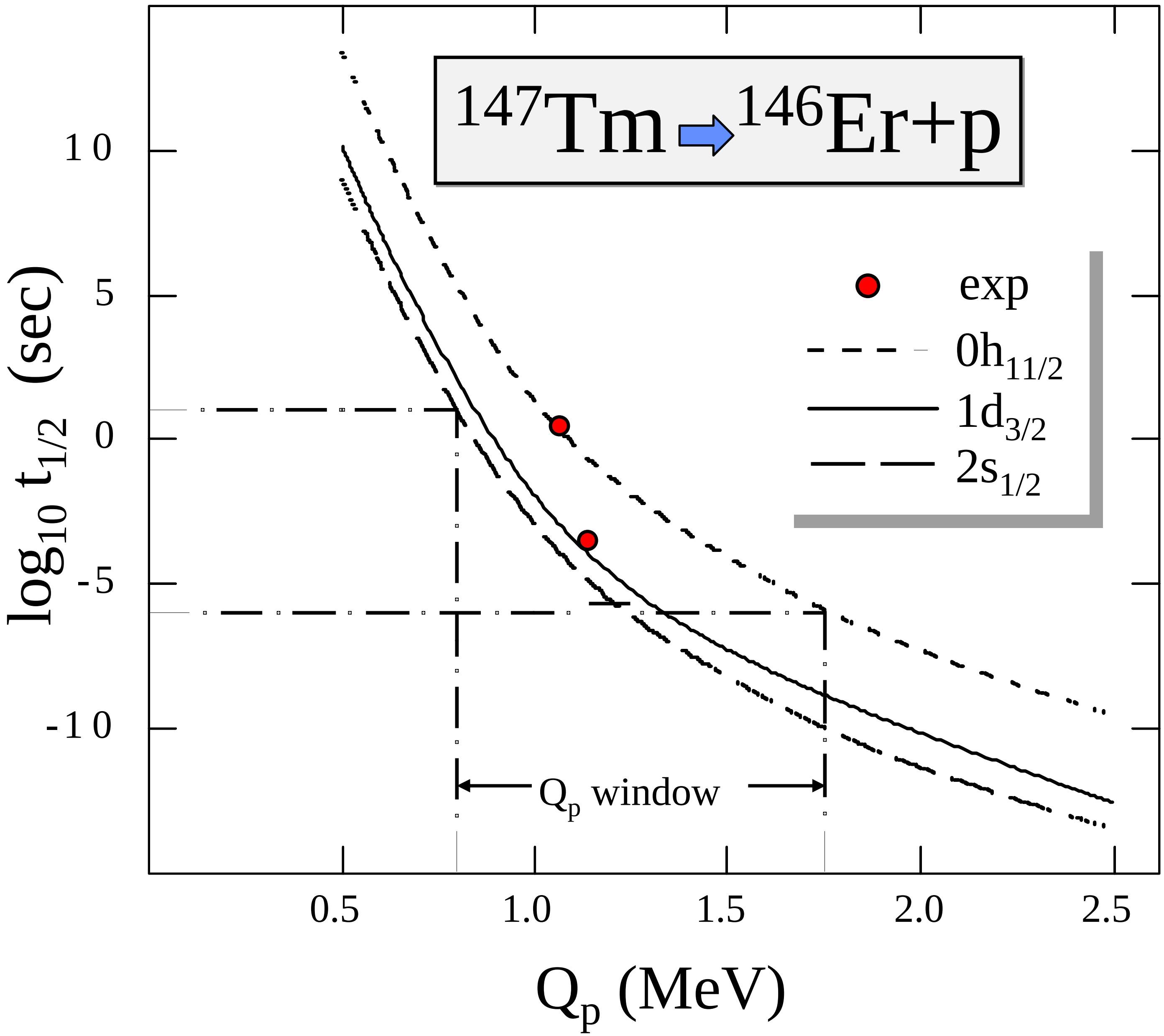
Other Known Proton Emitters
Proton Emitters Discovered
by the UNIRIB / HRIBF Collaboration
Stable Nuclei
Bi
Pb
Tl
Hg
Au
Pt
Ir
Os
Re
W
Ta
Hf
Lu
Yb
Tm
Er
Ho
Dy
Tb
Gd
Eu
Sm
Pm
Nd
Pr
Ce
La
Ba
Cs
Xe
I
Te
Sb
50 52 54 56 58 60 62 64 66 68 70 72 74 76 78 80 82 84 86 88 90 92 94 96 98 100 102
N
Predicted β 2
0.35
0.30
0.25
0.20
0.15
0.10
~0
-0.10
-0.15



V / MeV
30
20
10
1.25
0
-10
-20
-30
-40
20
40
60
80
100
Radius / fm
V_Coul
V_p,tot (l=5)
V_p,tot (l=0)
E_p = (1233 ± 3) keV
Δl T_1/2
0 3 μs
2 26 μs
5 81 ms
(85 ± 10) ms (Exp.)
151Lu(11/2−) → 150Yb(0+)

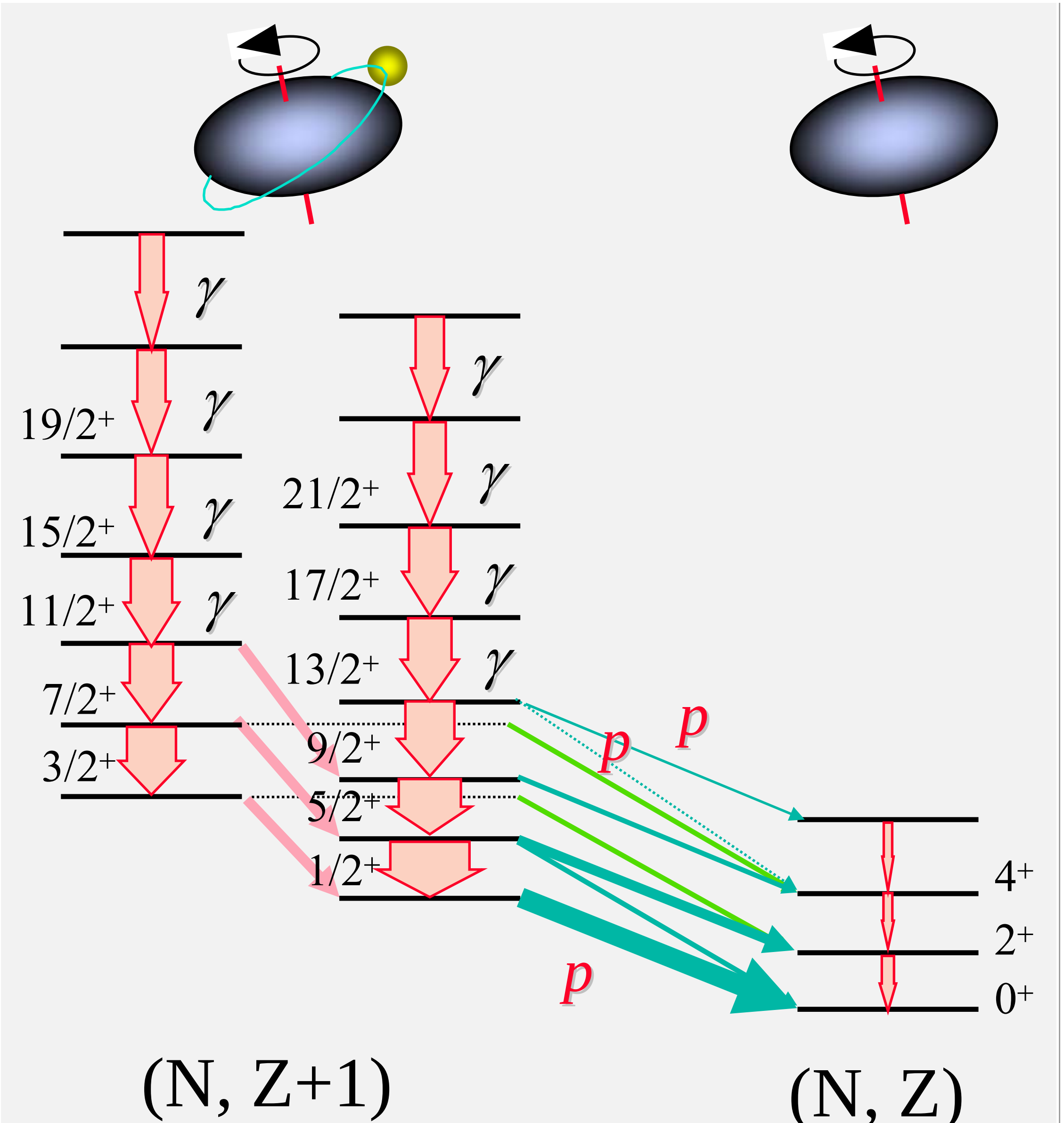


quasistationary 1d3/2 orbital
Qp=1139 keV
147Tm
log10|Φ(r)|
-2
-4
-6
-8
-10
r2
rB
0
20
40
60
80
100
120
140
160
180
r (fm)



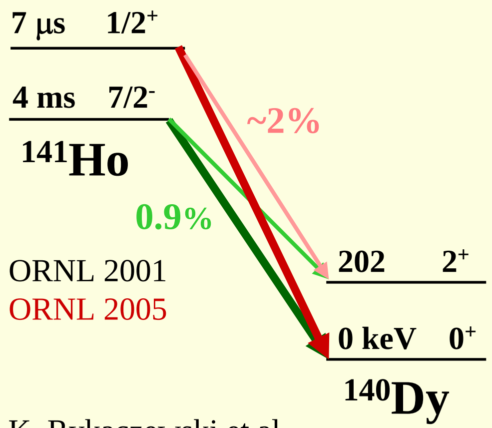
^{147}Tm → ^{146}Er+p
log$_{10}$ t$_{1/2}$ (sec)
10
5
0
-5
-10
exp
0h$_{11/2}$
1d$_{3/2}$
2s$_{1/2}$
Q$_p$ window
0.5
1.0
1.5
2.0
2.5
Q$_p$ (MeV)

Proton emission from deformed nuclei

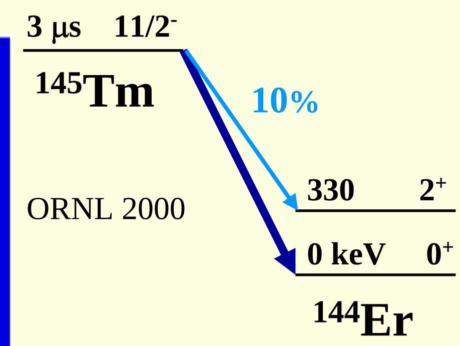


- Competition between gammas and protons (very narrow energy window!)
- Strong dependence of the proton width on:
  - deformation
  - angular momentum
  - residual interaction
- Angular momentum dependent coupling between bound states and narrow resonances
- Very clean three dimensional tunneling through the deformed barrier
- Non-standard approach to Nilsson orbitals!

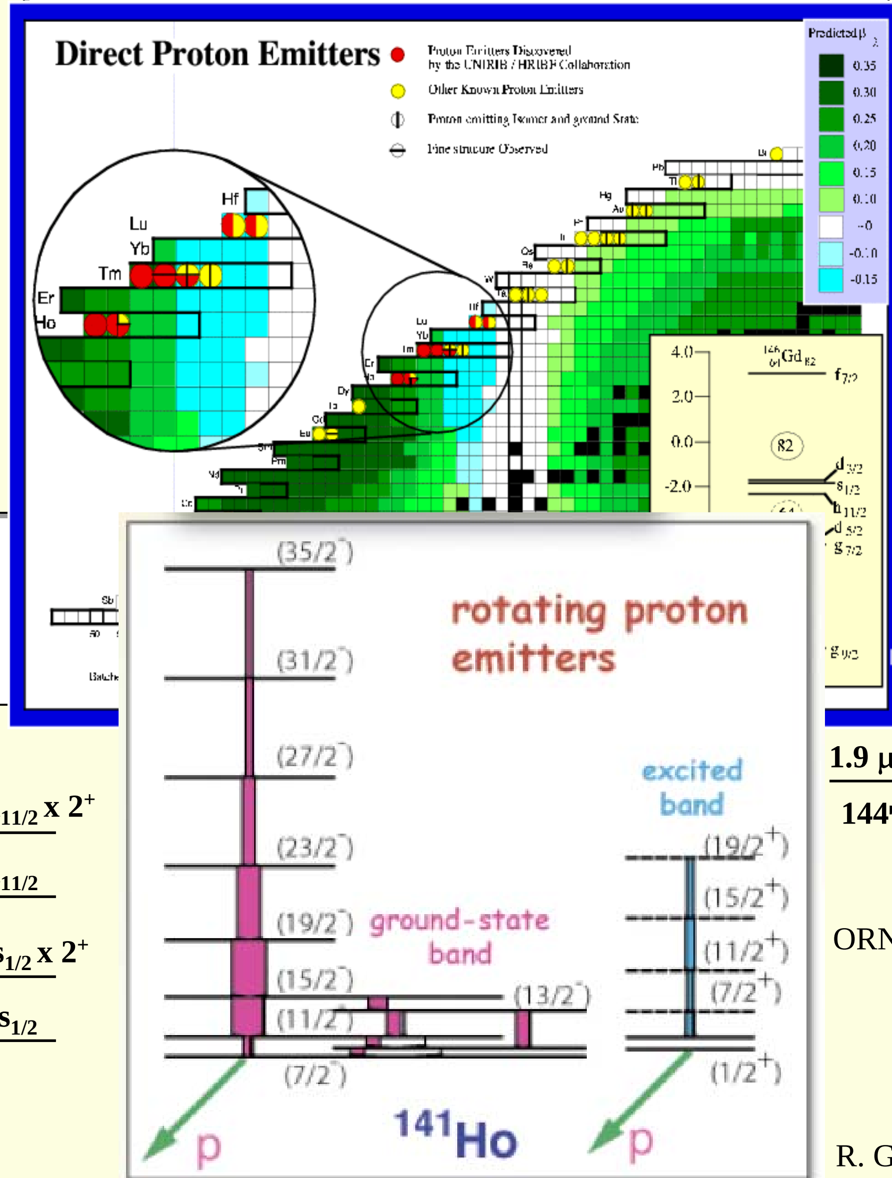
Fine structure in proton emission studied at the RMS?HRIBF

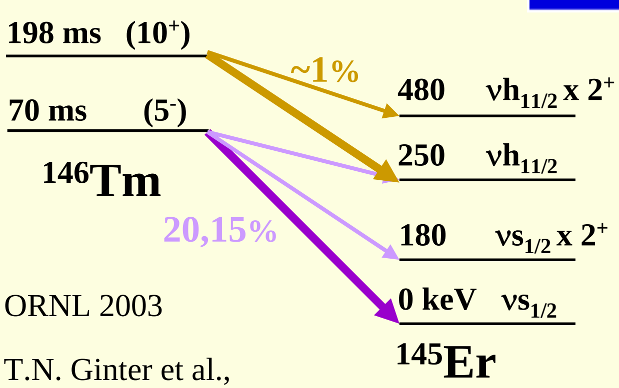


K. Rykaczewski et al.,
AIP 638, 149, 2002
M. Karny et al., PRL 2006

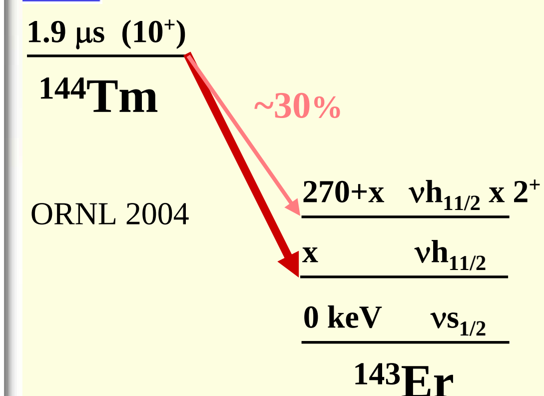


M. Karny et al.,
PRL 90, 012502, 2003





T.N. Ginter et al.,
PRC 68, 034330, 2003
M.N. Tantawy et al.,
PRC 73, 024316, 2006



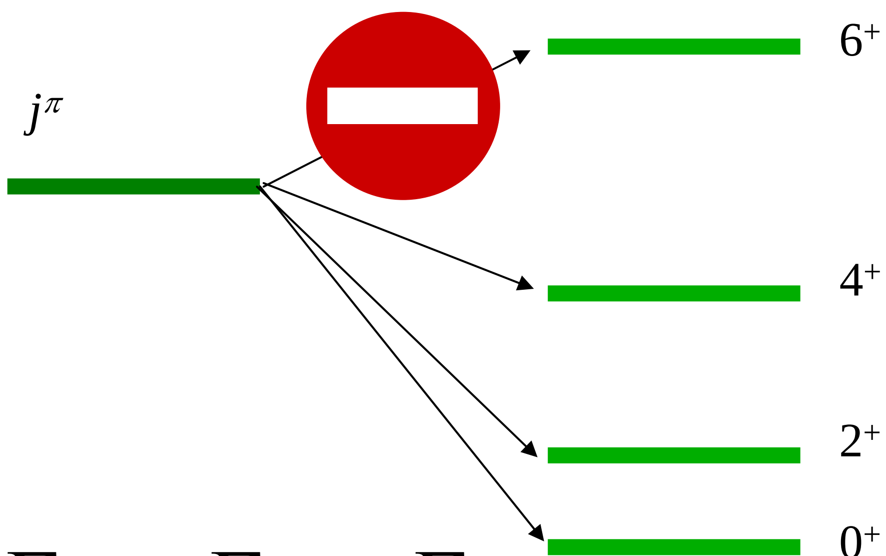
R. Grzywacz et al.,
EPJ A25, s01, 145, 2005

Fine structure in proton emission

$$\left\{-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}+\frac{\hbar^2}{2m}\frac{\ell(\ell+1)}{r^2}+V_N(r)+V_C(r)-Q_{J_f}\right\}\Psi_{J_f j\ell}(r)$$

$$+\sum_{J'_f j'\ell'} V_{J_f j\ell}^{J'_f j'\ell'}(r)\,\Psi_{J'_f j'\ell'}(r)=0$$

asymptotically depends on $k_{J_f} r$



$$\Gamma_{tot}=\Gamma_{0^+}+\Gamma_{2^+}+\Gamma_{4^+}$$

- Strong dependence on Q and l
- Coupling between channels not negligible!

Some References

Particle-core vibration coupling

C.N.Davids, H.Esbensen, Phys.Rev. C69, 034314 (2004)
M.Karny et al., Phys.Rev.Lett. 90, 012502 (2003)
K.Hagino, Phys.Rev. C64, 041304 (2001)
C.N.Davids, H.Esbensen, Phys.Rev. C64, 034317 (2001)
H. Esbensen and C.N. Davids, Phys. Rev. C64, 034317 (2001)

Deformed proton emitters, coupled-channels, R-matrix

B.Barmore et al., Phys.Rev. C62, 054315 (2000)
H.Esbensen, C.N.Davids, Phys.Rev. C63, 014315 (2001)
W.Krolas et al., Phys.Rev. C65, 031303 (2002)
A.T. Kruppa and W. Nazarewicz, Phys.Rev. C69, 054311 (2004)
M. Karny et al., Physics Letters B664, 52 (2008)

BCS treatment of pairing

G.Fiorin, E.Maglione, L.S.Ferreira, Phys.Rev. C67, 054302 (2003)

A.T. Kruppa and W. Nazarewicz, Phys. Rev. C69, 054311 (2004)

Test case: deformed WS+Coulomb potential for ^{141}Ho

β_2=0.244, k_{max}=2 fm^{-1}, l_{max}=11, N_{scat}=60

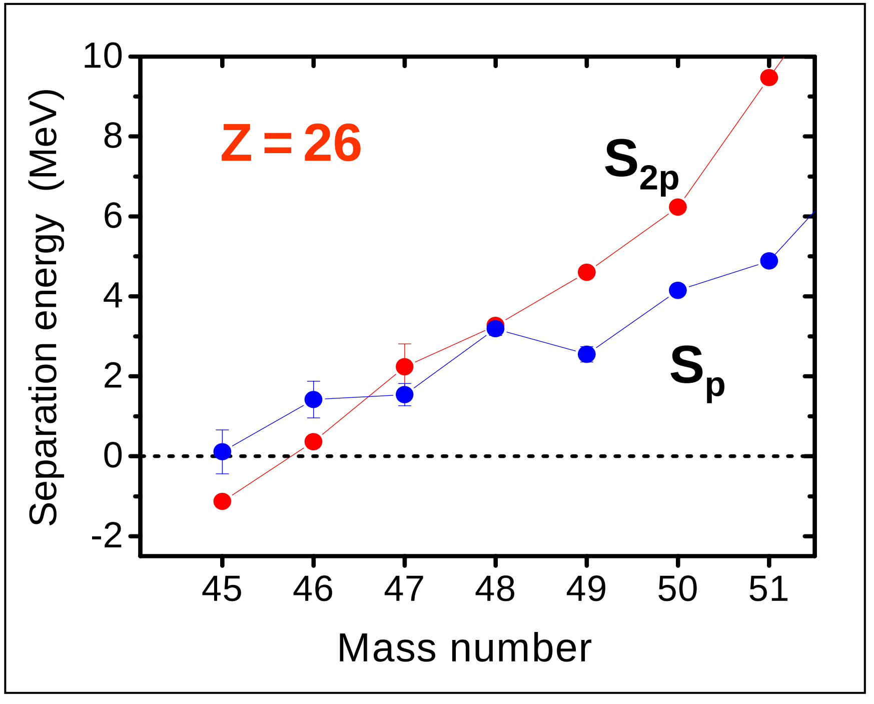
Ω^π	CC	HO diag.	Gamow diag.
$\frac{1}{2}^+$	0.756 ($1.98\cdot10^{-20}$)	0.756 ($2.018\cdot10^{-20}$)	0.758 ($2.191\cdot10^{-20}$)
	3.968 (0.053)	3.968 (0.050)	3.970 (0.051)
	5.454 (0.035)	5.454 (0.037)	5.465 (0.032)
	10.214 (833)	-	10.534 (237)
	11.686 (729)	-	11.692 (651)
	21.777 (527)	-	21.809 (544)
$\frac{7}{2}^-$	1.190 ($3.24\cdot10^{-16}$)	1.190 ($3.29\cdot10^{-16}$)	1.194 ($2.66\cdot10^{-16}$)
	8.789 (17.51)	-	8.790 (17.53)
	9.933 (178)	-	9.934 (178)
	15.360 (104)	-	15.375 (104)

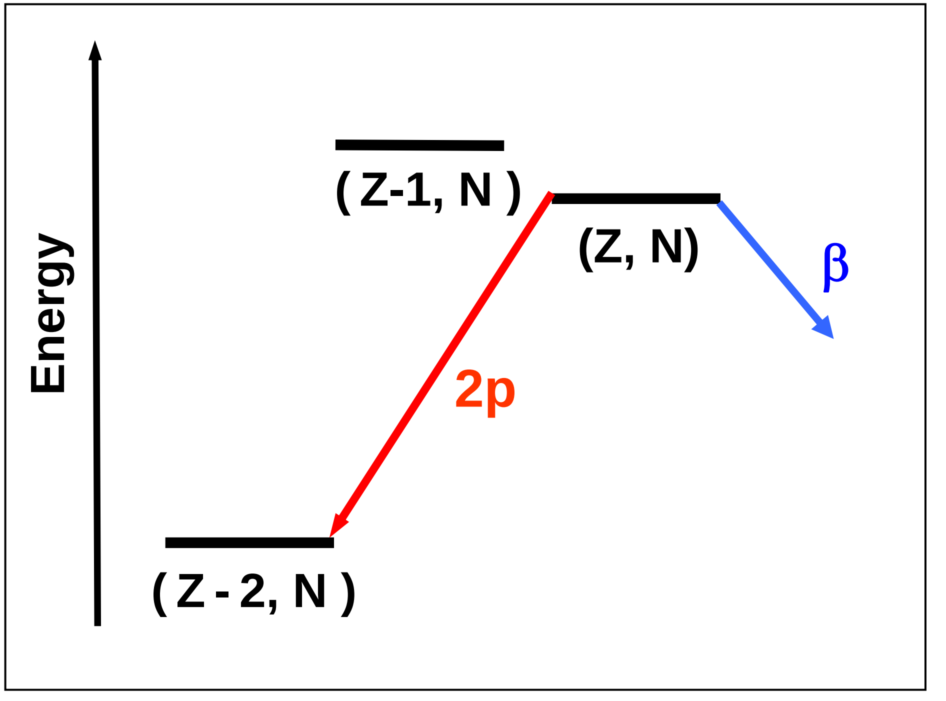
Two-proton radioactivity of ^{45}Fe

(from Z. Janas, Warsaw)

Ground-state *2p* radioactivity

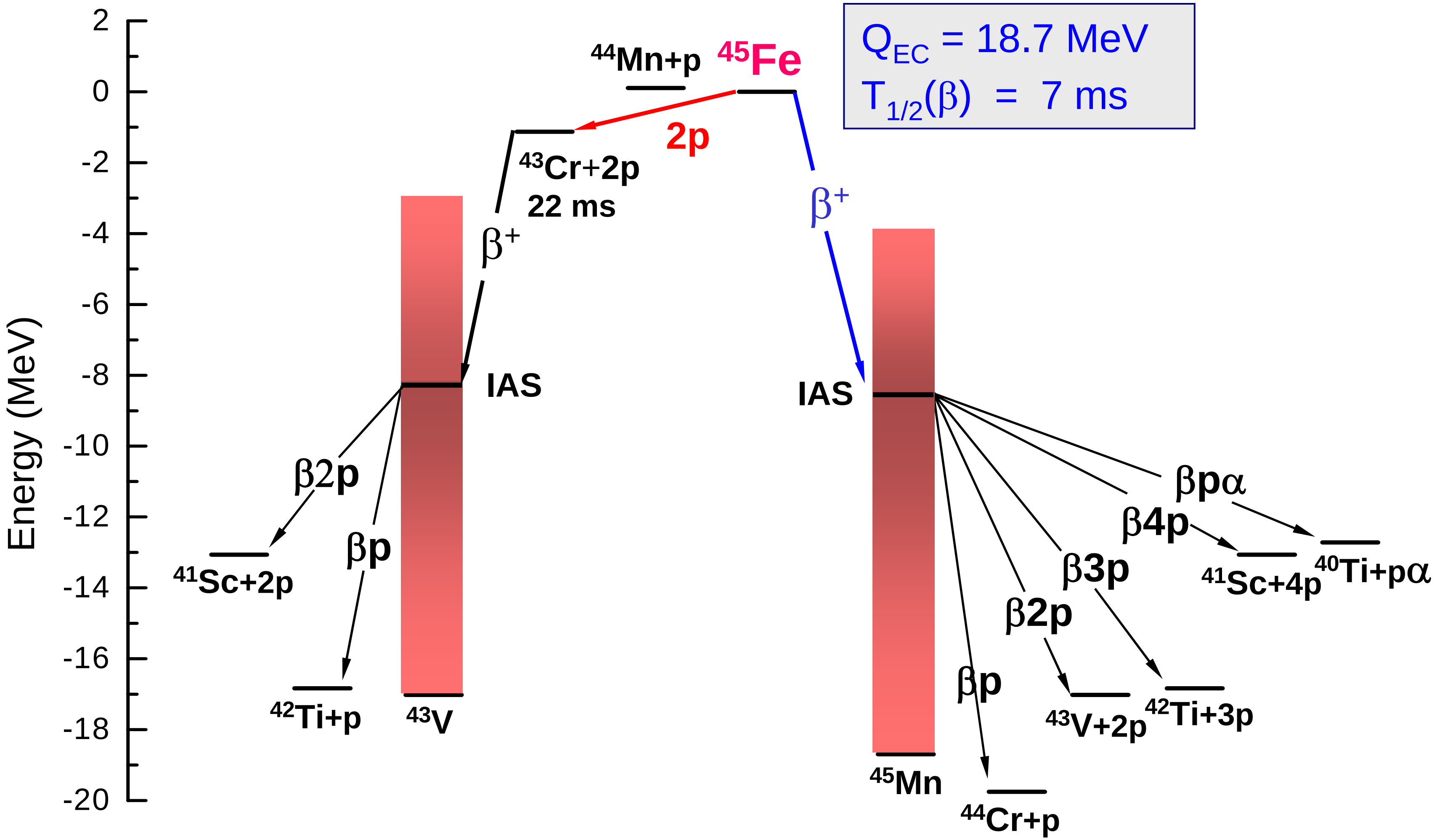
- **predicted by V. Goldansky in 1960**



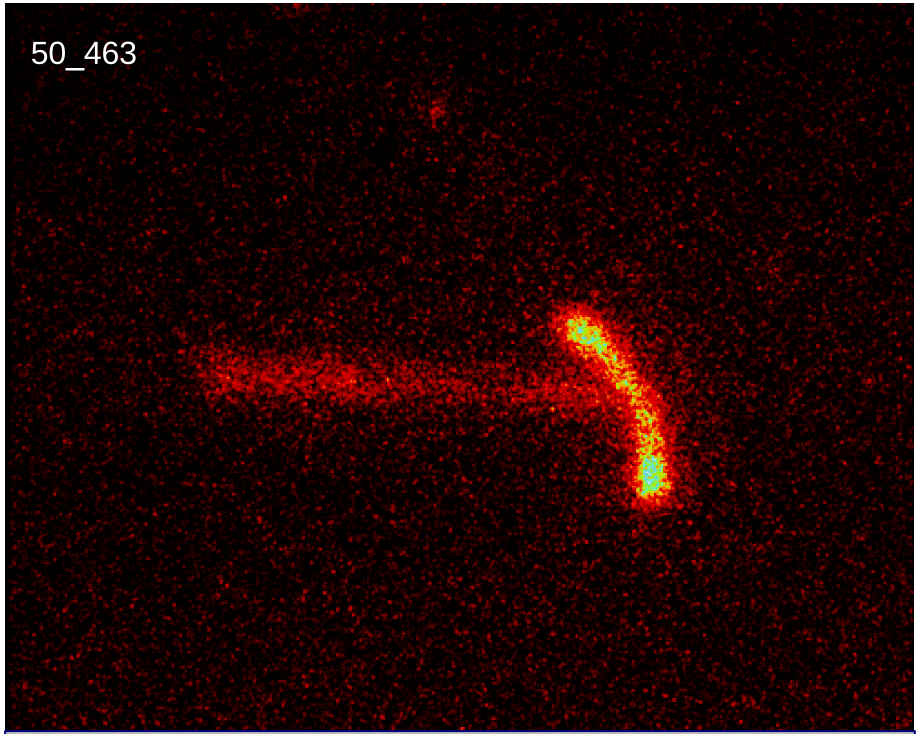


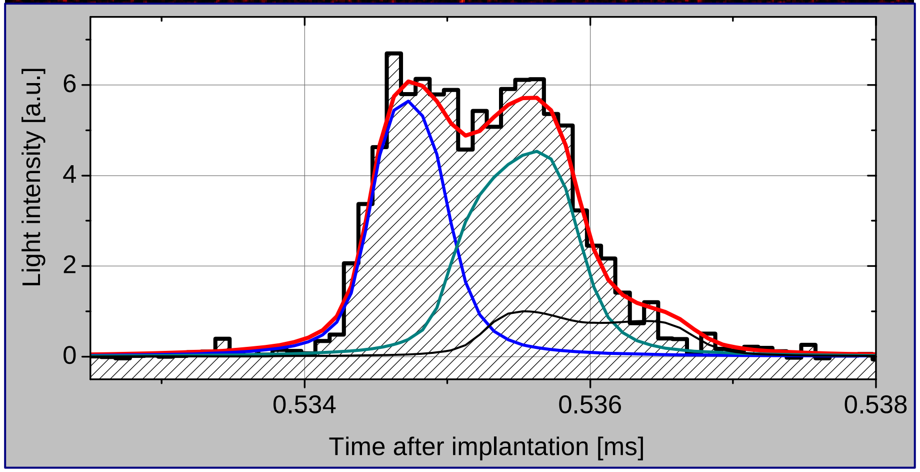
- single proton branch closed
- large Q_β value

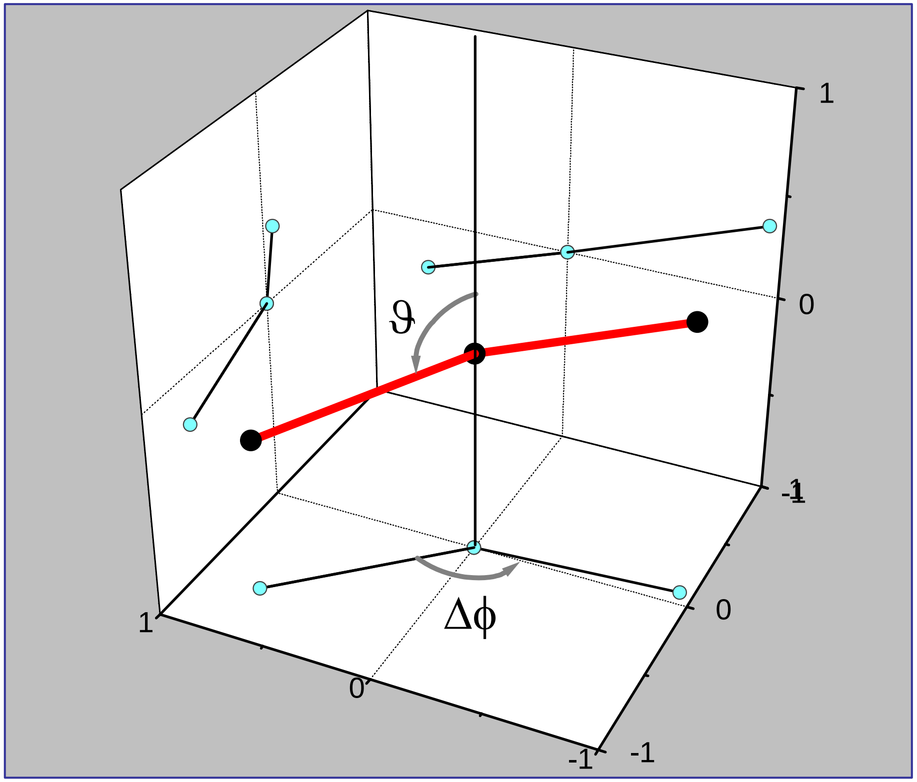
Expected decay scheme of ^{45}Fe



Event reconstruction



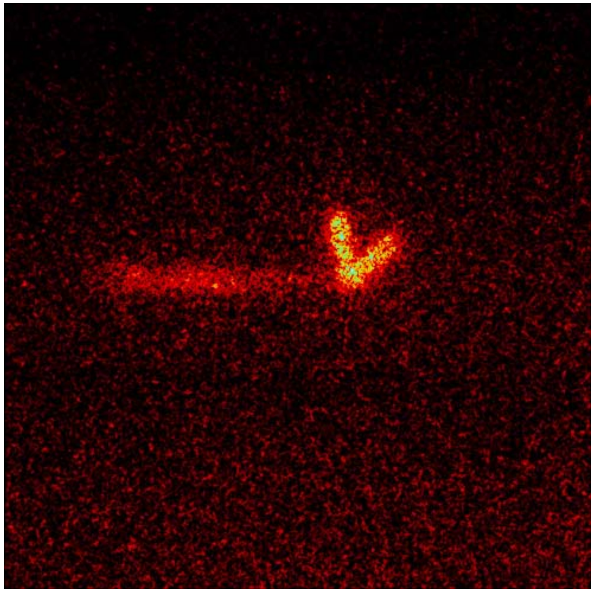


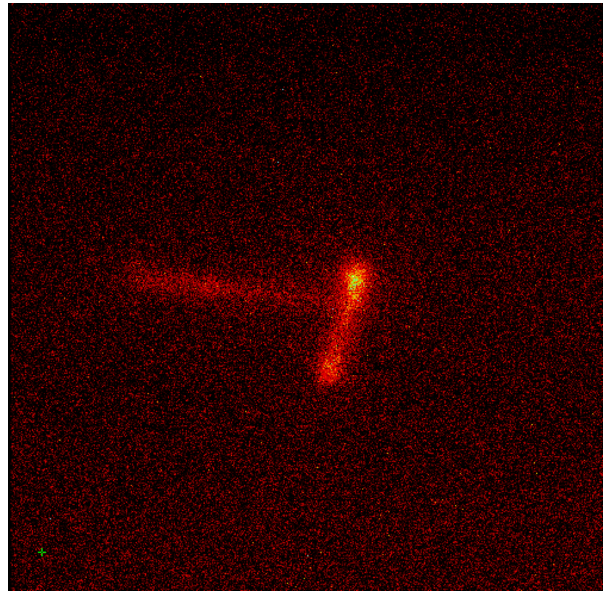


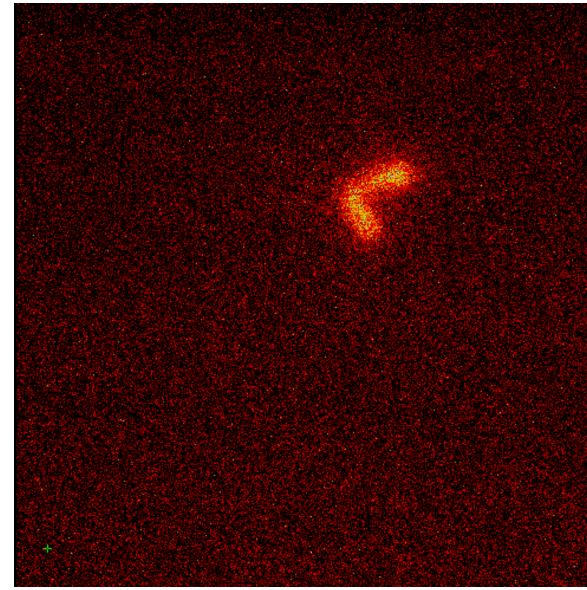
$\vartheta_1 = (104 \pm 2)°$, $\vartheta_1 = (70 \pm 3)°$

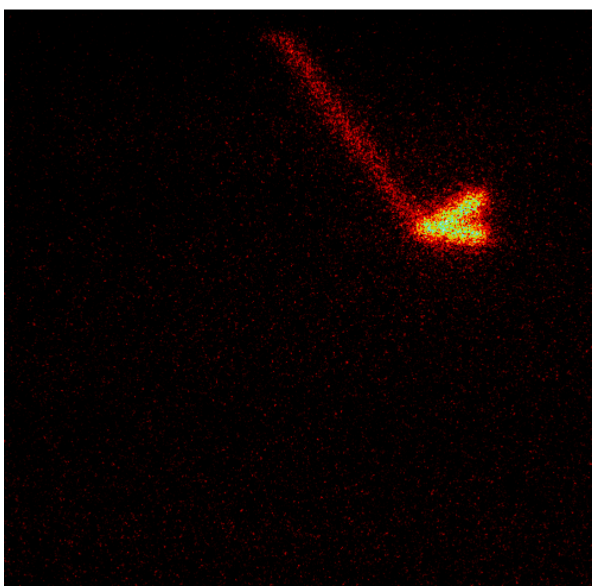
$\Delta\phi = (142 \pm 3)^o$ ➔ $\theta_{pp} = (143 \pm 5)^o$

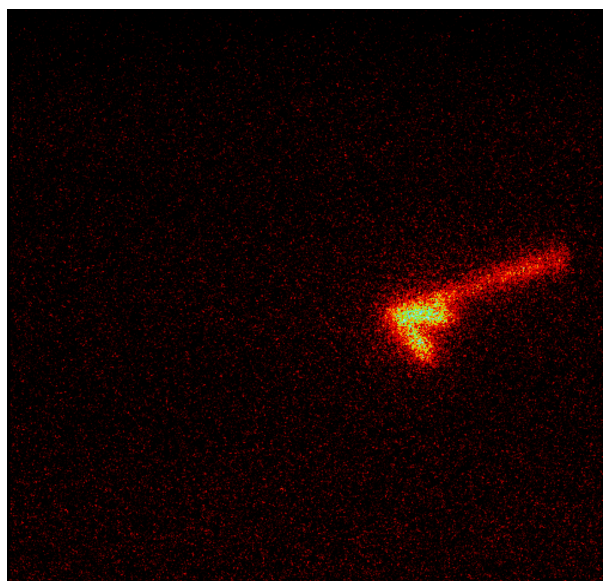
Gallery of 2*p* events





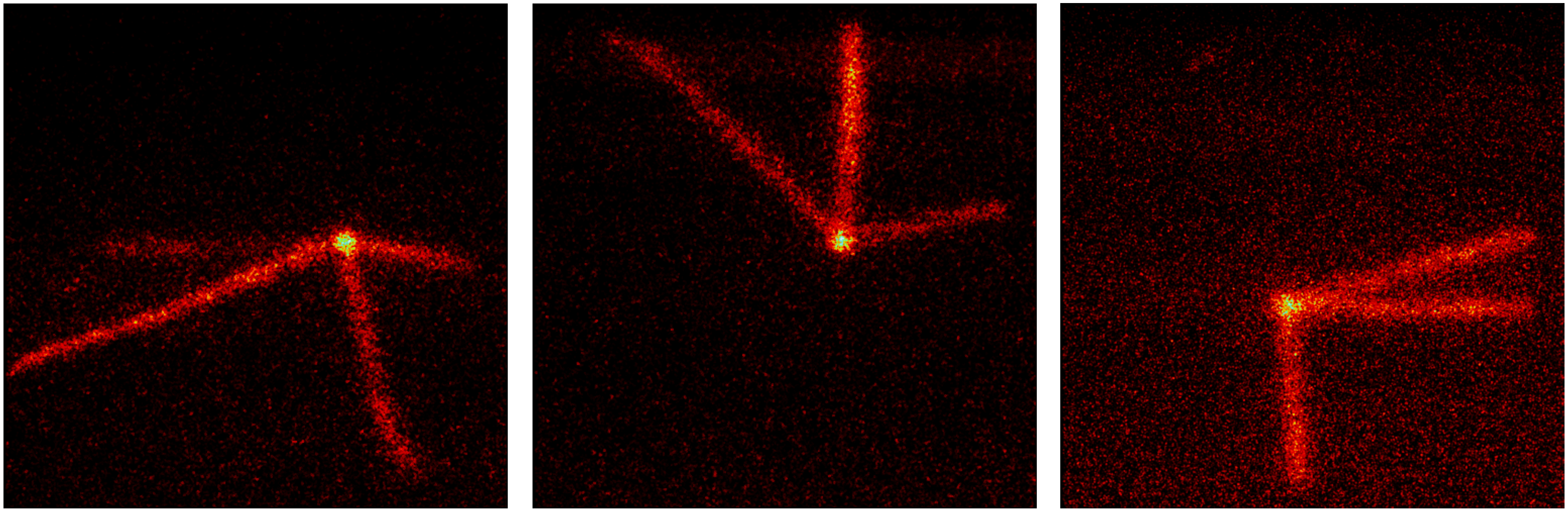






87 2*p* decay events

First observation of β3p !

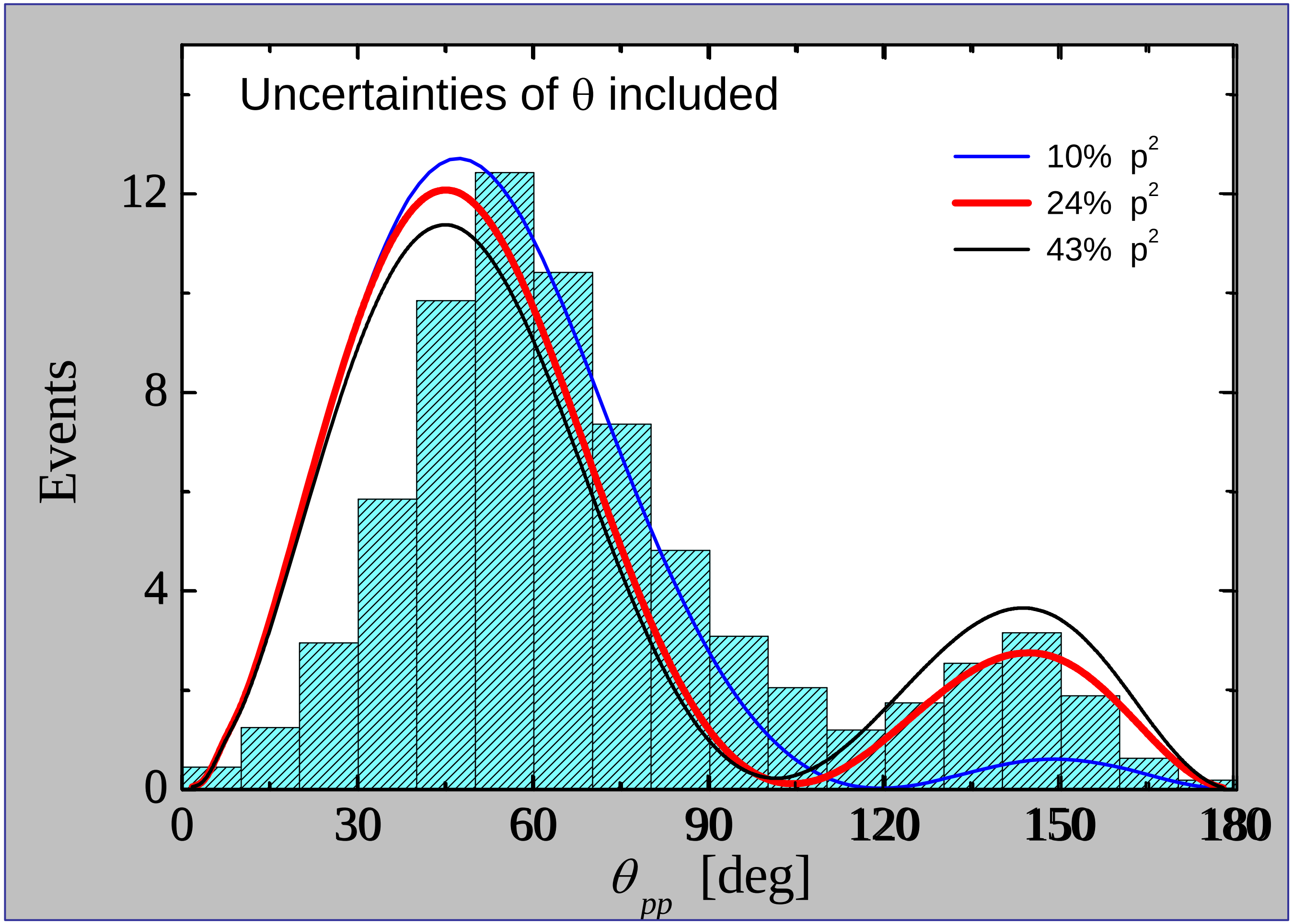


38 β - decay events:

- **24 – βp**
- **10 – β2p**
- **4 – β3p**

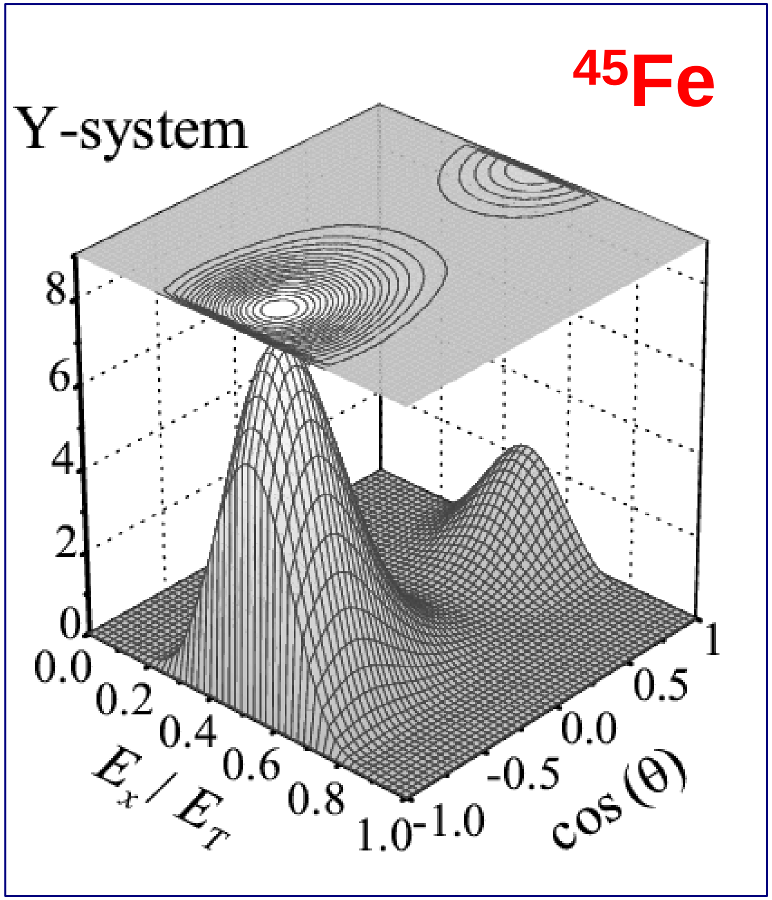
K. Miernik *et al*., PRC 76 (2007) 041304(R)

p-p angular correlation



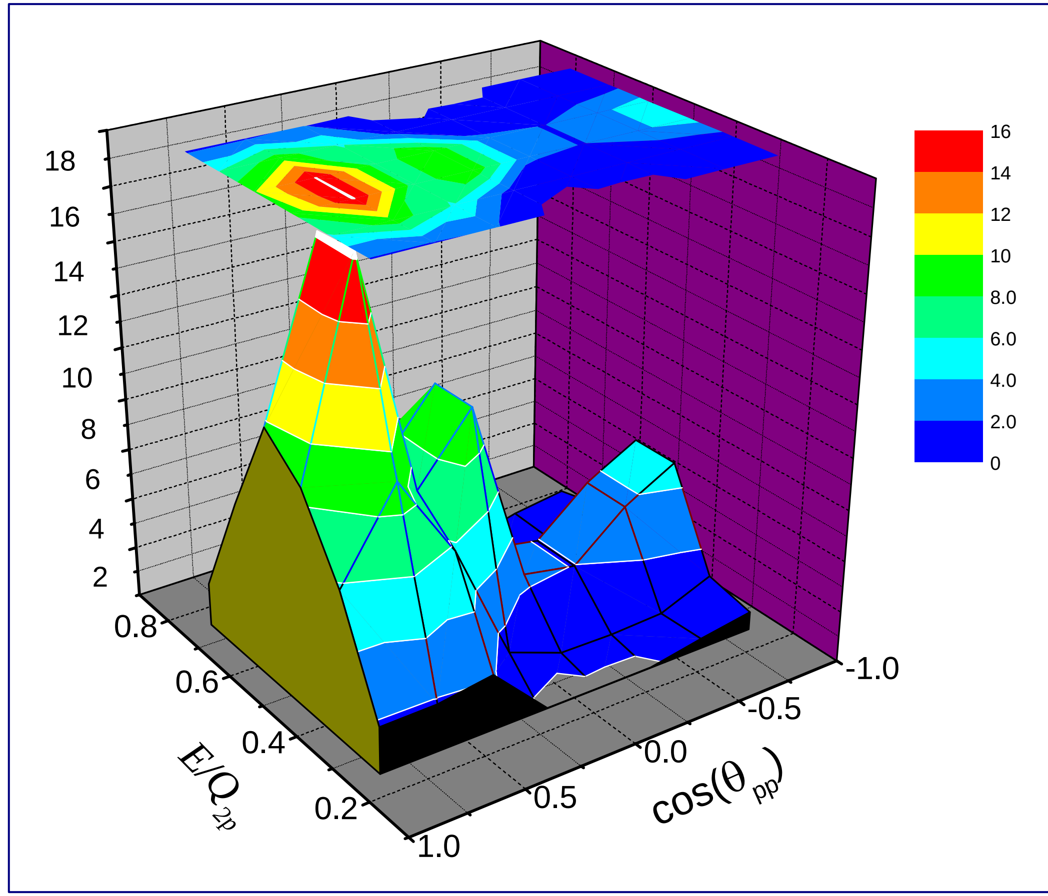
K. Miernik *et al*., PRL 99, 192501 (2007)
L.V. Grigorenko and M.V. Zhukov, PRC 68 (2003) 054005

Energy - angle 2D correlation



3-body model prediction

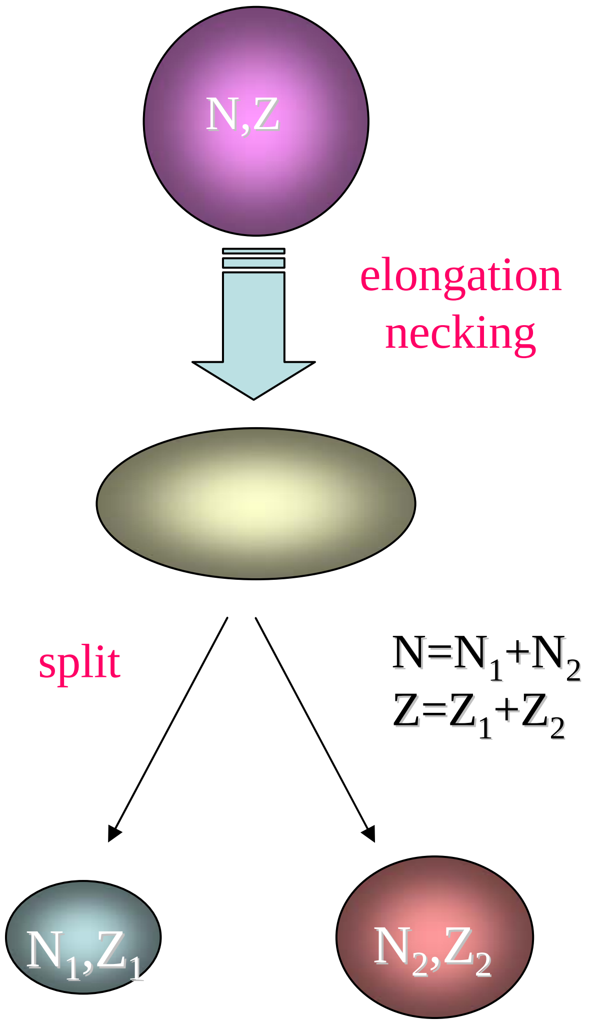
25% p^2 + 75% f^2

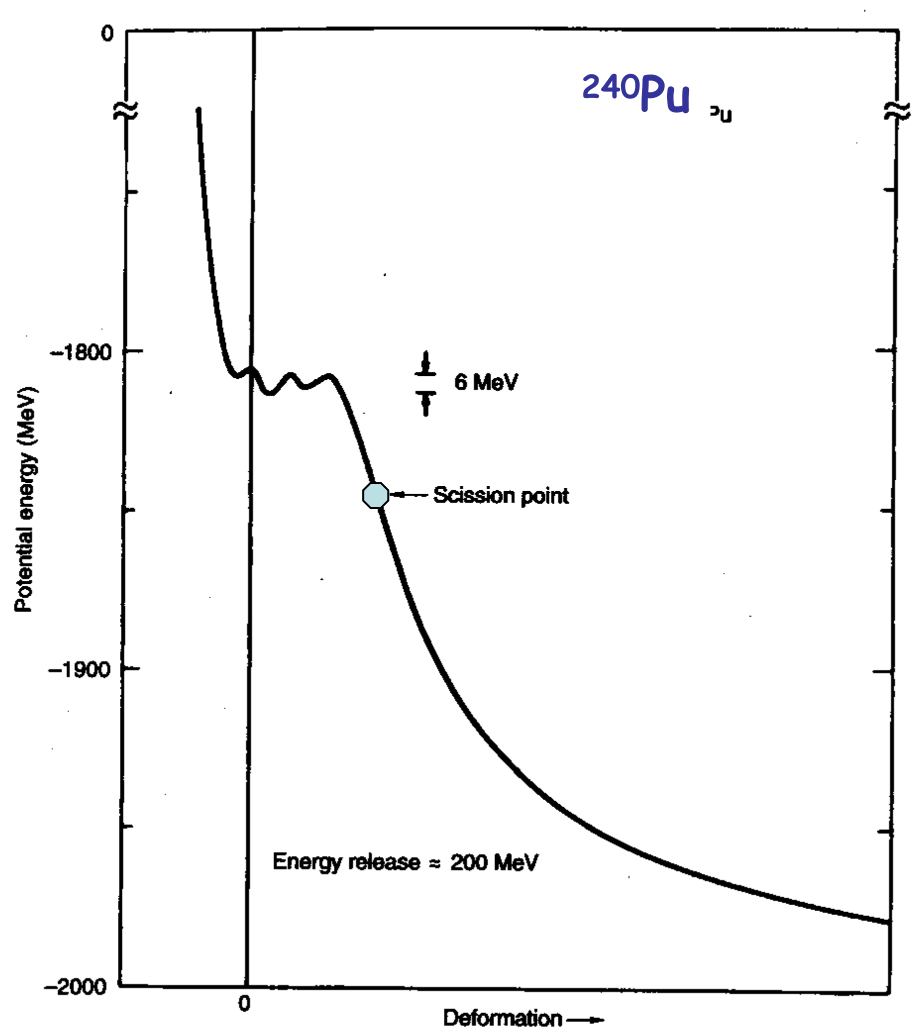


Experiment

Theoretical Description of the Fission Process

Fission


N,Z
elongation
necking
split
N=N1+N2
Z=Z1+Z2
N1,Z1
N2,Z2


240Pu
0
−1800
−1900
−2000
Potential energy (MeV)
6 MeV
Scission point
Energy release ≈ 200 MeV
0
Deformation →

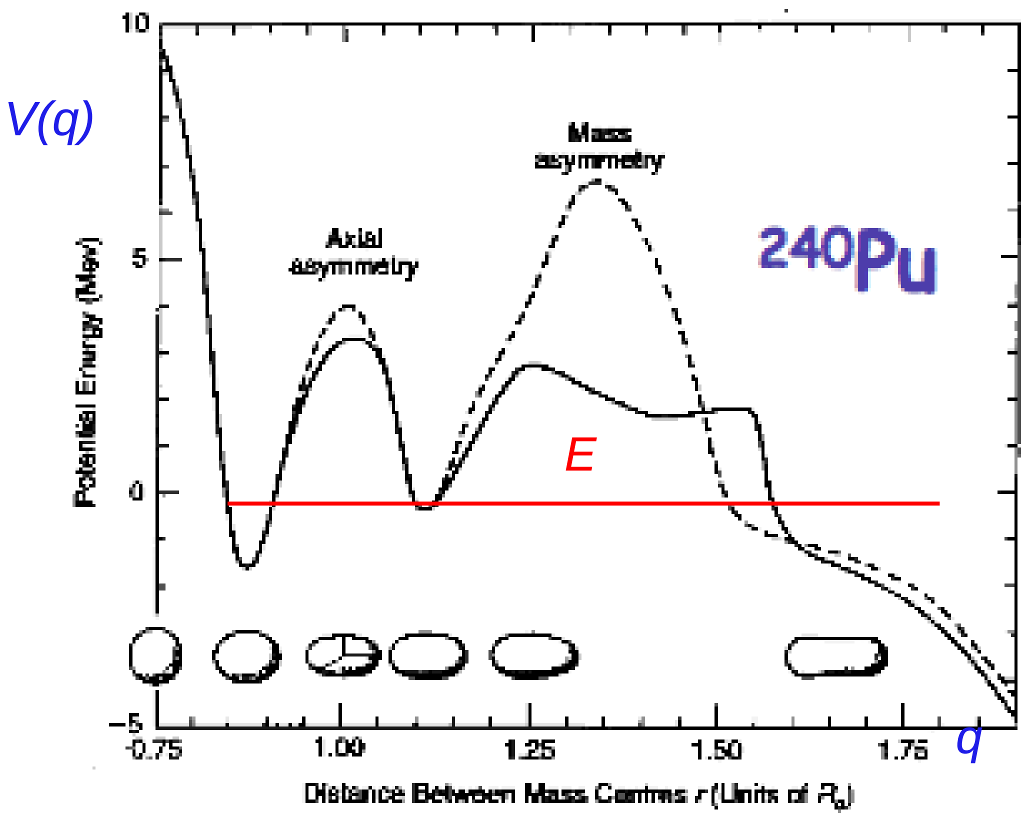
Adiabatic Approaches to Fission

WKB:

$$S = \int_{(s)} \left\{ 2\left[V(q) - E\right] \sum_{ij} B_{ij}(q) q'_i q'_j \right\}^{1/2} ds$$

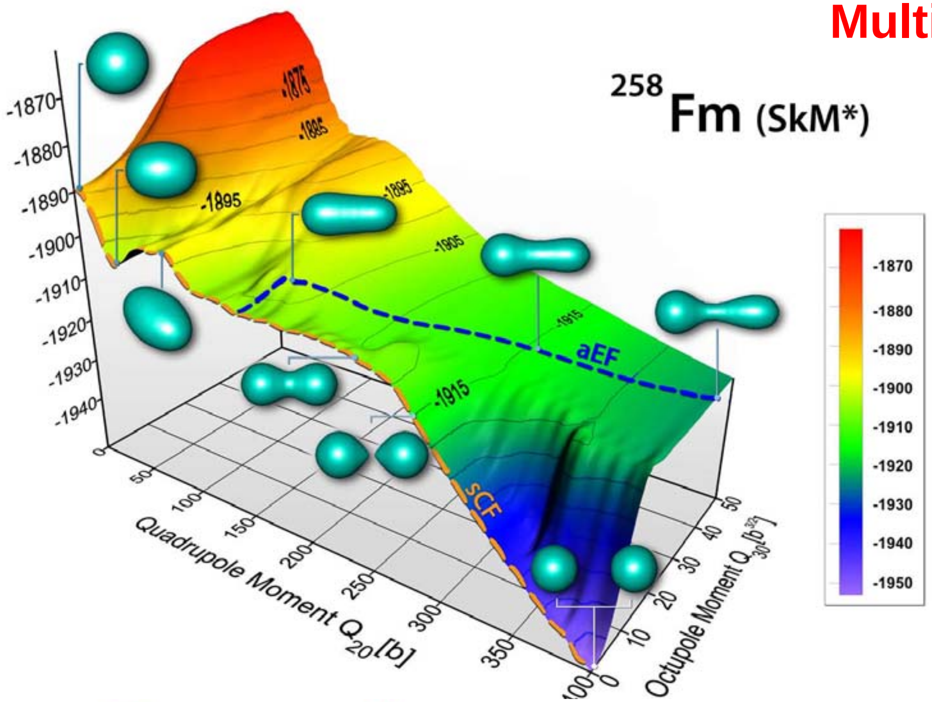
collective inertia (mass parameter) — $B_{ij}(q)$

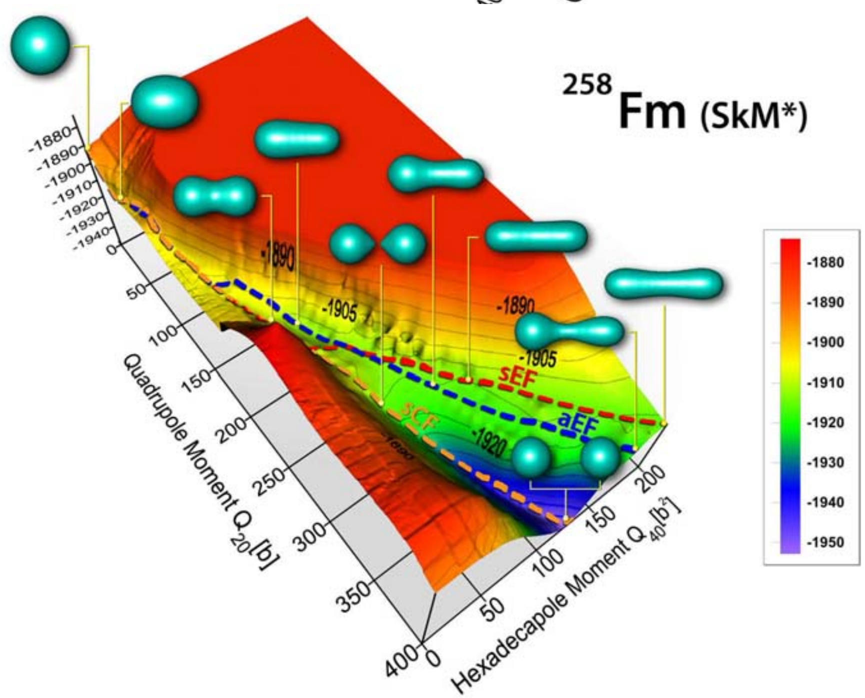
multidimensional space of collective parameters — $q'_i q'_j$



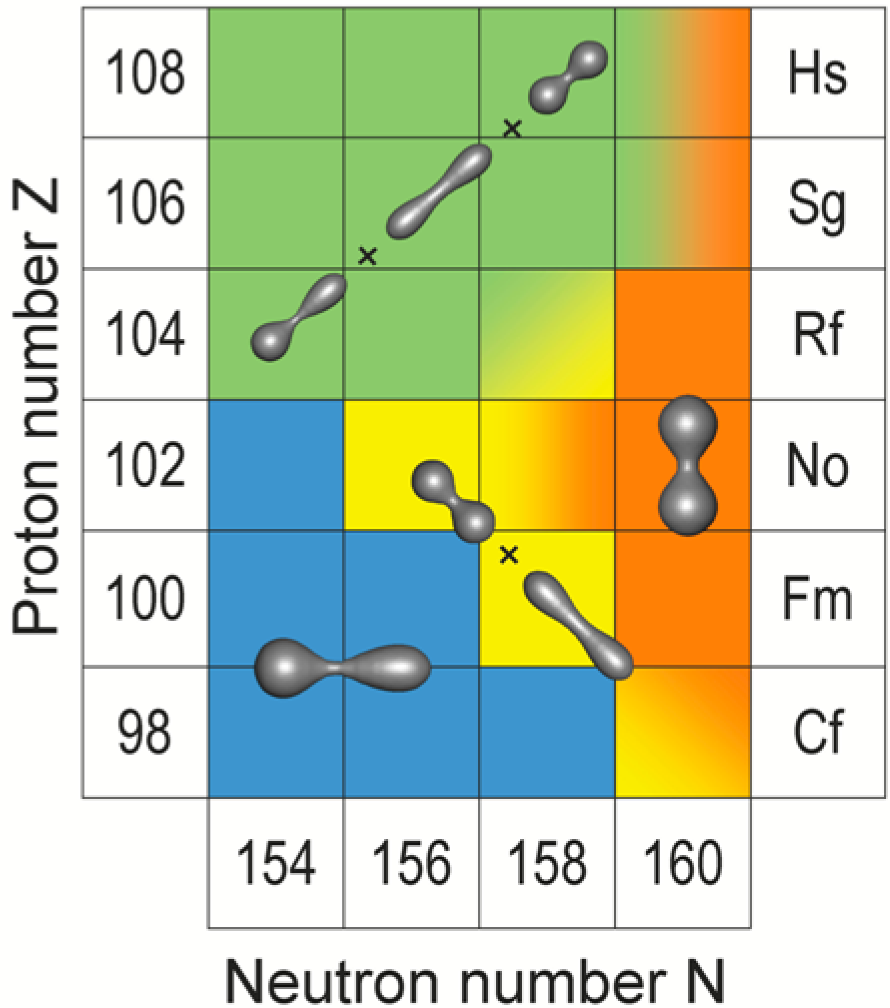
The action has to be minimized by, e.g., the dynamic programming method. It consists of calculating actions along short segments between adjacent, regularly spaced hyperplanes, perpendicular to the q-direction [A. Baran et al., Nucl. Phys. A361, 83 (1981)]

Multimodal fission in nuclear DFT





A. Staszczak, A.Baran,
J. Dobaczewski, W.N.

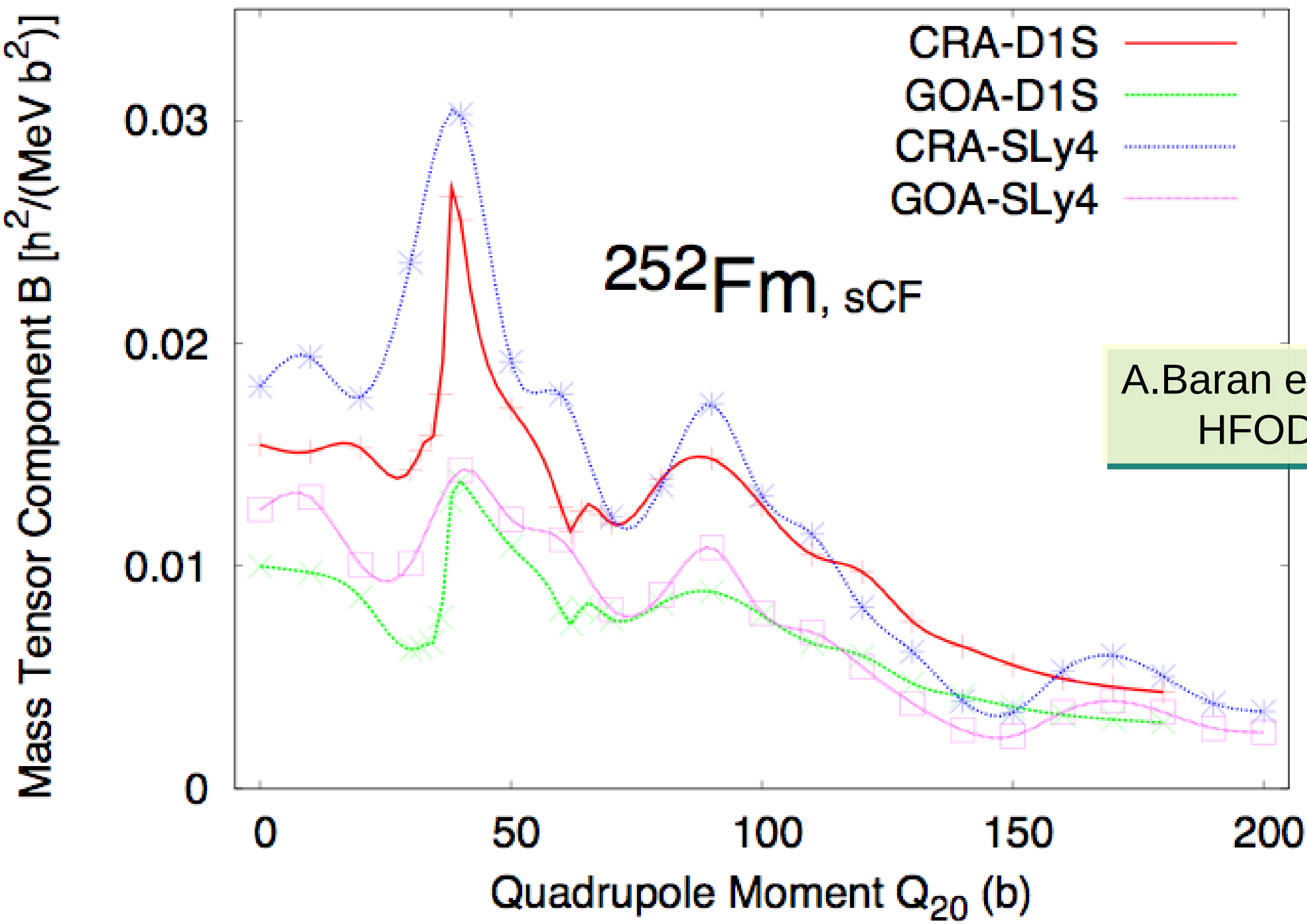


Collective inertia B(q) and ZPE

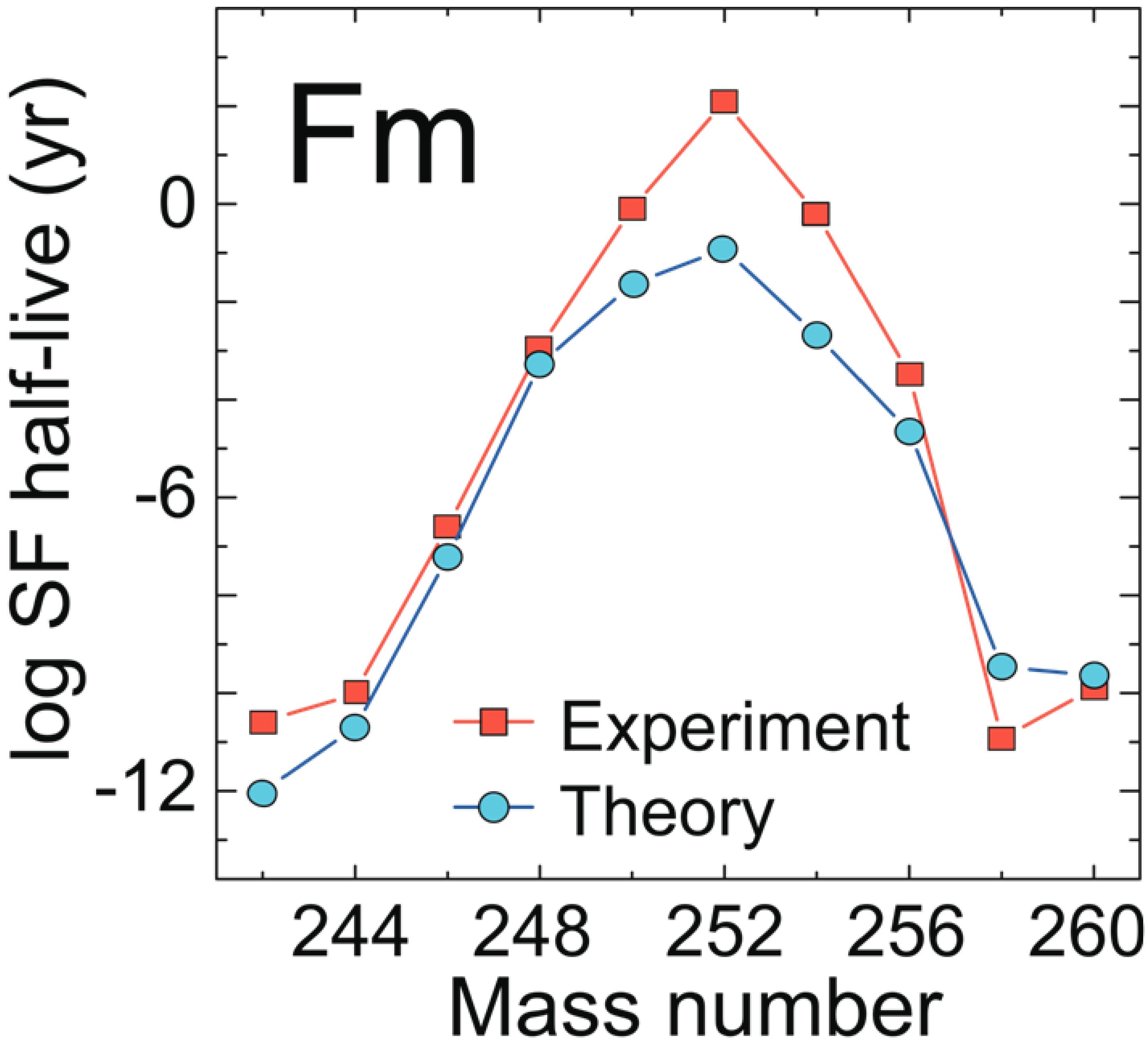
Various prescriptions for collective inertia and ZPE exist:

GOA of the GCM Ring and P. Schuck, The Nuclear Many-Body Problem, 1980

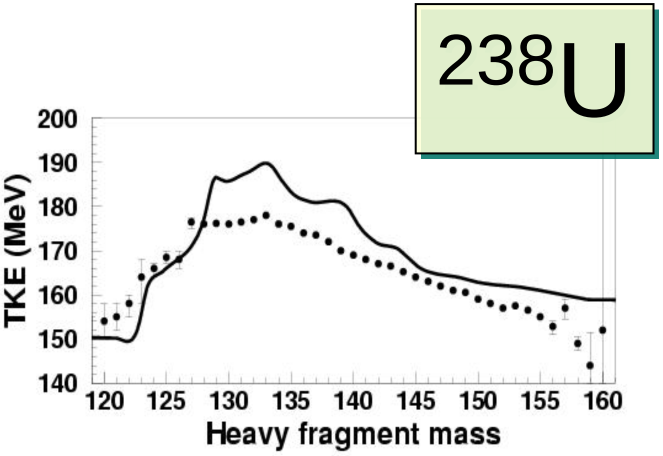
ATDHF+Cranking Giannoni and Quentin, Phys. Rev. C21, 2060 (1980);
Warda et al., Phys. Rev. C66, 014310 (2002)
Goutte et al., Phys. Rev. C71, 024316 (2005)

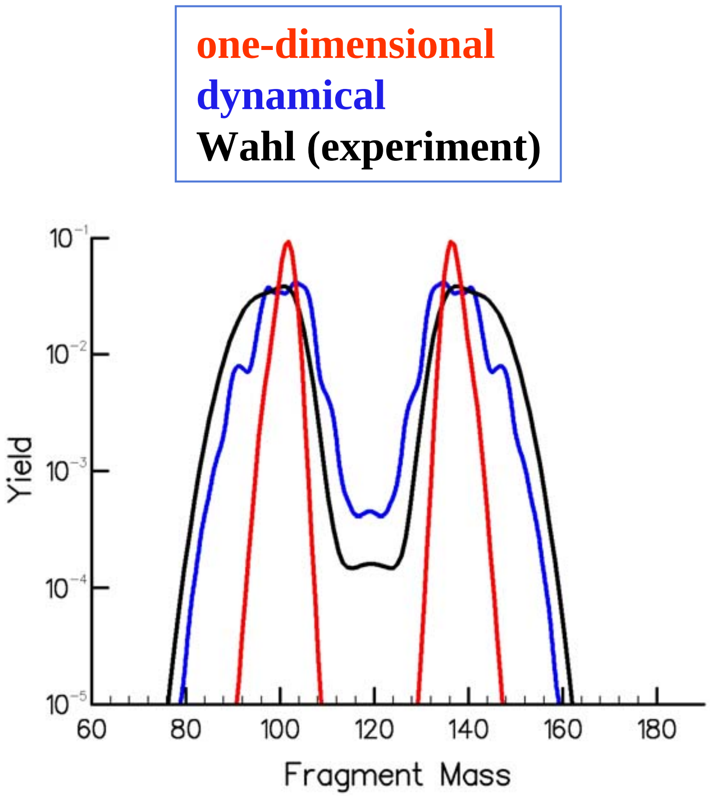


A.Baran et al., nucl-th/0610092
HFODD+BCS+Skyrme


Fm
log SF half-live (yr)
0
-6
-12
244
248
252
256
260
Mass number
Experiment
Theory

Kinetic Energy and Mass Distributions
in HFB+TDGCM(GOA)





- Time-dependent microscopic collective Schroedinger equation
- Two collective degrees of freedom
- TKE and mass distributions reproduced
- Dynamical effects are responsible for the large widths of the mass distributions
- No free parameters

HFB + Gogny D1S + Time-Dependent GOA

H. Goutte, P. Casoli, J.-F. Berger, D. Gogny, Phys. Rev. C71, 024316 (2005)

Actual Outline

- Introduction: nuclei as open systems
- Territory: nuclear landscape and the limits of nuclear existence
- Phenomena related to the openness
  - Impact of continuum on structural properties
- Recent highlights
- General comments on nuclear many-body theory
- Simple concepts and estimates
  - Halos; Driplines and pairing; Resonances; Gamow states
- Theoretical frameworks
  - Complex-energy quantum mechanics (Rigged Hilbert Space)
    - Resonant-state expansions
    - Gamow Shell Model and Complex Scaling
  - Real-energy quantum mechanics (Hilbert Space)
    - Continuum shell model
- Exotic nuclear decays
  - Proton emitters and two-proton emitters
  - Fission
- Perspectives

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Summary

Coupling of nuclear structure and reaction theory

Tying nuclear structure directly to nuclear reactions within a coherent framework applicable throughout the nuclear landscape is an important goal. For light nuclei, ab-initio methods hold the promise of direct calculation of low-energy scattering processes. In nuclear structure for heavier nuclei, the continuum shell model and modern mean field theories allow for the consistent treatment of open channels, thus linking the description of bound and unbound nuclear states and direct reactions. On the reaction side, better treatment of nuclear structure aspects is equally crucial. The battleground in this task is the territory of weakly bound nuclei where the structure and reaction aspects are interwoven.

Thank You